SEMESTER 1 EXAMINATION 2005/06

WAVE PHYSICS

Duration: 120 MINS

*Answer **all** questions in **Section A** and two **and only two** questions in **Section B**.*

Section A carries 1/3 of the total marks for the exam paper and you should aim to spend about 40 mins on it. Section B carries 2/3 of the total marks for the exam paper and you should aim to spend about 80 mins on it.

A Sheet of Physical Constants will be provided with this examination paper.

An outline marking scheme is shown in brackets to the right of each question.

Only university approved calculators may be used.

Section A

A1. Explain what is meant by *transverse* and *longitudinal* wave motions, and give an example of each. [3]

Give two examples of waves which are neither transverse nor longitudinal. [1]

A2. The equation of a sinusoidal transverse wave travelling along a string is given by $y = A \sin \pi(4x - 40t)$, where $A = 0.005$m, x and y are in metres and t is in seconds.

(a) Find the amplitude, wavelength, wavenumber, frequency, period and velocity of the wave. [3]

(b) Find the maximum *transverse* speed of any particle in the string. [1]

A3. Describe the Huygens model of wave propagation. [2]

Either

(a) Given that the speed v of shallow water waves is given by $v^2 = gh$, where g is the acceleration due to gravity and h is the water depth, explain why ocean waves are almost always nearly parallel to the shoreline when they break, [2]

or

(b) Explain why light rays, passing from air into a plate of glass, are refracted towards the normal to the interface. [2]

Illustrate your answer with a diagram.

A4. Explain briefly the phenomenon of wave *interference*. [1]

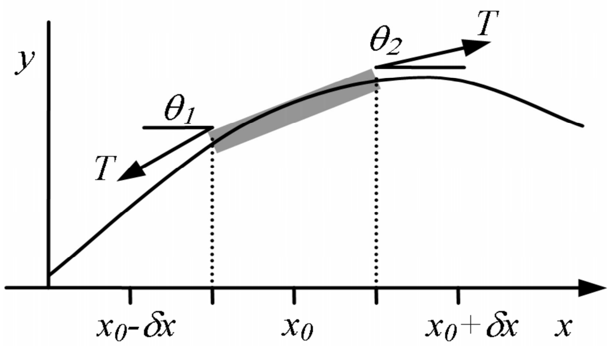
Two identical loudspeakers are placed 4 m apart, and emit a single frequency of 880 Hz with equal intensities. An observer stands facing the speakers so that she is exactly 100 m away from each of the two speakers. If she moves to either her left or her right, the sound diminishes, reaching a minimum at a distance of 4.8 m from her initial position. Sketch this arrangement, indicating the distances involved, and determine the wavelength of the sound waves. [3]

A5. Explain, with examples, the *boundary conditions* that may apply to a wave motion. [2]

By considering the different boundary conditions governing the instruments, explain why guitar and violin strings can support all harmonics of the fundamental frequency, but instruments like the clarinet produce only odd-numbered harmonics. [2]

Section B

B1. The figure below shows a section of a thin, flexible string of mass per unit length ρ and subject to a tension T.



By considering the net force acting on an element of the string (which, shown in grey, may be considered approximately rigid), derive the wave equation governing its transverse motion,

$$\frac{\partial^2 y}{\partial t^2} = \frac{T}{\rho}\frac{\partial^2 y}{\partial x^2}$$ [5]

Show that the wave equation has sinusoidal travelling wave solutions

$$f(x,t) = f_0 \cos(kx - \omega t + \phi)$$

and explain the significance of the parameters k, ω and ϕ. [5]

Determine the *dispersion relation* between k and ω. How does the wave speed depend upon ρ and T? [3]

State the boundary condition which must be met at a point where the string is fixed. Hence find the *standing wave* solutions to the wave equation, and determine the allowed oscillation frequencies when such a string of length L is fixed at its ends. [5]

If the top string of an acoustic guitar has a mass of 0.37 g.m^{-1}, and its length is determined by the distance 0.648 m from the *bridge* to the *nut*, find the tension required to tune the string to the note E_4 (a frequency of 329.6 Hz). [2]

B2. Explain what is meant by *Fraunhofer diffraction*. [3]

Show from first principles, with the aid of a diagram, that the dependence upon angle of the relative amplitude of a wave of wavelength λ, diffracted by a single slit of width a is given by the sinc function

$$a_1(\theta) \propto \frac{\sin\left\{\frac{\pi a}{\lambda}\sin\theta\right\}}{\frac{\pi a}{\lambda}\sin\theta}$$ [5]

State the *convolution theorem* and explain how it may be used to determine the diffraction patterns of regular arrays of a basic pattern. [3]

The diffraction pattern of an infinite regular array of narrow slits, whose centres are separated by a distance d, is given by

$$a_2(\theta) = a_0 \sum_{n=0}^{\infty} \delta\left(\theta - n\frac{\lambda}{d}\right)$$

where θ is the angle through which the incident beam is diffracted.

