## PHYS1011 Waves, Light and Quanta: 2013/14 exam report

Significant numbers of students ignored the instruction to submit answers to Section A and Section B in separate answer books. A good number also submitted answers to more than two questions in Section B, again ignoring the instruction in the exam rubric.

The numbers following each question label are the number of students attempting the question and the average mark achieved (out of 20).

Section A (153, 9.0) Section A contained short questions covering the whole syllabus of the module.

- A1 was a question on understanding the expression for a sinusoidal scalar wave in one dimension, using the standard relation between wavenumber and wavelength and the expression $v=\omega / k$ for the phase speed $v$ as the ratio of angular frequency over wavenumber.
- A2 was a question on Snell's Law and the critical angle for total internal reflection. This was mostly done quite well.
- A3 was a question on Bragg scattering, needing you to remember, or quickly reconstruct, the Bragg formula for scattering from parallel planes of atoms. With the setup described in the question, the Bragg condition is $2 d \sin \theta=m \lambda$ for the $m$ th order scattering at angle $\theta$ with wavelength $\lambda$ and planes separated by distance $d$. A common error was to omit the factor of 2 , although this cancelled in determining the second-order angle given the first-order angle.
- A4 was an application of the de Broglie relation, $\lambda=h / p$, for a highly relativistic particle with $E \approx p c$ (when $E \gg m c^{2}$ where $m$ is the particle's mass). Almost everyone knew the de Broglie relation, but after that it was important to use the relativistic expression for the momentum and observe that, to good precision, the proton's mass could be ignored
- A5 was a semi-quantitative application of the uncertainty principle, $\Delta x \Delta p \geq \hbar / 2$, using a nuclear confinement length to estimate a momentum. Given the momentum $p \sim \Delta p$, the energy is estimated from the nonrelativistic expression $p^{2} / 2 m$. The energy comes out small compared to the proton's rest energy, $m c^{2}$, justifying the use of nonrelativistic kinematics.

B1 Geometrical optics $(\mathbf{1 1 4}, \mathbf{1 1 . 0})$ This was a very popular question. The first two parts were bookwork, recalling what we mean by the paraxial and thin lens approximations. The paraxial approximation is more than simply quoting the small-angle approximations for sines and cosines. You also need to say why we can assume angles are small. We have rays which never make large angles with the optical axis and which also never get too far from the axis. Here 'far' is compared to the radii of curvature of lenses and mirrors in the system, so that rays always strike them at small angles of incidence.

- In part (c), many people wrote down the Gaussian lens formula, $1 / s+1 / s^{\prime}=1 / f$ and then replaced $s$ and $s^{\prime}$ by $f_{1}$ and $f_{2}$ respectively. This is incorrect. What you could do (and a good number of students did) is to combine

$$
\begin{aligned}
& 1 / s_{1}+1 / s_{1}^{\prime}=1 / f_{1} \\
& 1 / s_{2}+1 / s_{2}^{\prime}=1 / f_{2}
\end{aligned}
$$

and make the image of the first lens become the (virtual) object for the second. Using the fact that the lenses are at the same place (thin lenses), this means $s_{1}^{\prime}=-s_{2}$. Then you find that the image and object distances for the combined lens satisfy

$$
1 / s_{1}+1 / s_{2}^{\prime}=1 / f_{1}+1 / f_{2}
$$

Comparing this to the Gaussian lens equation shows that the focal length of the combined lens is given by $1 / f=1 / f_{1}+1 / f_{2}$.
An alternative is simply to multiply the matrices for two thin lenses and extract the combined focal length from the result.

- In part (d) to evaluate the combined focal length, it was much easier to substitute expressions for $1 / f_{1}$ and $1 / f_{2}$ into $1 / f=1 / f_{1}+1 / f_{2}$ and then invert the result, rather than substituting immediately into $f=f_{1} f_{2} /\left(f_{1}+f_{2}\right)$.
- In part (f), you had worked out that $2 \Delta n_{2}=\Delta n_{1}$, but then many students looked at the earlier result $f=R /\left(2 n_{2}-n_{1}-1\right)$ and incorrectly set $2 n_{2}=n_{1}$ in it. You needed to use $n_{2}$ and $n_{1}$ from either type of glass (both give the same answer).

B2 Wavefunctions and two-slit interference $(\mathbf{2 9}, \mathbf{1 1 . 1})$ This question was relatively unpopular. In part (c) a common error was to try to use the relation $E=h c / \lambda$, which applies for massless particles, when talking about (nonrelativistic) neutrons, for which the mass cannot be neglected. I find it best to know the de Broglie relation $\lambda=h / p$ to associate a wavelength with a momentum and then use a nonrelativistic or relativistic definition to relate momentum and energy as appropriate.
For this question, it was sufficient in part (c) to work just in terms of the de Broglie wavelength, which you already knew from part (b).

B3 Polarisation and birefringence (63, 8.3) In part (a), quite a few answers described how to produce different polarisation states, rather than describe the states themselves. You do not get credit for answering things not asked in the question.
A common error in (b) was to claim that a 'quarter-wave plate' is one-quarter of a wavelength thick. The name arises because the plate is set up to impose a quarter-wavelength phase shift between two perpendicular polarisation components. Answers to this question also produced some strange algebra, equivalent to $1 / a-1 / b \stackrel{?}{=} 1 /(a-b)$ which is never true for real $a$ and $b$.

B4 Thin-film interference (90, 8.7) It appeared that the notation ' $\pi$ (0) for $n_{a}<n_{b}\left(n_{a}>n_{b}\right)$ ) confused a few people. It should be read as $\pi$ when $n_{a}<n_{b}$ and 0 when $n_{a}>n_{b}$.
In part (b), quite a few people forgot that when calculating the optical path length for a light ray in a medium with refractive index $n$, you have to take account of $n$. A way to think about this is to consider that for a sinusoidal wave with wavenumber $k$, the phase change in distance $x$ is $k x=2 \pi x / \lambda=$ $2 \pi n x / \lambda_{o}$ where $\lambda_{o}$ is the wavelength in vacuum.
In answering part (c) many people wrote down an equation $d \sin \theta=m \lambda$ which normally appears in the context of diffraction from multiple narrow slits. Trying to massage this expression for use in this question usually meant missing a factor of 2 coming from considering a light ray traversing the air gap between the glass plate and the glass surface twice. Here, you wanted an expression to show that the path length from a double traversal of the air gap was equal to half-integer number of wavelengths (with the half accounting for a phase change of $\pi$ radians for the reflection from the lower glass surface).

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