SEMESTER 2 EXAMINATION 2013–2014

WAVES, LIGHT AND QUANTA

Duration: 120 MINS (2 hours)

This paper contains 9 questions.

Answers to Section A and Section B must be in separate answer books

Answer **all** questions in **Section A** and **only two** questions in **Section B**.

Section A carries 1/3 of the total marks for the exam paper and you should aim to spend about 40 mins on it.

Section B carries 2/3 of the total marks for the exam paper and you should aim to spend about 80 mins on it.

An outline marking scheme is shown in brackets to the right of each question.

A Sheet of Physical Constants is provided with this examination paper.

Only university approved calculators may be used.

A foreign language translation dictionary (paper version) is permitted provided it contains no notes, additions or annotations.

Geometric optics sign conventions

s	object distance	positive for real object, negative for virtual object
s'	image distance	positive for real image, negative for virtual image
R	radius of curvature	positive when centre of curvature is on the same side as the outgoing ray, negative otherwise (this means positive for concave mirror or convex refracting surface)

Ray-transfer matrices

These act on two-component vectors whose upper component is the distance of a ray from the optical axis and whose lower component is the angle the ray makes with the optical axis.

translation by distance t	$M = \begin{pmatrix} 1 & t \\ 0 & 1 \end{pmatrix}$	
refraction, spherical interface, radius R, from region n_1 to n_2	$M = \begin{pmatrix} 1 & 0 \\ \frac{1}{R}\left(\frac{n_1}{n_2} - 1\right) & \frac{n_1}{n_2} \end{pmatrix}$	
refraction, plane interface, from region n_1 to n_2	$M = \begin{pmatrix} 1 & 0 \\ 0 & \frac{n_1}{n_2} \end{pmatrix}$	
thin lens, radii R_1 and R_2, of refractive index n in medium with refractive index 1	$M = \begin{pmatrix} 1 & 0 \\ -\frac{1}{f} & 1 \end{pmatrix}$	$\frac{1}{f} = (n-1)\left(\frac{1}{R_1} - \frac{1}{R_2}\right)$
spherical mirror, radius R	$M = \begin{pmatrix} 1 & 0 \\ -\frac{1}{f} & 1 \end{pmatrix}$	$\frac{1}{f} = \frac{2}{R}$

Section A

A1. A transverse sinusoidal wave propagates on a stretched string. The disturbance is simultaneously a maximum at two points at $30\,\mathrm{m}$ and $40\,\mathrm{m}$ along the string measured from some reference point. If the wavespeed is $340\,\mathrm{m\,s^{-1}}$, what are the *two* lowest possible frequencies of the wave? [5]

A2. A light bulb is set in the horizontal bottom surface of a $3.0\,\mathrm{m}$ deep swimming pool. What is the diameter of the circle on the surface from which light can escape into the air above? Take the index of refraction of the water in the pool to be 1.33 and assume the water is still. [4]

A3. X-rays with wavelength $0.12\,\mathrm{nm}$ undergo first-order diffraction from a crystal at an angle of 22° between the beam direction and the scattering planes. What is the angle for second order diffraction from these same planes? [4]

A4. Calculate the de Broglie wavelength of a proton in one of the beams of the Large Hadron Collider at CERN, whose energy is $7\,\mathrm{TeV}$. [3]

A5. Estimate the minimum energy in MeV of a proton which is confined to be inside a nucleus of radius $5 \times 10^{-15}\,\mathrm{m}$. [4]

TURN OVER

Section B

B1. (a) Explain what is meant by the *paraxial approximation* in geometrical optics. What is the consequence of this approximation for ray angles and angles of incidence, reflection and refraction on optical components? [4]

(b) What is meant by the *thin lens approximation*? [2]

(c) A *doublet* lens comprises two lenses placed as close together as possible along the optical axis. If the individual focal lengths of the two lenses are f_1 and f_2 respectively, show that the combined focal length of the doublet is $f_1 f_2/(f_1 + f_2)$ [3]

(d) A doublet uses two lenses, 1 and 2, made from materials with distinct refractive indices n_1 and n_2. Lens 1 has one flat face and one concave face with radius of curvature R. Lens 2 has two convex faces, each with radius of curvature R. Show that the focal length f of the doublet is given by

$$f = \frac{R}{2n_2 - n_1 - 1}$$ [4]

