UNIVERSITY OF SOUTHAMPTON PHYS2023W1

SEMESTER 1 EXAMINATION 2009/10

WAVE PHYSICS

Duration: 120 MINS

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*Answer **all** questions in **Section A** and two **and only two** questions in **Section B**.*

Section A carries 1/3 of the total marks for the exam paper and you should aim to spend about 40 mins on it. Section B carries 2/3 of the total marks for the exam paper and you should aim to spend about 80 mins on it.

A sheet of physical constants will be provided with this examination paper.

An outline marking scheme is shown in brackets to the right of each question.

Only University-approved calculators may be used.

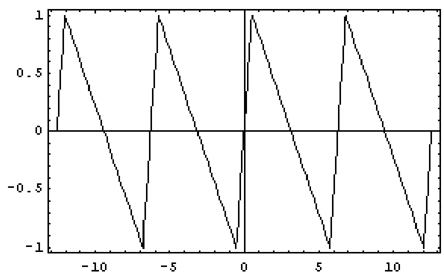
Section A

A1. A microwave oven operates by injecting electromagnetic waves with a frequency of 2.45 GHz into the cavity formed by the conducting metal shields on each face of the oven box. Explain, with the aid of sketches and simple calculations, why it is generally necessary to rotate the food using a turntable during operation. [4]

A2. Show that the operator $\hat{\omega} \equiv i\frac{\partial}{\partial t}$ when applied to a single frequency complex wave $y(x,t) = a \exp i(kx - \omega t)$ correctly yields the angular frequency ω of the wave. [2]

Explain why the same operator does not give the same result for real sinusoidal waves $y(x,t) = a\cos(kx - \omega t + \phi)$. [2]

A3. The figure below represents the oscilloscope trace of a signal within a laser light show. The horizontal scale is microseconds, and the vertical scale is volts.



Deduce which of the following could represent the first four harmonic components of the signal, and explain your choice.

a. $\frac{36}{5\pi^2}\left(\sin\omega_0 t + \frac{\sqrt{3}}{4}\sin 2\omega_0 t + \frac{2}{9}\sin 3\omega_0 t + \frac{\sqrt{3}}{16}\sin 4\omega_0 t\right)$

b. $\frac{36}{5\pi^2}\left(\cos\omega_0 t + \frac{\sqrt{3}}{2}\cos 2\omega_0 t + \frac{2}{3}\cos 3\omega_0 t + \frac{\sqrt{3}}{4}\cos 4\omega_0 t\right)$

c. $\frac{36}{5\pi^2}\left(\sin\omega_0 t - \frac{\sqrt{3}}{4}\sin 2\omega_0 t + \frac{2}{9}\sin 3\omega_0 t - \frac{\sqrt{3}}{16}\sin 4\omega_0 t\right)$

d. $\frac{36}{5\pi^2}\left(\cos\omega_0 t - \frac{\sqrt{3}}{2}\cos 2\omega_0 t + \frac{2}{3}\cos 3\omega_0 t - \frac{\sqrt{3}}{4}\cos 4\omega_0 t\right)$ [4]

A4. The equation for sound waves through air of density ρ and elastic constant E may be taken to be

$$\frac{\partial^2 \xi}{\partial t^2} = \frac{E}{\rho}\frac{\partial^2 \xi}{\partial x^2}$$

where the elastic constant may in turn be written in terms of the air pressure P and the specific heat ratio γ as $E = \gamma P$. Given that $\gamma = 1.4$ and the density and pressure at sea level are respectively $1.29\,\mathrm{kg\,m^{-3}}$ and $1.013 \times 10^5\,\mathrm{Pa}$, calculate the speed of sound at sea level and the time taken for the sound of a thunder clap to travel a kilometre at that altitude. [4]

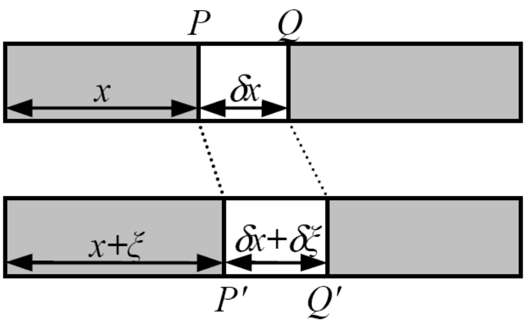
A5. Given that, in question A4, the density may be written in terms of the molar mass m and the molar volume V as $\rho = m/V$, and that the molar volume is determined at a given pressure and temperature by the ideal gas equation $pV = RT$, where R is the molar gas constant, show that the speed of sound depends only upon the temperature of the atmosphere. [2]

On a thundery summer afternoon the temperature of the atmosphere is found to decrease with altitude except for a region close to the surface which is occupied by the cold downdraught from the clouds. Soundings indicate temperatures of 15 C at ground level, 25 C at 500 m altitude and 0 C at 3500 m. Indicate with a sketch how refraction will affect the propagation of the thunder clap through the atmosphere. [2]

Section B

B1. What are meant by *transverse* and *longitudinal* wave motions? Give an example of each, and an example of a wave that is neither. [5]

The figure below shows the *longitudinal* motion of a volume element of an elastic medium which is displaced (bottom) by $\xi(x,t)$ from its rest position (top).



