

SEMESTER 2 EXAMINATION 2014-2015

ENERGY AND MATTER

Duration: 120 MINS (2 hours)

This paper contains 8 questions.

Answers to Section A and Section B must be in separate answer books

Answer **all** questions in **Section A** and **only two** questions in **Section B**.

Section A carries 1/3 of the total marks for the exam paper and you should aim to spend about 40 mins on it.

Section B carries 2/3 of the total marks for the exam paper and you should aim to spend about 80 mins on it.

An outline marking scheme is shown in brackets to the right of each question.

A Sheet of Physical Constants is provided with this examination paper.

Only university approved calculators may be used.

A foreign language translation dictionary (paper version) is permitted provided it contains no notes, additions or annotations.

Useful formulae

Conversions

$$0 \text{ K} = -273.15 \text{ }^\circ\text{C}$$

$$1 \text{ atm} = 101\,325 \text{ Pa} = 760 \text{ Torr}$$

$$1 \text{ Pa} = 10^{-5} \text{ bar} = 9.8686 \times 10^{-6} \text{ atm} = 7.5 \times 10^{-3} \text{ Torr}$$

Thermodynamic potentials

Internal Energy: U

Enthalpy: $H = U + PV$

Helmholtz Free Energy: $F = U - TS$

Gibbs Free Energy: $G = U - TS + PV$

Availability: $A = U - T_0S + P_0V$

Atomic weights

Hydrogen: 1

Helium: 4

Nitrogen: 14

Section A

- A1.** The International Space Station has an internal volume $V = 916 \text{ m}^3$ and is pressurised to $P = 101.3 \text{ kPa}$ at a temperature of 20°C . What is the total mass and RMS velocity of the air contained (assuming air is molecular nitrogen N_2 which behaves as an ideal gas)?

[6]

Bookwork:

Ideal gas: $PV = Nk_B T$ [1]

Seen similar:

$$N = PV/(k_B T) \text{ [1]} = 2.3 \times 10^{28} \text{ [1]}$$

$$M = N \times m = 2.3 \times 10^{28} \times 28u = 1070 \text{ kg [1]}$$

$$v_{\text{RMS}} = \sqrt{\frac{3k_B T}{m}} \text{ [1]} = 510 \text{ m/s [1]}$$

- A2.** An ideal gas at pressure P_1 , volume V_1 and temperature T_1 undergoes a free (Joule) expansion to volume V_2 . Find the new temperature T_2 and pressure P_2 . Define the terms ‘adiabatic’ and ‘isochoric’ and state whether either of these can be used to describe the Joule expansion.

[4]

Bookwork:

In a free expansion, temperature does not change $T_2 = T_1$ [1].

Seen similar:

For fixed temperature $P_1 V_1 = P_2 V_2$ [$\frac{1}{2}$] $\Rightarrow P_2 = (V_1/V_2) P_1$ [$\frac{1}{2}$].

Bookwork:

Adiabatic: no heat flow into or out of the gas [$\frac{1}{2}$].

Isochoric: no change in volume [$\frac{1}{2}$].

The Joule expansion is adiabatic, but not isochoric. [1].

TURN OVER

- A3.** A reversible heat engine is used as a heat pump to run a domestic freezer. It consumes 20 W of electrical power and maintains the cold reservoir at -10°C in a room at $+20^{\circ}\text{C}$. Draw a heat engine diagram depicting the flow of heat and work, find the efficiency of this engine, and calculate the rate at which it extracts heat from the cold reservoir. [5]

Bookwork:

Diagram [1]

Reversible \Rightarrow efficiency $\eta = 1 - \frac{T_{\text{cold}}}{T_{\text{hot}}}$ [1]

