SEMESTER 1 EXAMINATION 2016-2017

WAVE PHYSICS

Duration: 120 MINS (2 hours)

This paper contains 9 questions.

Answers to Section A and Section B must be in separate answer books.

Answer **all** questions in **Section A** and **only two** questions in **Section B**.

Section A carries 1/3 of the total marks for the exam paper and you should aim to spend about 40 mins on it.

Section B carries 2/3 of the total marks for the exam paper and you should aim to spend about 80 mins on it.

An outline marking scheme is shown in brackets to the right of each question.

A Sheet of Physical Constants is provided with this examination paper.

Only university approved calculators may be used.

A foreign language dictionary is permitted ONLY IF it is a paper version of a direct 'Word to Word' translation dictionary AND it contains no notes, additions or annotations.

8 page examination paper.

Section A

A1. The equation of a sinusoidal transverse wave travelling along a string is

$$\psi(x,t) = A\sin(\phi(x,t)) = A\sin(5\pi x - 40\pi t),$$

where $A = (10/\pi)$ m, x and ψ are measured in metres and t is in seconds.

(a) Find the wavelength and frequency of the wave. [1]

(b) Find the maximum *transverse* speed of any particle in the string. [1]

(c) By writing the phase $\phi(x,t)$ in terms of the angular frequency ω and wavenumber k, or otherwise, show that the *phase velocity* v_p of a point of constant phase ϕ is given by $v_\mathrm{p} = \omega/k$. [2]

A2. What is meant by *dispersion*? [1]

Give an expression for the *group velocity* of a wave whose angular frequency is given as a function of the wavenumber k by $\omega(k)$. [1]

Determine the relationship between the phase velocity $v_\mathrm{p} = \omega/k$ and the group velocity for shallow water capillary waves whose dispersion relation is

$$\omega^2 = \frac{h_0\sigma}{\rho}k^4$$

where σ is the surface tension, h_0 the depth of the water, and ρ its density. [2]

A3. Explain the meanings of *transverse* and *longitudinal* wave motions, and give an example of each. [3]

Give an example of a wave that is neither transverse nor longitudinal. [1]

A4. Explain, with examples, the *boundary conditions* that may apply to a wave motion. [2]

By considering the different boundary conditions governing the instruments, explain why violin strings can support all harmonics of the fundamental frequency, but instruments like the clarinet produce only odd-numbered harmonics. [2]

A5. Outline how the *mean* frequency, and the *standard deviation* of the frequency, may be determined for a complex wave motion,

(a) from its expression as a function of time, $\psi(t)$, and [2]

(b) from its frequency spectrum, $g(\omega)$. [2]

TURN OVER

Section B

B1. (a) Explain briefly the phenomenon of wave *interference*, and what is meant by *constructive* and *destructive* interference. [4]

(b) Outline the *Huygens description* of wave propagation, and explain how it can be used to calculate the diffraction pattern of an illuminated object. [4]

The light of a collimated laser beam of wavelength λ falls at normal incidence upon a flat screen. Between the screen and the laser, in the middle of the beam at a distance d from the screen, the beam is scattered from a small particle.

(c) Show that, at the screen, the interference between the scattered light and the unperturbed laser beam will result in a series of bright rings whose radii are given, for integer values of n, by

$$r_n^2 = n\left(n + \frac{2d}{\lambda}\right)\lambda^2 \,.$$ [4]

Photochemical etching techniques are used to make the screen transparent at the positions of the interference maxima, and the scattering particle is removed.

(d) Show that, when illuminated with the original collimated laser beam, the light transmitted by the etched screen comprises three components:

(i) the plane wavefronts of the original illumination [1]

(ii) curved wavefronts that converge upon a point a distance d after the screen; and [2]

(iii) curved wavefronts that appear to diverge from a point a distance d before the screen. [2]

(e) Explain how the positions of the points to and from which the curved wavefronts convergence and divergence will vary if the wavelength of the illuminating laser is changed, assuming that $n \ll 2d/\lambda$. [2]

(f) Suggest an optical application for this etched screen. [1]

B2. (a) What is meant by the *Fourier transform*? How may it be defined mathematically? [4]

A yacht's pulsed radar emits microwave bursts of duration T of a single frequency $f_0 = \omega_0/2\pi$ (where $\omega_0 T \gg 1$), so that the wave amplitude is

$$a(t) = \begin{cases} \sin\omega_0 t & (-T/2 \le t \le T/2)\,; \\ 0 & (t < -T/2,\ t > T/2)\,. \end{cases}$$

