UNIVERSITY OF SOUTHAMPTON PHYS1011W1

SEMESTER 2 EXAMINATION 2014–2015

WAVES, LIGHT AND QUANTA

Duration: 120 MINS (2 hours)

This paper contains 9 questions.

Answers to Section A and Section B must be in separate answer books

Answer **all** questions in **Section A** and **only two** questions in **Section B**.

Section A carries 1/3 of the total marks for the exam paper and you should aim to spend about 40 mins on it.

Section B carries 2/3 of the total marks for the exam paper and you should aim to spend about 80 mins on it.

An outline marking scheme is shown in brackets to the right of each question.

A Sheet of Physical Constants is provided with this examination paper.

Only university approved calculators may be used.

A foreign language word to word® translation dictionary (paper version) is permitted provided it contains no notes, additions or annotations..

Geometric optics sign conventions

s	object distance	positive for real object, negative for virtual object
s'	image distance	positive for real image, negative for virtual image
f	focal length	positive for converging lens or concave mirror, negative for diverging lens or convex mirror
R	radius of curvature	positive when centre of curvature is on the same side as the outgoing ray, negative otherwise (this means positive for concave mirror or convex refracting surface)

Ray-transfer matrices

These act on two-component vectors whose upper component is the distance of a ray from the optical axis and whose lower component is the angle the ray makes with the optical axis.

translation by distance t	$M = \begin{pmatrix} 1 & t \\ 0 & 1 \end{pmatrix}$	
refraction, spherical interface, radius R, from region n_1 to n_2	$M = \begin{pmatrix} 1 & 0 \\ \frac{1}{R}\left(\frac{n_1}{n_2} - 1\right) & \frac{n_1}{n_2} \end{pmatrix}$	
refraction, plane interface, from region n_1 to n_2	$M = \begin{pmatrix} 1 & 0 \\ 0 & \frac{n_1}{n_2} \end{pmatrix}$	
thin lens, radii R_1 and R_2, of refractive index n in medium with refractive index 1	$M = \begin{pmatrix} 1 & 0 \\ -\frac{1}{f} & 1 \end{pmatrix}$	$\frac{1}{f} = (n-1)\left(\frac{1}{R_1} - \frac{1}{R_2}\right)$
spherical mirror, radius R	$M = \begin{pmatrix} 1 & 0 \\ -\frac{1}{f} & 1 \end{pmatrix}$	$\frac{1}{f} = \frac{2}{R}$

Section A

A1. A transverse sinusoidal wave with wave speed v and wavelength λ propagates on a stretched string. The wave motion takes place in a vertical plane. An ant of negligible mass is standing on the string. What is the maximum amplitude of the wave for which the ant can remain in contact with the string if the ant does not actively cling on? [5]

A2. At what angle above the horizontal is the sun if sunlight reflected from the surface of a calm lake is completely linearly polarised? Take the refractive indices of air and water to be 1 and 1.33 respectively. [4]

A3. An object is placed $15\,\mathrm{cm}$ in front of a converging lens. If the lens has focal length $22\,\mathrm{cm}$, where is the image? Is the image real or virtual? Is the image upright or inverted? [4]

A4. Electrons are accelerated from rest through a potential difference of $100\,\mathrm{V}$. What is their de Broglie wavelength after the acceleration? [3]

A5. A radar emits a pulse of electromagnetic radiation of duration $2\,\mathrm{ms}$. If the position uncertainty of a photon in the pulse is the length of the pulse, estimate the uncertainty in the frequency of the photon. [4]

TURN OVER

Section B

B1. (a) An object is placed in a medium of refractive index n_1 a distance s in front of a spherical interface with a medium of refractive index n_2. If the spherical surface has radius of curvature R and an image is formed at distance s' beyond it, show that the object and image distances are related by

$$\frac{n_1}{s} + \frac{n_2}{s'} = \frac{n_2 - n_1}{R}$$ [5]

and that the lateral magnification of the image compared to the object is

$$-\frac{n_1}{n_2}\frac{s'}{s}.$$ [3]

(b) A straight cylindrical glass rod of length $24\,\mathrm{cm}$ and refractive index 1.60 sits in air. Its ends are sections of spherical surfaces with radii of curvature $6.0\,\mathrm{cm}$ and $12.0\,\mathrm{cm}$ respectively, with both surfaces bulging outwards. A small object of height $1.5\,\mathrm{mm}$ is placed outside and along the axis of the cylinder, at a distance of $23.0\,\mathrm{cm}$ from the end with radius $6.0\,\mathrm{cm}$.

