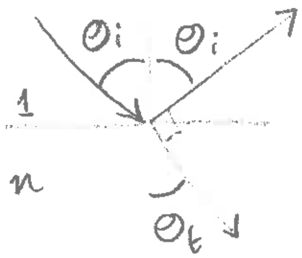
PHYS1011 SEMESTER 2 EXAM 2014/15

A1 Transverse displacement $y = A\sin(kx - \omega t + \phi)$ ① with $\omega = kv$ ① $= \frac{2\pi v}{\lambda}$

acceleration $\ddot{y} = -\omega^2 y$ ① $\quad |\ddot{y}|_{max} = A\omega^2$

we want $|\ddot{y}|_{max} \leq g$ ① $\longrightarrow A \leq \frac{g}{\omega^2} = \boxed{\frac{g\lambda^2}{4\pi^2 v^2}}$ ① [5]

A2



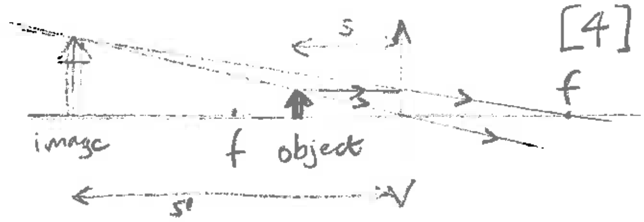
- Snell for refraction: $\sin\theta_i = n\sin\theta_t$ ①
- Brewster condition: $\theta_i = \frac{\pi}{2} - \theta_t$ ①
- angle above horizontal is θ_t
- $\tan\theta_t \overset{①}{=} \frac{1}{n} \longrightarrow \theta_t = \tan^{-1}\left(\frac{1}{1.33}\right) = \boxed{36.9°}$ ① [4]

A3
- can quote Gaussian lens equation $\frac{1}{s} + \frac{1}{s'} = \frac{1}{f}$ ①
- object distance $s = 15\,\text{cm}$
- focal length $f = 22\,\text{cm}$
- image distance $s' = \frac{1}{1/f - 1/s} = \frac{sf}{s-f}$ ① $= \frac{15 \times 22\,\text{cm}}{(15-22)} = \boxed{-47.1\,\text{cm}}$

① sign, image same side as object

→ image is 47.1 cm in front of lens, same side as object.

- image is virtual and upright. ① [4]



A4
- electron kinetic energy $T = 100\,\text{eV}$
- much less than rest energy (0.511 MeV) so nonrelativistic ①
- de Broglie $\lambda = \frac{h}{p} = \frac{h}{\sqrt{2mT}}$ ① $= \frac{hc}{\sqrt{2mc^2T}} = \frac{1240\,\text{eV nm}}{\sqrt{2 \times 0.511 \times 10^8\,\text{eV}^2}}$

$= \boxed{0.12\,\text{nm}}$ ① [3]

A5
- semi-quantitative use of uncertainty principle
- from question $\Delta x = c\Delta t$ ① where $\Delta t = 2\,\text{ms}$
- Heisenberg $\Delta p = \frac{\hbar}{2\Delta x}$ ①
- for a photon $E = pc = hf$ ① $\longrightarrow \Delta f = \frac{c}{h}\Delta p = \frac{c}{h}\frac{h}{4\pi c\Delta t}$ ①

$= \frac{1}{4\pi\Delta t} = \frac{10^3}{8\pi}\,\text{Hz} \sim 40\,\text{Hz}$ [4]

A questions: bookwork and application; similar questions in coursework bookwork plus seen problem

B1 (a)

- using results from front of paper, matrix for propagation over distance s, refraction and propagation over distance s' is

$$\begin{pmatrix} 1 & s' \\ 0 & 1 \end{pmatrix}\begin{pmatrix} 1 & 0 \\ \frac{1}{R}\left(\frac{n_1}{n_2}-1\right) & \frac{n_1}{n_2} \end{pmatrix}\begin{pmatrix} 1 & s \\ 0 & 1 \end{pmatrix} \equiv \begin{pmatrix} A & B \\ C & D \end{pmatrix} \quad (2)$$