(a) Write the transmission of a real diffraction grating - of width b, composed of narrow slits of width c spaced by a distance d - as a combination of products and convolutions of simple functions; [2]

(b) Hence determine the diffraction pattern of such a grating, and [2]

(c) Sketch the pattern. [2]

Given that for small angles ($\theta \sim \sin\theta$), adjacent diffraction orders are separated in angle by λ/d, and the width of each order is around λ/b, estimate the resolution of a grating spectrograph for use in first order at $\lambda = 500$ nm when the grating parameters are $b = 50$ mm, $d = (1/600)$ mm, $c = 0.3\ \mu$m. [3]

B3. What is meant by the *Fourier transform*? How may it be defined? [4]

An experiment to investigate human hearing involves playing short bursts of duration T of a single tone with frequency $f_0 = \omega_0/2\pi$ (where $\omega_0 T \gg 1$),

$$a(t) = \begin{cases} \cos\omega_0 t & (-T/2 \le t \le T/2) \\ 0 & (t < -T/2,\ t > T/2) \end{cases}$$

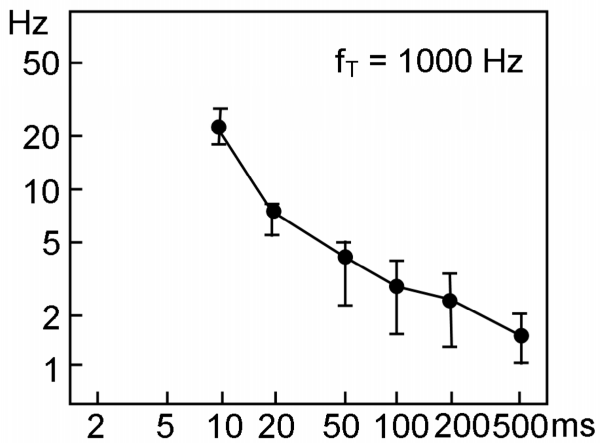
Show that the Fourier transform of a single burst is given by

$$b(\omega) \propto \frac{\sin\{(\omega_0 - \omega)T/2\}}{\omega_0 - \omega} + \frac{\sin\{(\omega_0 + \omega)T/2\}}{\omega_0 + \omega}$$

and sketch the amplitude and intensity spectra corresponding to your result for frequencies around ω_0, where the second term may be neglected. [8]

Given that $\{\sin(a)/a\}^2 = 1/2$ when $a = 1.392$, derive the full width at half maximum intensity of the spectrum of each burst. [4]

Experimentally, it is found that the smallest difference in tone that the ear can distinguish depends upon the duration. The figure below (from *Psychoacoustics*, by Zwicker and Fastl) shows the minimum distinguishable frequency difference as a function of pulse duration T for frequencies around 1 kHz.



Compare, qualitatively and quantitatively, the measured frequency sensitivity (shown above) with the spectral widths of the various pulses used, and comment on your results. [4]

B4. Explain what is meant by *dispersion*. Give examples of practical manifestations of dispersion, and of an application that exploits it. [5]

Show that the wave equation

$$i\,m\frac{\partial y}{\partial t} = -\frac{\partial^2 y}{\partial x^2}$$

where m is a constant, has complex exponential travelling wave solutions of the form

$$f(x,t) = f_0 \exp i(kx - \omega t)$$

Explain the significance of the parameters k and ω, and determine the *dispersion relation* between k and ω. [5]

What is meant by the *phase* and *group velocities*? Give, for the above example, an expression for the phase velocity v_p in terms of ω. [3]

A travelling wave has two components, equal in magnitude, with frequencies $\omega_0 \pm \delta\omega$ and wavenumbers $k_0 \pm \delta k$. Show that the wave may be written in the form

$$f(x,t) = f_1 \exp i(k_0 x - \omega_0 t)\ \cos(\delta k\, x - \delta\omega\, t)$$

and thus takes the form of a complex exponential travelling wave that is modulated by a slowly-varying, real periodic function. [4]

By considering how δk depends upon $\delta\omega$, show that the phase velocity of the wave differs from the group velocity of the modulating envelope by a factor of two. [3]

END OF PAPER