New question:

$$\eta = 1 - \frac{-10+273}{+20+273} \left[\frac{1}{2}\right] = 10\% \left[\frac{1}{2}\right]$$

$$\eta = \frac{\text{Work}}{Q_{\text{H}}} \Rightarrow Q_{\text{H}} = \frac{20 \text{ W}}{\eta} = 200 \text{ W} \left[1\right]$$

$$Q_{\text{C}} + \text{Work} = Q_{\text{H}} \Rightarrow Q_{\text{C}} = 200 \text{ W} - 20 \text{ W} = 180 \text{ W} \left[1\right]$$

- A4.** A cup of water at T_{initial} cools to room temperature T_{room} . Derive an expression for the change in entropy of 1) the water, and 2) the room. Comment on the sign of the total change in entropy. [5]

Seen similar:

Differential change in entropy of water: $dS = \frac{dQ}{T}$ [1] = $C \frac{dT}{T}$ [1] where C is the heat capacity.

$$\Rightarrow \Delta S = C \int_{T_{\text{initial}}}^{T_{\text{room}}} \frac{dT}{T} = C \ln \left(\frac{T_{\text{room}}}{T_{\text{initial}}} \right) \left[1\right]$$

Entropy of room: (constant temperature) $\Delta S = C \frac{T_{\text{initial}} - T_{\text{room}}}{T_{\text{room}}}$ [1]

The total change in entropy is positive $\left[\frac{1}{2}\right]$, as it must be for any irreversible process $\left[\frac{1}{2}\right]$

Section B

- B1.** (a) Write down the equation of state for an ideal gas of N particles. Derive an expression for the number density of particles n in terms of pressure and temperature. [3]

Bookwork:

$$PV = Nk_B T \quad [1]$$

$$n = N/V \quad [1]$$

$$n = k_B T / P \quad [1]$$

- (b) State the equipartition theorem and explain carefully what is meant by a degree of freedom. [4]

Bookwork:

The Classical Equipartition Theorem states that, for a system in thermal equilibrium at temperature T , the average energy per particle is $\frac{1}{2}k_B T$ per degree of freedom [2].

A degree of freedom is any dynamical variable which contributes a quadratic term to the total energy of the particle [2].

- (c) List the degrees of freedom for a diatomic molecule. [3]

Bookwork:

Three translational kinetic [1]; two rotational [1]; two vibrational [1].

- (d) Explain why the molar heat capacity at constant volume of the diatomic molecule H_2 is $3R/2$ below 100 K and $5R/2$ above 200 K. [2]

Bookwork:

Below 100 K, thermal energies are not sufficient to excite the rotational degrees of freedom [1], and only translational degrees of freedom are excited [1].

TURN OVER

- (e) 1 mol of H_2 in a container of constant volume increases in temperature from 50 K to 300 K. Using the heat capacities discussed above, calculate the heat supplied by the environment to the gas. [3]

Seen similar:

Constant volume: $dW = 0 \Rightarrow dU = dQ$ [1].

$U_{\text{initial}} = 3R/2 \times 50 \text{ K}$, $U_{\text{final}} = 5R/2 \times 300 \text{ K}$ [1].

$\Rightarrow \Delta Q = 675R = 5.6 \text{ kJ}$ [1].

(Mark for either answer in terms of R or S.I.)

- (f) Calculate the RMS velocities of H_2 at 300 K and at 50 K. [3]

Seen similar:

$$v_{\text{RMS}} = \sqrt{\frac{3k_B T}{m}} \quad [1]$$

50K:

$$v_{\text{RMS}} = \sqrt{\frac{3 \times 1.38 \times 10^{-23} \times 50}{4 \times 1.66 \times 10^{-27}}} \text{ m/s} = 790 \text{ m/s} \quad \left[\frac{1}{2}\right]$$

300K:

$$v_{\text{RMS}} = 1930 \text{ m/s} \quad \left[\frac{1}{2}\right]$$

- (g) Find the ratio of the rate of effusion at the two temperatures given in part (f), noting that the volume remains constant. [2]