(e) Because of dispersion, the refractive indices of each of the lenses 1 and 2 in a doublet are different for red and blue light. Define $\Delta n_1 = n_1^{\text{blue}} - n_1^{\text{red}}$ for lens 1 and define Δn_2 similarly for lens 2. Find the relation between Δn_1 and Δn_2 that will make the focal lengths of the doublet the same (i.e. *achromatic*) for red and blue light. [2]

(f) Using the information in the table below, explain which type of glass should be used for lens 1 and lens 2 to make an achromatic doublet. [3]

	n^{blue}	n^{red}
crown glass	1.525	1.517
flint glass	1.632	1.616

For such a crown/flint glass doublet, what value of R will give a focal length of $10\,\text{cm}$ for red and blue light? [2]

B2. A neutron of mass m moves freely inside a one-dimensional box of length L. The neutron is in a state of fixed energy and is described by a spatial wavefunction $\psi(x)$ for $0 \leq x \leq L$. The wavefunction must vanish at the ends of the box.

(a) Describe how the wavefunction is related to a probability density for finding the neutron at different positions in the box. [3]

(b) Show that the allowed energies of the neutron are

$$E_n = \frac{n^2 h^2}{8mL^2} \qquad \text{for } n = 1, 2, 3, \ldots$$ [7]

(c) Free neutrons, each with energy equal to the lowest allowed energy ($n = 1$) in the box, are fired one by one so that they impinge normally on a double slit and are detected on a screen $2\,\mathrm{m}$ beyond. A fringe pattern of hits builds up on the screen with successive maxima separated by $0.1\,\mathrm{mm}$.

If the slit separation is $15\,\mu\mathrm{m}$, what is the length L of the box? Explain any approximations you make in obtaining your result. [8]

(d) What would your answer be for the length of the box if neutrons were replaced by electrons throughout this question? [2]

TURN OVER

B3. (a) For a plane monochromatic light wave, what is the direction of the electric field vector relative to the direction of propagation? By considering such a plane light wave, explain what is meant by *linear* and *circular* polarisation. [4]

(b) A birefringent crystalline material has refractive indices n_1 and n_2 for the two perpendicular components of linearly polarised light passing through it.

If a crystal of the material is to be used as a quarter-wave plate, the number of wavelengths of each component in the material must differ by $1/4$. Show that the minimum thickness d for such a plate is

$$d = \frac{\lambda_0}{4|n_1 - n_2|}$$ [5]

where λ_0 is the vacuum wavelength of the incident light.

(c) What is the minimum thickness of a quarter-wave plate if the indices of refraction are 1.875 and 1.635 and the wavelength in vacuum is $\lambda_0 = 589\,\mathrm{nm}$? [2]

(d) Explain how a quarter-wave plate can transform linearly polarised light into circularly polarised light. [7]

(e) Does a quarter-wave plate transform *any* linearly polarised beam into a circularly polarised one? Explain why or why not. [2]

B4. (a) A *very thin* soap film (refractive index $n = 1.33$) in air, whose thickness is much less than a wavelength of visible light, reflects hardly any light. In contrast, an equally thin layer of soapy water ($n = 1.33$) on glass ($n = 1.50$) appears quite shiny. Why is there a difference? [5]

[When light in a medium of refractive index n_a is incident on one of index n_b, the reflected light has a phase shift of π (0) relative to the incident light if $n_a < n_b$ $(n_a > n_b)$.]

(b) What is the thinnest coating of $\mathrm{MgF_2}$, with refractive index $n_g = 1.39$, on glass ($n = 1.50$) that produces a strong reflection for orange light with wavelength $600\,\mathrm{nm}$ incident from air onto the coated glass? [5]

(c) A plate of glass $10.0\,\mathrm{cm}$ long is placed with one end in contact with a flat glass surface and the other end resting on a thin metal strip, creating a thin wedge of air between the plate and the glass surface. The system is illuminated from above by light of wavelength $650\,\mathrm{nm}$.

Explain, with the help of a sketch diagram, why a series of parallel, regularly spaced, light and dark fringes is seen if the system is viewed from above. [4]

The number of bright fringes seen within a length of $1\,\mathrm{cm}$ perpendicular to the fringe lines is found to be 27. Estimate, quoting an error, the thickness of the metal strip. [6]

END OF PAPER