- a ray $\begin{pmatrix} y \\ \alpha \end{pmatrix}$ from object becomes $\begin{pmatrix} y' \\ \alpha' \end{pmatrix}$ at image with $y' = Ay + B\alpha$
- For image formation, y' should not depend on α so we demand $B=0$ (1)
- Evaluating the matrix

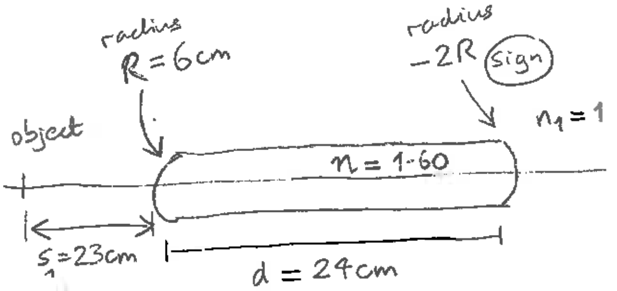
$$\begin{pmatrix} A & B \\ C & D \end{pmatrix} = \begin{pmatrix} 1 & s' \\ 0 & 1 \end{pmatrix}\begin{pmatrix} 1 & s \\ \frac{1}{R}\left(\frac{n_1}{n_2}-1\right) & \frac{s}{R}\left(\frac{n_1}{n_2}-1\right)+\frac{n_1}{n_2} \end{pmatrix} \overset{(1)}{=} \begin{pmatrix} 1+\frac{s'}{R}\left(\frac{n_1}{n_2}-1\right) & s+\frac{ss'}{R}\left(\frac{n_1}{n_2}-1\right)+\frac{s'n_1}{n_2} \\ \frac{1}{R}\left(\frac{n_1}{n_2}-1\right) & \frac{s}{R}\left(\frac{n_1}{n_2}-1\right)+\frac{n_1}{n_2} \end{pmatrix}$$

so $B=0$ gives $\quad s + \frac{s'n_1}{n_2} = -\frac{ss'}{R}\left(\frac{n_1}{n_2}-1\right)$

divide by ss', multiply by n_2 : $\boxed{\frac{n_1}{s} + \frac{n_2}{s'} = \frac{n_2-n_1}{R}}$ (1) [5]

- For magnification $\overset{(1)}{m} = \frac{y'}{y} = A = 1 + \frac{s'}{R}\left(\frac{n_1-n_2}{n_2}\right) \overset{(1)}{=} 1 - \frac{s'}{n_2}\left(\frac{n_1}{s}+\frac{n_2}{s'}\right)$

$$= \boxed{-\frac{n_1}{n_2}\frac{s'}{s}} \quad (1)$$ [3]



(b)

(i) use result above

$$\frac{1}{s_1} + \frac{n}{s_1'} = \frac{n-1}{R} \quad (1)$$

$$s_1' = \frac{n}{\frac{n-1}{R} - \frac{1}{s_1}} \overset{(1)}{=} \frac{1.6 \text{ cm}}{\frac{0.6}{6} - \frac{1}{23}} = \boxed{28.3 \text{ cm}} \ (1)$$ [3]

(ii) First image is beyond (1) the other end of the rod $\rightarrow$ it becomes a virtual object for the second refraction at distance (1) [2]

$s_2 = d - s_1' = -4.3\text{cm}$

(iii) for second refraction $\quad \frac{n}{s_2} + \frac{1}{s_2'} = \frac{1-n}{-2R} \rightarrow s_2' = \frac{1}{\frac{n-1}{2R} - \frac{n}{s_2}}$

(1) ↖ sign! (1)

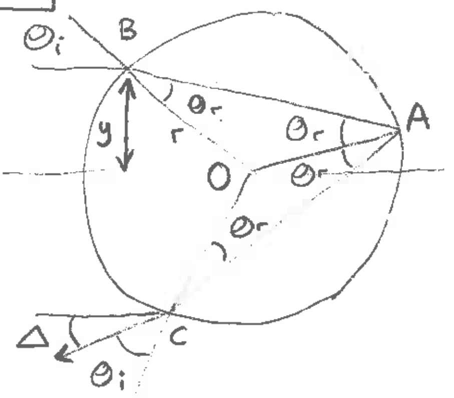
$$s_2' = \frac{1 \text{ cm}}{\frac{0.6}{12} + \frac{1.6}{4.3}} = \boxed{2.4 \text{ cm}} \ (1)$$ $\rightarrow$ 2.4cm beyond far end of rod [3]