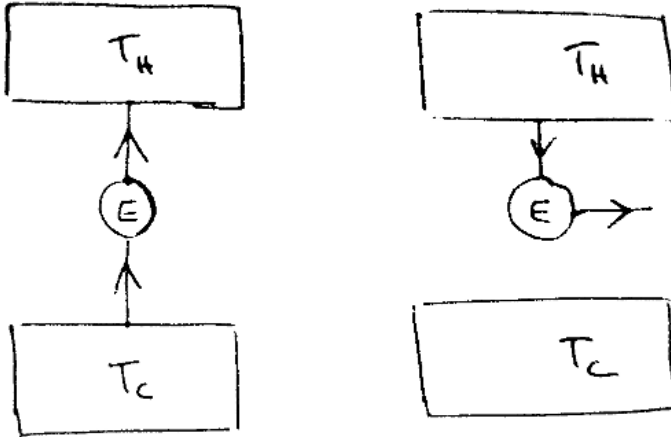
Seen similar:

Effusion flux: $\Phi = \frac{n\langle s \rangle}{4} \propto \sqrt{T}$ (because n is constant) [1]

Hence, ratio is $\sqrt{\frac{300}{50}} = \sqrt{6} = 2.45$ [1]

- B2.** (a) Using **two** heat engine diagrams, describe the two processes which are forbidden according to the Clausius and Kelvin formulations of the Second Law of Thermodynamics. State these two formulations in words. [4]

Bookwork:



Diagrams: [1] + [1].

Clausius: 'No process is possible whose sole result is the transfer of heat from a colder to a hotter body'. [1]

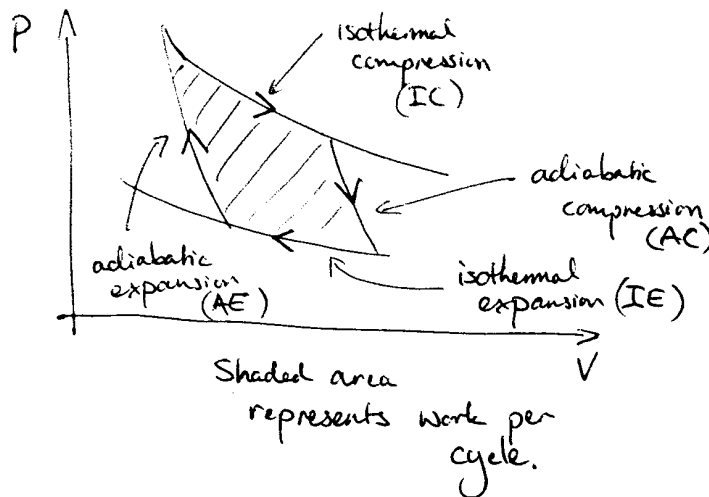
Kelvin: 'No process is possible whose sole result is the complete conversion of heat into work'. [1]

(Marks given for meaning, not for verbatim recall.)

TURN OVER

- (b) (i) Draw a P–V diagram for the Carnot cycle. [1]
- (ii) Label and describe each of the four stages in this cycle. [2]
- (iii) Indicate which feature of the diagram represents the per cycle work. [1]

Bookwork:



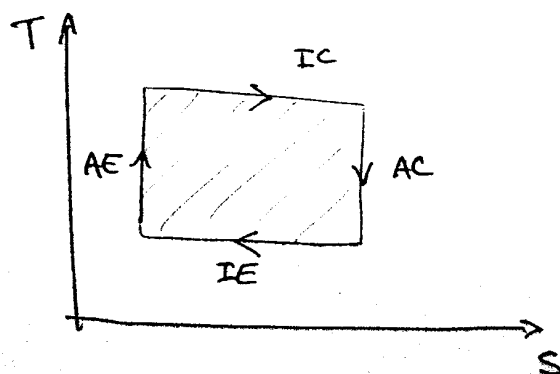
Axes labels [$\frac{1}{2}$]. Cyclic operation described using e.g. arrows [$\frac{1}{2}$].

