SEMESTER 2 EXAMINATION 2012–2013

WAVES, LIGHT AND QUANTA

Duration: 120 MINS (2 hours)

This paper contains 10 questions.

Answers to Section A and Section B must be in separate answer books

Answer **all** questions in **Section A** and **only two** questions in **Section B**.

Section A carries 1/3 of the total marks for the exam paper and you should aim to spend about 40 mins on it.

Section B carries 2/3 of the total marks for the exam paper and you should aim to spend about 80 mins on it.

An outline marking scheme is shown in brackets to the right of each question.

A Sheet of Physical Constants is provided with this examination paper.

Only university approved calculators may be used.

A foreign language translation dictionary (paper version) is permitted provided it contains no notes, additions or annotations.

Geometric optics sign conventions

s	object distance	positive for real object, negative for virtual object
s'	image distance	positive for real image, negative for virtual image
R	radius of curvature	positive when centre of curvature is on the same side as the outgoing ray, negative otherwise (this means positive for concave mirror or convex refracting surface)

Ray-transfer matrices

These act on two-component vectors whose upper component is the distance of a ray from the optical axis and whose lower component is the angle the ray makes with the optical axis.

translation by distance t

$$M = \begin{pmatrix} 1 & t \\ 0 & 1 \end{pmatrix}$$

refraction, spherical interface, radius R, from region n_1 to n_2

$$M = \begin{pmatrix} 1 & 0 \\ \frac{1}{R}\left(\frac{n_1}{n_2} - 1\right) & \frac{n_1}{n_2} \end{pmatrix}$$

refraction, plane interface, from region n_1 to n_2

$$M = \begin{pmatrix} 1 & 0 \\ 0 & \frac{n_1}{n_2} \end{pmatrix}$$

thin lens, radii R_1 and R_2, of refractive index n in medium with refractive index 1

$$M = \begin{pmatrix} 1 & 0 \\ -\frac{1}{f} & 1 \end{pmatrix} \qquad \frac{1}{f} = (n-1)\left(\frac{1}{R_1} - \frac{1}{R_2}\right)$$

spherical mirror, radius R

$$M = \begin{pmatrix} 1 & 0 \\ -\frac{1}{f} & 1 \end{pmatrix} \qquad \frac{1}{f} = \frac{2}{R}$$

Section A

A1. A wavefunction $\psi(x,t)$ satisfies a wave equation

$$\frac{\partial^2\psi(x,t)}{\partial x^2} = \frac{1}{v^2}\frac{\partial^2\psi(x,t)}{\partial x^2},$$

where v is a constant. Show that $\psi(x,t) = f(x \pm vt)$ is a solution of this equation for any function f. You may assume that f can be differentiated as often as needed. [2]

What is the physical meaning of v? [1]

A2. A ray of light in diamond (refractive index 2.42) is incident on a plane interface with air. What is the largest angle the ray can make with the normal to the interface and not be totally reflected back into the diamond? [2]

A3. Explain what is meant by the *paraxial approximation* in geometrical optics. What is the consequence of this for trigonometric functions of ray angles and of angles of incidence, reflection and refraction on optical components? [4]

A4. A thin lens forms an image of an object. The object is $15.0\,\mathrm{cm}$ from the lens and the image is $10.0\,\mathrm{cm}$ from the lens on the same side as the object. What is the focal length of the lens? Is the lens converging or diverging? [4]

A5. The photoelectric work function of potassium is $2.3\,\mathrm{eV}$. If light of wavelength $250\,\mathrm{nm}$ falls on potassium, find the stopping potential needed to prevent any photocurrent flowing. [3]

A6. An electron (e) and a proton (p) are moving relativistically and have the same energy E. Show that the ratio of their de Broglie wavelengths is given approximately by

$$\frac{\lambda_e}{\lambda_p} \approx 1 - \frac{M^2c^4}{2E^2}$$ [3]

where M is the proton mass. Why has the electron mass been neglected here? [1]

TURN OVER

Section B

B1. A transparent sphere of radius R is made of a material with refractive index n and sits in air. Consider paraxial rays near or on a diameter of the sphere.

(a) Write down the ray-transfer matrices for (i) the refraction of a ray entering the sphere, (ii) propagation of the ray across the diameter of the sphere and (iii) the refraction of a ray leaving the sphere. Note any sign conventions that you apply. [6]

(b) Hence show that the matrix for a ray passing through the sphere is

$$\begin{pmatrix} \frac{2}{n} - 1 & \frac{2R}{n} \\ -\frac{2}{R}\frac{(n-1)}{n} & \frac{2}{n} - 1 \end{pmatrix}$$

[4]

(c) In a van Leeuwenhoek microscope an object is placed against a small glass sphere and viewed through the sphere. What value of the refractive index of the glass will give a virtual image at infinity? [5]

If the glass sphere has this refractive index and its diameter is $2\,\mathrm{mm}$, what is the resulting angular magnification (ratio of the angle subtended by the image to the angle subtended by the object when viewed by the unaided eye at a near-point distance of $25\,\mathrm{cm}$)? [5]

B2. (a) A beam of light in a transparent medium of refractive index n_a is partially reflected and partially transmitted at a plane interface with a second medium of index n_b.