(iv) magnification $m = -\frac{1}{n}\frac{s_1'}{s_1} \cdot \left(-n\frac{s_2'}{s_2}\right) \overset{(1)}{=} \frac{s_1's_2'}{s_1s_2} = \frac{s_1's_2'}{s_1(d-s_1')} \overset{(1)}{=} \boxed{-0.68}$ [2]

final image is real (1) and inverted (1) [2]

Bookwork and seen problem

B2



- First add to figure from question to show how angles can be determined from equality of angles of incidence and reflection at A and reflection symmetry about line OA

(a) using Snell's law ① [2]

$$\boxed{\sin\theta_i = n\sin\theta_r} \ ①$$

(b) add up angles turned through at B, A, C

$$\theta_{turn} = \underbrace{\theta_i - \theta_r}_{B \ ①} + \underbrace{\pi - 2\theta_r}_{A \ ①} + \underbrace{\theta_i - \theta_r}_{C} = \pi + 2\theta_i - 4\theta_r$$

+② for completing figure as above

$$\Delta = \pi - \theta_{turn} = 4\theta_r - 2\theta_i \ ①$$

but $\sin\theta_i = \frac{y}{r} = x$, so ① [6]

$$\boxed{\Delta = 4\arcsin\left(\frac{x}{n}\right) - 2\arcsin x}$$

(c) from (a) $\cos\theta_i = n\cos\theta_r \frac{d\theta_r}{d\theta_i}$ ① [2]

$$\longrightarrow \boxed{\frac{d\theta_r}{d\theta_i} = \frac{1}{n}\frac{\cos\theta_i}{\cos\theta_r}} \ ①$$

since θ_i increases monotonically with x in $0 \le x < 1$, we differentiate Δ wrt θ_i ①

$$\frac{d\Delta}{d\theta_i} = 4\frac{d\theta_r}{d\theta_i} - 2 = \frac{4}{n}\frac{\cos\theta_i}{\cos\theta_r} - 2 \ ①$$

stationary pt. where $\cos\theta_r = \frac{2}{n}\cos\theta_i$ ① $\rightarrow \frac{n^2}{4}\left(1 - \frac{x^2}{n^2}\right) = 1 - x^2$ ①

$$\frac{n^2}{4} - \frac{x^2}{4} = 1 - x^2 \rightarrow \frac{3x^2}{4} = \frac{4-n^2}{4} \rightarrow \ ① \ \boxed{x = \sqrt{\frac{4-n^2}{3}}} \quad [5]$$

[at the stationary pt can check that $\left.\frac{d^2\Delta}{d\theta_i^2}\right|_{stat.pt.} = -\frac{3}{2}\tan\theta_i < 0$ so is indeed a maximum, but not required]

(d) When looking down-sun, reflected rays from raindrops will bunch up around the maximum value for Δ, so you'll see a bright disk, brighter at its outside edge, with darker outside it. ②

Using the values given: $x_{red} = 0.862$ $\Delta_{max,red} = 42.5°$ ②
$x_{violet} = 0.856$ $\Delta_{max,violet} = 40.8°$

The brighter arc for red light will be outside that for violet light, so the primary rainbow has violet on the inside and red on the outside. ①

[5]

Unseen problem but single, double and multiple slits covered in lectures

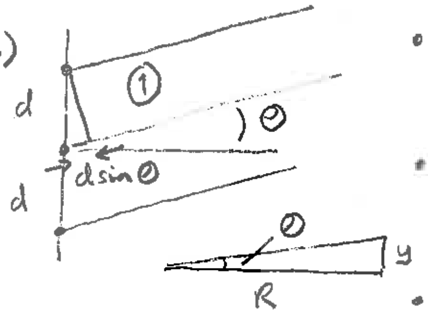
B3

(a) Use narrowness to assume a very wide (1) central diffraction maximum from each slit, compared to the spacing of maxima from the interference between different slits. (1) [2]

(b) Use long-distance approximation to allow us to consider parallel outgoing rays from each slit (Fraunhofer) [could use a lens to focus parallel rays...] (1) [2]