Following 4 stages identified and which is which indicated on diagram: adiabatic compression [$\frac{1}{2}$], isothermal compression [$\frac{1}{2}$], adiabatic expansion [$\frac{1}{2}$], isothermal expansion [$\frac{1}{2}$].

Area inside loop identified as representing work per cycle [1].

- (iv) Draw a T–S diagram for the Carnot cycle and indicate the correspondence between stages in the P–V diagram. [2]

Bookwork:



Correctly drawn T–S diagram [$\frac{1}{2}$] with axes labels [$\frac{1}{2}$]. Correspondence between all four stages correctly identified [1]

- (c) (i) Show that the work necessary to compress one mole of ideal gas **adiabatically** from volume V_1 and temperature T_1 to volume V_2 is

$$W = \frac{RT_1}{1-\gamma} \left[\left(\frac{V_2}{V_1} \right)^{1-\gamma} - 1 \right].$$

[5]

Bookwork:

Adiabatic: $PV^\gamma = \text{constant} = P_1 V_1^\gamma$ [1]

$$\begin{aligned} W &= \int_{V_1}^{V_2} P dV = P_1 V_1^\gamma \int_{V_1}^{V_2} V^{-\gamma} dV \quad [1] \\ &= \frac{P_1 V_1^\gamma}{1-\gamma} [V^{1-\gamma}]_{V_1}^{V_2} = \frac{P_1 V_1^\gamma}{1-\gamma} (V_2^{1-\gamma} - V_1^{1-\gamma}) \quad [1] \\ &= \frac{P_1 V_1}{1-\gamma} V_1^{\gamma-1} (V_2^{1-\gamma} - V_1^{1-\gamma}) \quad [1] \\ &= \frac{RT_1}{1-\gamma} \left[\left(\frac{V_2}{V_1} \right)^{1-\gamma} - 1 \right] \quad [1] \end{aligned}$$

- (ii) Find an expression for the work necessary to perform the same compression **isothermally** and explain whether the work is larger in the adiabatic or the isothermal case.

[3]

Seen similar:

$PV = RT \Rightarrow P = RT/V$ (constant temperature) [1]

$$W = \int_{V_1}^{V_2} P dV = RT_1 \int_{V_1}^{V_2} \frac{dV}{V} = RT_1 \ln \left[\frac{V_2}{V_1} \right] \quad [1]$$

Work is larger for adiabatic case $[\frac{1}{2}]$ because in isothermal case heat also contributes to increase in internal energy $[\frac{1}{2}]$.

TURN OVER

(iii) For a Joule expansion, describe **qualitatively**

- i. the change in entropy
- ii. the change in temperature.

[2]

Bookwork:

- 1) Entropy increases $[\frac{1}{2}]$ because there are more possible microscopic configurations $[\frac{1}{2}]$
- 2) Temperature stays constant $[\frac{1}{2}]$ because no work is done by the gas $[\frac{1}{2}]$

- B3.** (a) A blackbody spectrum is observed to be maximum at a wavelength of 800 nm and the total power output is measured to be 1 kW. What is the temperature and area of the emitter? [4]

New question:

$$\text{Wien's law: } T = 2.9 \times 10^{-3} \text{ m K} / \lambda_{\text{max}} = 3625 \text{ K} \quad [2]$$

$$\text{Stefan-Boltzmann law: } P/A = \sigma T^4 \quad [1]$$

$$\Rightarrow A = P/(\sigma T^4) = 1.02 \times 10^{-6} \text{ m}^2 \quad [1]$$

- (b) The luminosity of the Sun is $L = 3.85 \times 10^{26} \text{ W}$. Treating Pluto (radius $r = 1.2 \times 10^6 \text{ m}$) as a blackbody in equilibrium with the Sun, estimate its temperature at closest approach to the Sun ($d = 4.4 \times 10^{12} \text{ m}$). [6]