(b) Show that the Fourier transform of a single burst is given by

$$b(\omega) \propto \frac{\sin\{(\omega_0 - \omega)T/2\}}{\omega_0 - \omega} - \frac{\sin\{(\omega_0 + \omega)T/2\}}{\omega_0 + \omega}$$

and show that, for frequencies around ω_0, the second term may be neglected. [4]

(c) Sketch the amplitude and intensity spectra corresponding to your result for frequencies around $\omega = \omega_0$, labelling any significant frequencies. [4]

(d) Given that $[\sin(a)/a]^2 = 1/2$ when $a = 1.392$, derive the full width at half-maximum intensity of the spectrum of each burst. [4]

(e) The radar, which operates by reflecting microwave bursts from other vessels or obstacles, is required to measure the positions of the reflecting objects with a precision of 30 m. Taking the speed of electromagnetic waves to be 3×10^8 m s^{-1}, estimate the maximum allowable duration of each radar burst, and hence estimate the minimum bandwidth of the transducers used to generate it. [4]

TURN OVER

B3. (a) Explain what is meant by a *continuity condition* in the context of wave propagation, and state the conservation laws with which they are generally associated. [4]

The wave equation for the longitudinal motion within a material of density ρ and elasticity E is

$$\frac{\partial^2 \xi}{\partial t^2} = \frac{E}{\rho} \frac{\partial^2 \xi}{\partial x^2},$$

where ξ is the longitudinal displacement of the material from its rest position.

(b) Express the *dispersion relation* between the frequency ω and wavenumber k of a sinusoidal sound wave in the medium in terms of ρ and E. [2]

The continuity conditions for the longitudinal displacement ξ of sound waves as they pass, at $x = x_0$, from a region A ($x < x_0$) of density ρ_A and elasticity E_A into another region B ($x > x_0$) of density ρ_B and elasticity E_B are

$$\begin{aligned} \xi_A(x_0, t) &= \xi_B(x_0, t), \\ E_A \frac{\partial \xi_A}{\partial x}(x_0, t) &= E_B \frac{\partial \xi_B}{\partial x}(x_0, t). \end{aligned}$$

(c) State the physical reason for each of these conditions. [2]

The displacements of travelling sinusoidal sound waves in the two regions A and B may be written in the form

$$\xi_{A,B}(x, t) = a_{A,B} \cos\left(\omega t - k_{A,B}(x - x_0)\right) .$$

(d) For the situation in which a wave of angular frequency ω approaches x_0 through medium A, state the possible values for the wavenumber $k_{A,B}$ in the two regions. [2]

(e) Hence show that when a sound wave of amplitude a_i passes from region A to region B it results in a reflected wave of amplitude a_r where

$$\left|\frac{a_r}{a_i}\right| = \left|\frac{Z_A - Z_B}{Z_A + Z_B}\right|$$

and the acoustic impedance Z is given in terms of the density ρ and elasticity E by $Z = \sqrt{E\rho}$. [5]

(f) Ultrasound imaging depends upon the reflection of high frequency sound at the interfaces between different media. Given the data in the table below, and assuming breast and tumourous tissue to be approximately homogeneous in their acoustic properties, find the fraction of the wave *intensity* that is reflected when ultrasound is normally incident upon the interface between breast tissue and a tumour within it. [2]

material	density (kg m^{-3})	elasticity (kPa)
breast tissue	1020	25
tumour	1040	93

(g) What other considerations would determine the overall strength of the ultrasound signal obtained when imaging a tumour within breast tissue? [3]

TURN OVER

B4. The Boeing CH-47 *Chinook* helicopter has two synchronized, counter-rotating rotors, with axes separated by 11.9 m, which rotate at 225 revolutions per minute. Each of the overlapping rotors comprises three rotor blades.

Describe, qualitatively and when possible quantitatively, the production and propagation of sound waves from the helicopter's rotors as the aircraft flies over a stationary observer, and the sound that will be heard by the observer below. You may wish to consider

(a) the origin and nature of the sound waves, [4]

(b) the pattern of waves emitted, [6]

(c) the effects of the aircraft's motion, and [5]

(d) the consequences of having a pair of rotors. [5]

You may assume that the aircraft flies at a steady speed of 80 m s^{-1} and constant height of 160 m, and take the speed of sound in air to be 330 m s^{-1}.

END OF PAPER