(i) Draw a diagram showing the incident, reflected and transmitted beams and indicate the angles they make with the normal to the interface. [2]

(ii) What is the angle between the reflected and transmitted beams for which the reflected light will be completely linearly polarized? [2]

(iii) Show that the angle of incidence, θ_p, for which this happens is given by

$$\tan\theta_p = \frac{n_b}{n_a}$$ [3]

(iv) Light reflected from a glass plate (refractive index 1.65) immersed in ethanol (refractive index 1.36) is found to be completely linearly polarized. At what angle does the partially polarized *transmitted* beam travel into the plate? [3]

(b) An ideal linear polarizer is rotated at rate ω between a similar pair of stationary crossed polarizers. A beam of unpolarized light of intensity I_0 is incident on one of the stationary polarizers and passes through all three of them.

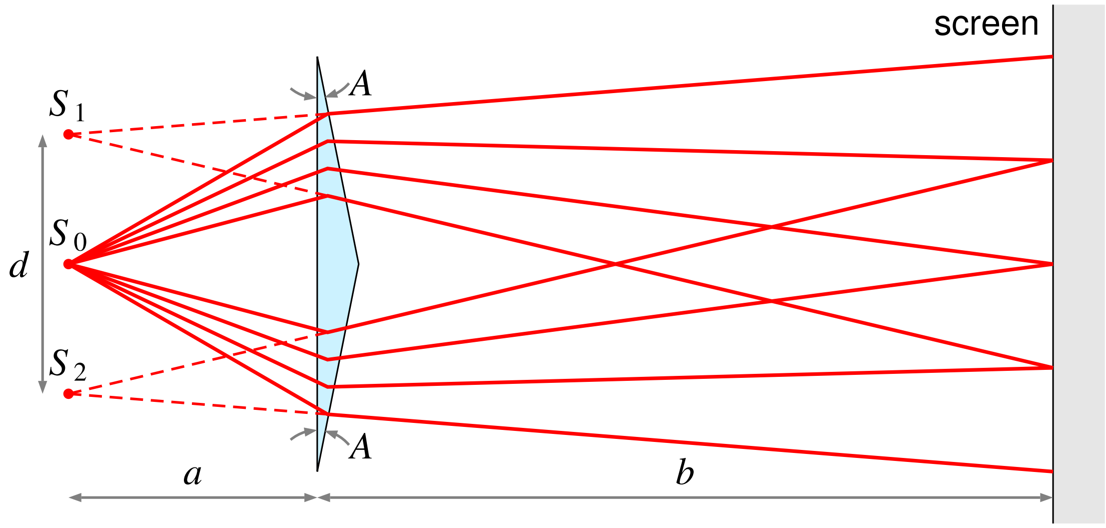
(i) If the polarization axis of the rotating polarizer is initially parallel to the axis of the polarizer on the incident side, show that the emerging beam has intensity which varies as a function of time t according to

$$I = \frac{I_0}{16}\big(1 - \cos(4\omega t)\big)$$ [8]

(ii) What is the intensity of the beam emerging from the *rotating* polarizer when $t = \pi/(6\omega)$? [2]

TURN OVER

B3. The figure below shows a Fresnel biprism interferometer. The prism is made of material with refractive index n and the magnitude of the prism angle A is extremely small. Light from a source S_0 is refracted by the prism, giving rise to two virtual coherent sources S_1 and S_2. The prism is at distance a from S_0 and a screen is placed at distance b from the prism.



(a) By first calculating, in terms of A and n, the angles that the incoming and outgoing rays make with the symmetry axis, or otherwise, show that the separation of the virtual sources S_1 and S_2 is given by

$$d = 2aA(n - 1).$$ [10]

Remember that the light rays are refracted both on entering and leaving the prism and note where you make approximations.

(b) If the source S_0 is monochromatic, with wavelength λ, show, noting any approximations you make, that the spacing between neighbouring bright fringes near the centre of the interference pattern on the screen is given by

$$\frac{\lambda(a + b)}{d}.$$ [8]

(c) Calculate the fringe spacing observed on the screen for green light of wavelength $500\,\text{nm}$. Take $a = 0.200\,\text{m}$, $b = 2.00\,\text{m}$, $A = 3.50\,\text{mrad}$ and $n = 1.50$. [2]

B4. A photon of wavelength λ scatters off an electron which is at rest. The photon emerges with wavelength λ' with its momentum making an angle ϕ relative to its initial direction.

(a) What quantities are conserved in this collision? [2]

(b) If the scattering angle ϕ is non-zero, the three-momenta of the initial and scattered photon define a plane. What is the orientation of the electron's three-momentum after the scattering relative to this plane? [2]

(c) Show that the change in wavelength for the photon is given by

$$\lambda' - \lambda = \frac{h}{mc}(1 - \cos\phi)$$ [9]

(d) X rays with wavelength $0.0665\,\mathrm{nm}$ scatter off electrons at rest. What is the longest wavelength found in the scattered X rays? At which scattering angle is this wavelength observed? [3]

(e) What would be the maximum change in wavelength for a visible photon of initial wavelength $500\,\mathrm{nm}$ if it scattered off an electron at rest? Explain whether or not you would expect to be able to detect this shift with your unaided eye. [4]

END OF PAPER