Classical Mechanics - exercise sheet 1

This first set of exercises covers previous material – mainly mathematical techniques - that you'll need during the course. This should therefore all be familiar to you, and the exercises should simply serve as revision. If there are things with which you're not comfortable, though, you should revise or study them properly, and ask for help if necessary: it will subsequently be assumed that you're able to use everything fluently.

No marks will be awarded for simply stating the answer: logical derivation/working is required in all but the most straightforward cases. Remember to distinguish clearly between scalars and vectors throughout.

To submit your solutions, please scan or photograph them using e.g. *Microsoft Lens* to produce a single PDF, and submit it through the Blackboard site by 3pm on **Friday 16th October**.

Reading

Read about vectors, vector products and different coordinate systems in your favourite textbook. e.g.

Fowles & Cassiday	*Analytical Mechanics* (7th ed.)	chapter 1
Chow	*Classical Mechanics* (2nd ed.)	sections 1.1, 1.2, 1.4-1.7
French & Ebison	*Introduction to Classical Mechanics*	chapter 1
Kibble & Berkshire	*Classical Mechanics* (5th ed.)	sections A1-4
Thornton & Marion	*Classical Dynamics* (5th ed.)	chapter 1

1 Scalar and vector products **(5 marks)**

Given the three vectors,

$$\mathbf{a} = 2\mathbf{j} + \mathbf{k}$$
$$\mathbf{b} = \mathbf{i} + \mathbf{j}$$
$$\mathbf{c} = 4\mathbf{k}$$

where **i**, **j** and **k** are the unit vectors parallel to the positive Cartesian (x, y and z) axes, find:

1	$\mathbf{a} \cdot (\mathbf{b} + \mathbf{c})$	**3**	$\mathbf{a} \cdot (\mathbf{b} \times \mathbf{c})$	**5**	$\mathbf{a} \times (\mathbf{b} \times \mathbf{c})$
2	$(\mathbf{a} + \mathbf{b}) \cdot \mathbf{c}$	**4**	$(\mathbf{a} \times \mathbf{b}) \cdot \mathbf{c}$	**6**	$(\mathbf{a} \times \mathbf{b}) \times \mathbf{c}$

2 Vector geometries **(5 marks)**

a) Find the angle between the vectors

$$\mathbf{a} = c\mathbf{j} + 7c\mathbf{k}, \qquad \mathbf{b} = 5c\mathbf{i} + c\mathbf{j} + 7c\mathbf{k}$$

b) Describe the regular six-sided polyhedron of which they are the face-diagonal and body-diagonal.

c) For what value or values of c are the following vectors perpendicular to each other?

$$\mathbf{d} = \mathbf{i} + c\mathbf{j} + 3\mathbf{k}, \qquad \mathbf{e} = 2\mathbf{i} + c\mathbf{j} - c\mathbf{k}$$

3 Energy conservation **(5 marks)**

A cricket ball (mass 0.16 kg) is dropped from rest, level with the top of the Forth Railway Bridge, and falls into the sea 110 m below. Assuming unrealistically that there is no air resistance, calculate

a) the initial potential energy of the cricket ball

b) its kinetic energy after 1/4 of the descent

c) its kinetic energy after 1/2 of the descent

d) its final kinetic energy, just before it enters the water.

4 Velocity & acceleration **(5 marks)**

The position vector of a particle at time t is given by

$$\mathbf{r} = b \cos \omega t \, \mathbf{j} - b \sin \omega t \, \mathbf{k}$$

where b and ω are constants.

a) Calculate the vectorial velocity **v** and vectorial acceleration **a** and show that they are perpendicular.

b) Sketch a diagram showing the path followed by the particle in the y-z plane and draw at a generic position P along this path the three vectors **r**, **v** and **a**.

c) With which kind of motion is this example concerned, and what do the constants b and ω represent?