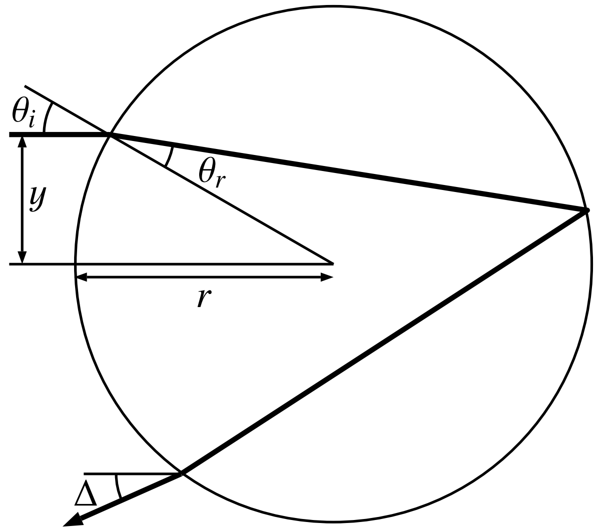
(i) What is the position of the image formed by refraction at the surface with radius $6.0\,\mathrm{cm}$? [3]

(ii) This image becomes the new object for refraction in the second surface. Is it a real or virtual object? [2]

(iii) What is the position of the final image after the second refraction? [3]

(iv) What is the lateral magnification of the final image? [2]
Is it real or virtual and upright or inverted? [2]

B2. For a *primary* rainbow a horizontal light ray refracts, reflects and refracts again as it passes into and out of a spherical raindrop of radius r and refractive index n, as shown in the figure below. The offset of the incoming ray from a parallel diameter of the drop is y. Let $x = y/r$ be the scaled offset. The angles of incidence and refraction as the ray enters the drop are θ_i and θ_r respectively. We wish to investigate the *exit angle*, Δ, below the horizontal between the outgoing ray and a line antiparallel to the incoming ray.



(a) State the relation between θ_i and θ_r. [2]

(b) By considering the angle turned through by the ray at the successive refraction, reflection and refraction, or otherwise, show that

$$\Delta = 4\theta_r - 2\theta_i = 4\arcsin(x/n) - 2\arcsin(x).$$ [6]

(c) Using the result from (a), show that

$$\frac{d\theta_r}{d\theta_i} = \frac{\cos\theta_i}{n\cos\theta_r}$$ [2]

and hence that the maximum of Δ occurs when

$$x = \sqrt{\frac{4-n^2}{3}}.$$ [5]

(d) Given $n = 1.330$ for red light and $n = 1.342$ for violet light, explain which colour of light you will see on the outside arc of the primary rainbow. [5]

TURN OVER

B3. A mask with three equally-spaced narrow parallel slits is illuminated at normal incidence by a monochromatic plane wave of light with wavelength λ. The spacing between neighbouring slits is d. The transmitted light is viewed on a screen.

(a) If we want to calculate the pattern on the screen, how does the assumption that the slits are very *narrow* simplify our calculation? [2]

(b) What further approximations can we make if the screen is a *long distance* R from the slits and we observe the intensity as a function of small transverse displacements y across the screen (where y is zero at the intersection of a line normal to the mask with the screen)? [2]

(c) If the three slits have equal widths, show that intensity I_1 of light on the screen as a function of y is

$$I_1(y) = \frac{I_0}{9}\left[1 + 2\cos\left(2\pi\frac{dy}{R\lambda}\right)\right]^2 \qquad [\ 6\ ]$$

where I_0 is a constant.

(d) Now suppose the width of the central slit is increased to make it twice as wide as the other two slits. Show that the intensity becomes

$$I_2(y) = \frac{16 I_0}{9}\cos^4\left(\pi\frac{dy}{R\lambda}\right) \qquad [\ 4\ ]$$

where I_0 is the same constant as before.

(e) Sketch the curves $I_1(y)$ and $I_2(y)$ for $-R\lambda/d \leq y \leq R\lambda/d$. [6]

B4. In the photoelectric effect, electromagnetic radiation of a fixed wavelength strikes the surface of a metal cathode and ejects electrons. The electrons are collected as a photocurrent i at an anode at relative potential $V = V_{\text{anode}} - V_{\text{cathode}}$.

(a) Sketch a graph of the dependence of the photocurrent i on V and add curves to show the effect of (i) increasing the radiation intensity at fixed wavelength and (ii) reducing the wavelength at fixed intensity. [3]

(b) Explain the terms *stopping potential* and *work function* and give an equation relating them to the wavelength of the illuminating radiation. [6]

(c) Explain the following features of the photoelectric effect: (i) the individual electron energies are independent of the intensity of the illumination; (ii) there is no photocurrent at all below a certain threshold frequency. [3]

A calcium cathode with surface area $5\,\text{mm}^2$ was irradiated with light of wavelength $275\,\text{nm}$. The stopping potential was found to be $1.3\,\text{V}$. When the same cathode was irradiated with light of wavelength $200\,\text{nm}$, the stopping potential was $3.0\,\text{V}$.

(d) Use the information above to determine the value of Planck's constant and the work function for calcium. [4]

(e) For irradiation with $200\,\text{nm}$ light of intensity $100\,\text{W}\,\text{m}^{-2}$, the photocurrent was $4\,\mu\text{A}$. Determine the photoelectric efficiency of the cathode (the number of photoelectrons emitted per incident photon). [4]

END OF PAPER