(a) By considering the extension of the longitudinal element of density ρ, elasticity E and cross-sectional area A, show that the tension differs from the tension of the medium at rest by $T(x,t)$ where

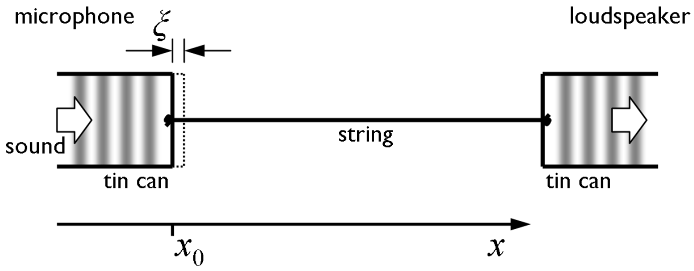
$$T(x,t) = EA\frac{\partial \xi}{\partial x}$$ [2]

(b) Hence, by considering the net force acting upon the element, show that the wave equation governing its longitudinal motion will be

$$\frac{\partial^2 \xi}{\partial t^2} = \frac{E}{\rho}\frac{\partial^2 \xi}{\partial x^2}\,.$$ [2]

Explain, with examples, what are meant by the *continuity conditions* for wave motion at the interface between two media. How does a discontinuity in the characteristics of a medium affect the wave propagating through it? [3]

A 'tin can telephone' is made by stretching a length of string between the bases of two tin cans, as shown below, so that the base of each can acts as a microphone diaphragm whose motion generates longitudinal waves along the taut string (and vice-versa for the loudspeaker).



(i) Assuming that the whole of the can base moves with the same displacement, and that its mass can be neglected, show that the continuity conditions at the air-can-string interface are

$$\begin{aligned} \xi_1(x_0, t) &= \xi_2(x_0, t) \\ E_1 A_1 \frac{\partial \xi_1}{\partial x}(x_0, t) &= E_2 A_2 \frac{\partial \xi_2}{\partial x}(x_0, t), \end{aligned}$$

where A_1 and A_2 are the cross-sectional areas of the tin can and the string respectively and $\xi_{1,2}$, $E_{1,2}$ are the wave displacement and medium elasticity in region 1 ($x < x_0$) and region 2 ($x > x_0$). [3]

(ii) Hence show that for all of the incident sound energy to be transferred to the string, the cross-sectional areas must satisfy

$$A_1 Z_1 = A_2 Z_2$$

where the impedances per unit cross-sectional area, $Z_{1,2}$, are in each region given by $Z = \sqrt{E\rho}$. [3]

(iii) Thus suggest the best diameter of tin can to use if you have a piece of string with a diameter of 1.5 mm, density $125\,\mathrm{kg\,m^{-3}}$ and elastic modulus $7 \times 10^8\,\mathrm{N\,m^{-2}}$, assuming air to have a density of $1.29\,\mathrm{kg\,m^{-3}}$ and elastic modulus $1.4 \times 10^5\,\mathrm{N\,m^{-2}}$. [2]

B2. (a) Describe what are meant by *travelling* and *standing* waves. [2]

(b) Show that travelling waves of the form $y(x,t) = f(u)$, where $u = x - c\,t$, are general solutions for wave systems described by wave equations of the type

$$\frac{\partial^2 y}{\partial t^2} = A\frac{\partial^2 y}{\partial x^2}$$

where y is the wave displacement at position x and time t. Derive the relationship between the constants c and A. [4]

(c) Explain what is meant by a *boundary condition*. Give two examples of boundary conditions and describe how they affect wave propagation in everyday or scientific situations. [4]

(d) The Doppler shift of a moving source may be determined by considering the source to provide a moving boundary condition upon the system in which the radiated wave propagates. Given that, for a source of sinusoidal waves of angular frequency ω, moving in the positive x direction with speed v, the boundary condition may be written as

$$y(vt,t) = a\cos\omega t\,,$$

show that the specific travelling wave solution will be

$$f(u) = a\cos\omega\left(\frac{-u}{c-v}\right)$$

and thus that

$$y(x,t) = a\cos\left(\frac{c}{c-v}\omega t - \frac{\omega}{c-v}x\right)\,,$$

and find the frequency shift between the source and the resulting wave. [4]