Using small transverse displacements on screen allows us to use small angle approx ($\sin\theta \sim \theta$, $\tan\theta \sim \theta$, $\cos\theta \sim 1$) (1)

(c)



- path-length difference between neighbouring slits is $d\sin\theta \approx d\theta \simeq \frac{dy}{R}$ (1)
- corresponding phase difference
$$\phi = 2\pi \frac{dy}{R\lambda} \quad (1)$$
- add contributions of equal amplitude from the 3 slits

$$A\sin X + A\sin(X+\phi) + A\sin(X-\phi) \quad (1)$$ where X is a common term containing time-dependence

$$= A\sin X + A(\sin X\cos\phi + \cos X\sin\phi \quad (1)$$
$$+ \sin X\cos\phi - \cos X\sin\phi)$$

[or use $Ae^{iX} + Ae^{i(X+\phi)} + Ae^{i(X-\phi)} = Ae^{iX}(1+2\cos\phi)$]

$$= A\sin X(1+2\cos\phi) \qquad (*)$$

- square and time-average for intensity $\longrightarrow$

$$\boxed{I_1(y) = \frac{I_0}{9}\left(1 + 2\cos\left(2\pi\frac{dy}{R\lambda}\right)\right)^2} \quad (1)$$

where I_0 is intensity in fwd direction ($y=0$) [6]

(d) When we double the width of the central slit we double (1) its contribution in the amplitude $\longrightarrow$ so $(*)$ above becomes

$$A\sin X(2 \overset{(1)}{+} 2\cos\phi) = 4A\sin X\cos^2\left(\frac{\phi}{2}\right) \quad (1)$$

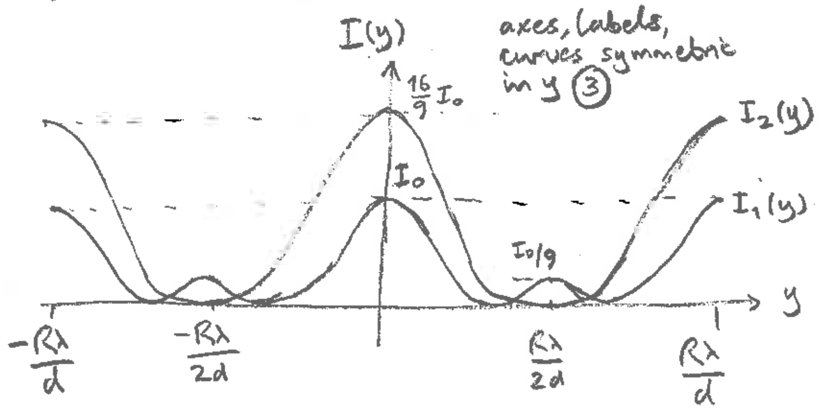
The total fwd amplitude is $\frac{4}{3}$ what it was before, so for the intensity,

$$\boxed{I_2(y) = \frac{16 I_0}{9}\cos^4\left(\pi\frac{dy}{R\lambda}\right)} \quad (1)$$

[4]

(e)

axes, labels, curves symmetric in y (3)



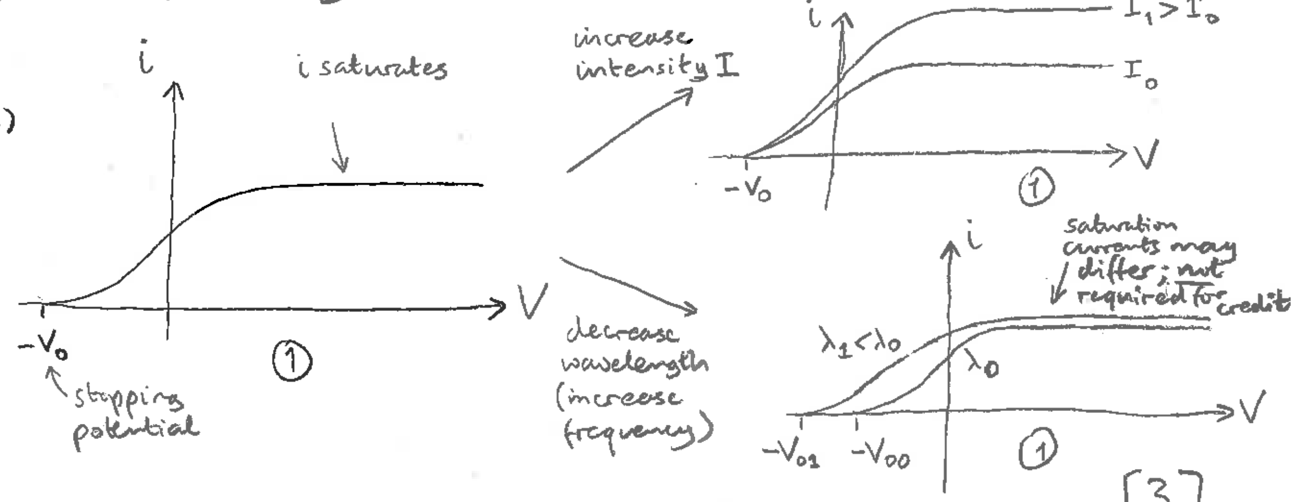
- both have maxima at $0, \pm\frac{R\lambda}{d}$ (1)
- I_1 has max at $\pm\frac{R\lambda}{2d}$ (1) whereas I_2 has min
- I_1 has further minima (1) when $\cos\left(2\pi\frac{dy}{R\lambda}\right) = -\frac{1}{2}$

[6]

B4

Bookwork plus partly-seen problem

(a)



[3]

(b) First write consequence of energy conservation (1)

$$eV_0 \overset{(1)}{=} hf - \phi = \frac{hc}{\lambda} - \phi$$

- V_0 is stopping potential
- $hf = \frac{hc}{\lambda}$ is photon energy (1)
- ϕ is work function

- stopping potential is magnitude of negative potential difference (1) between anode and cathode needed to shut off photocurrent, eV_0 is maximum kinetic energy of emitted electrons (1)
- work function is minimum energy needed to eject an electron from the cathode. (1)

[6]

(c) (i) An electron is emitted by a single photon (1) so electron energy depends on the photon energy and hence on frequency (1) but not intensity

(ii) If photon frequency is too low, its energy is less than the work function and no electron can be emitted (1)

[3]

(d) $\lambda_1 = 275\,\text{nm}$ $V_{01} = 1.3\,\text{V}$ From above $h = \frac{e(V_{02} - V_{01})}{c(1/\lambda_2 - 1/\lambda_1)}$ (1)

$\lambda_2 = 200\,\text{nm}$ $V_{02} = 3.0\,\text{V}$

$$hc = \frac{(3.0 - 1.3)\,\text{eV nm}}{(1/200 - 1/275)} = 1247\,\text{eV nm} \longrightarrow h = \frac{1.247 \times 10^{-6}\,\text{m} \times 1.6 \times 10^{-19}\,\text{J}}{3.0 \times 10^{8}\,\text{ms}^{-1}}$$

$$= \boxed{6.65 \times 10^{-34}\,\text{Js}} \quad (1)$$

to get ϕ for calcium

$$\phi = \frac{hc}{\lambda_1} \overset{(1)}{-} eV_{01} = \frac{1247\,\text{eV nm}}{275\,\text{nm}} - 1.3\,\text{eV} \overset{(1)}{=} \boxed{3.23\,\text{eV} = 5.17 \times 10^{-19}\,\text{J}}$$

[4]

(e)

- Intensity $I = 100\,\text{Wm}^{-2}$ on area $A = 5\,\text{mm}^2$; wavelength $\lambda = 200\,\text{nm}$
  Number of incident photons per second is
  $$n_\gamma = \frac{IA}{hc/\lambda} \quad (1)$$
- With photocurrent $i = 4\,\mu\text{A}$, number of emitted electrons per second is
  $$n_e = \frac{i}{e} \quad (1)$$
- photoelectric efficiency $\frac{n_e}{n_\gamma} = \frac{i}{e}\overset{(1)}{\frac{hc}{IA\lambda}} = \frac{4 \times 10^{-6}\,\text{Cs}^{-1}}{e} \frac{1247\,\text{eV nm}}{100\,\text{Wm}^{-2} \times 5 \times 10^{-6}\,\text{m}^2 \times 200\,\text{nm}}$

(using h calculated above) $= \boxed{5.0\%}$ (1)

[4]