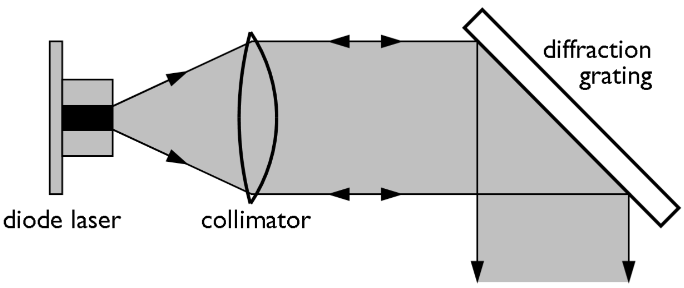
(e) Hence find the signal which will be measured by a detector a constant distance L downstream of the source, i.e. at a position $x = v\,t + L$. [2]

(f) Explain how such an arrangement could be used to measure the speed of water flowing from an acoustic sounder to a nearby microphone, and suggest suitable values of ω and L if the device is to be used to measure boat speeds up to $10\,\mathrm{m\,s^{-1}}$, given that the speed of sound in water is around $1500\,\mathrm{m\,s^{-1}}$. [4]

B3. Explain the *Huygens* description of wave propagation, and how it accounts for the phenomenon of *diffraction*. [4]

A diode laser may be considered equivalent to an infinitely broad gain medium that lies behind a mask containing an aperture with the same cross-section as the gain region of the real device. For a diode laser designed to operate at wavelengths around 780 nm, the gain region has a rectangular cross-section measuring 5 μm by 10 μm. Calculate the far-field divergence angle of the laser beam in the two directions parallel to the axes of the rectangle, stating clearly any assumptions or approximations made. [4]

To be used in a practical device, the diverging laser beam is collimated using a small convex lens whose focus is positioned around the output face of the diode laser. If the lens has a focal length of 10 mm, calculate (a) the minimum diameter of the lens, if it is to pass all of the central diffraction order, and (b) the difference in thickness of the lens between its centre and the edge, given that the refractive index of the lens is 1.53. [4]



To stabilize the diode laser to a single wavelength, a reflective diffraction grating is used to feed light of the desired wavelength back into the gain region, as illustrated above. For light with the correct wavelength, the first diffracted order is reflected back along its incident path, while the zeroth order (ordinary reflection) provides the output from the combined device.

Show, with the aid of a diagram, that, in this *Littrow* configuration, the diffraction grating should be set to an incidence angle θ given by

$$\sin\theta = \frac{\lambda_0}{2d}$$

where λ_0 is the laser wavelength and d is the spacing of the grating rulings. [3]

Show also that any light with a slightly different wavelength $\lambda = \lambda_0 + \delta\lambda$ will be returned from the grating at an angle $\delta\theta$ to the incident light, where

$$\delta\theta \approx \frac{\delta\lambda}{d\cos\theta}\,.$$ [3]

Hence, taking the lens focal length to be 10 mm and the gain region width to be 5 μm (as above), and assuming the grating incidence angle to be 45°, find the detuning $\delta\lambda$ for which none of the reflected light will strike the gain region. [2]

B4. Explain what is meant by *dispersion* in wave propagation. Give two examples of its practical consequences. [4]

The wave functions of quantum particles are described by the Schrödinger equation

$$\alpha \frac{\partial y}{\partial t} = \beta \frac{\partial^2 y}{\partial x^2} + \gamma\, y$$

where y is the quantum wave function at position x and time t and the constants α, β and γ are equal to $i\hbar$, $-\hbar^2/2m$ and the potential energy V respectively, where $\hbar$ is Planck's constant$/2\pi$ and m the mass of the particle.

(a) Demonstrate that travelling waves of the form $y(x,t) = f(u)$, where $u = x - c\,t$, are not general solutions to the Schrödinger wave equation. [4]

(b) Show that, in contrast, single-frequency complex exponential waves of the form

$$y(x,t) = y_0 \exp i(kx - \omega t)$$

are possible solutions. Derive the *dispersion relation* between ω and k in terms of $\hbar$, m and V, and deduce the phase velocity ω/k if $V = 0$. [4]

An interesting class of solutions to the Schrödinger equation are Gaussian wavepackets of the form

$$y(x,t) = \sqrt{\frac{\pi}{d + at}} \exp i(kx - \omega t) \exp -\frac{(x - v\,t)^2}{4(d + at)}$$

where $a = \beta/\alpha$ and the constant d defines the width of the wavepacket at $t = 0$.

(c) Describe or sketch the shape of the wavepacket at time $t = 0$, and explain how it evolves with time. [4]

(d) Show that the wavepacket is indeed a solution to the Schrödinger equation, and find how the group velocity v depends upon the wavenumber k. [4]

END OF PAPER