Seen similar:

$$\text{Absorbing area of Pluto: } \pi r^2 \quad [\frac{1}{2}]$$

$$\text{Emitting area of Pluto: } 4\pi r^2 \quad [\frac{1}{2}]$$

$$\text{Solar intensity at Pluto: } \frac{L}{4\pi d^2} \quad [1]$$

$$\text{Blackbody power emitted by Pluto: } 4\pi r^2 \sigma T^4 \quad [1]$$

Equating:

$$4\pi r^2 \sigma T^4 = \frac{L}{4\pi d^2} \pi r^2 \quad [1]$$

Hence:

$$T = \left(\frac{L}{16\pi d^2 \sigma} \right)^{1/4} \quad [1] = 51 \text{ K} \quad [1]$$

- (c) Explain what it means for entropy to be a function of state and write down the expression the change in entropy for a system at temperature T when a small amount of heat is added in a reversible process. [2]

Bookwork:

Entropy being a function of state means it represents a property of the system, regardless of how that state was reached [1]

$$dS = \frac{dQ}{T} \quad [1]$$

TURN OVER

- (d) State the Fundamental Equation of Thermodynamics and show that the differential change in entropy of an ideal gas is $dS = c_V \frac{dT}{T} + R \frac{dV}{V}$ [3]

Bookwork:

$$dU = TdS - PdV \quad [1]$$

$$\text{Using } dU = c_V dT \text{ and } PV = RT \quad [1]$$

$$dS = \frac{1}{T} dU + \frac{P}{T} dV = c_V \frac{dT}{T} + R \frac{dV}{V} \quad [1]$$

- (e) Heat is delivered to a heat engine operating between $T_H = 500 \text{ K}$ and $T_C = 300 \text{ K}$ via a cylindrical steel rod (thermal conductivity $\kappa = 400 \text{ W m}^{-1} \text{ K}^{-1}$) of area $A = 0.1 \text{ m}^2$ and length $L = 1 \text{ m}$. What is the maximum rate at which the engine can do **work**? [5]

New question:

$$\text{Fourier's law: } \frac{dQ}{dt} = A\kappa \frac{dT}{dx} \quad [1]$$

$$\frac{dT}{dx} = 200 \text{ K/m} \quad \left[\frac{1}{2}\right]$$

$$\Rightarrow \frac{dQ}{dt} = (0.1 \times 400 \times 200) \text{ W} = 8 \text{ kW} \quad [1]$$

$$\text{Engine maximum efficiency: } \eta = 1 - \frac{T_C}{T_H} \quad [1] = 0.4 \quad \left[\frac{1}{2}\right]$$

$$\text{Maximum work output: } W = \eta \frac{dQ}{dt} = 3.2 \text{ kW} \quad [1]$$

B4. Throughout this question you may assume the gas is ideal.

- (a) Starting with the availability A , find the appropriate thermodynamic potential for a process which is open to and in good thermal contact with the environment, and identify the name given to this potential from the list at the start of the examination paper. [4]

Seen similar:

$$P = P_0 \text{ and } T = T_0 \text{ which implies } dP = 0 \text{ and } dT = 0 \text{ [1]}$$

$$\text{Hence } dA = dU - TdS + PdV = d(U - TS + PV) \text{ [1]}$$

$$\text{Identify } G = U - TS + PV \text{ [1] as 'Gibbs free energy' [1]}$$

- (b) The molar heat capacity of a gas **at constant pressure** is observed to be $c_p = 5R/2$, independent of temperature. Find the molar heat capacity at constant volume for this gas, and state the type of gas this is likely to be. [2]

Seen similar:

$$c_v = c_p - R = 3R/2 \text{ [1]}$$

This is likely to be a monatomic gas [1]

- (c) One mole of monatomic gas undergoes Joule expansion from volume V_1 to $V_2 > V_1$. It is then compressed isothermally back to V_1 . Show that the work done during this compression is $W = RT \ln [V_2/V_1]$. [3]

Seen similar:

$$dW = -PdV \text{ [1]}$$

$$P = RT/V \text{ [1]}$$

$$W = -RT \int_{V_2}^{V_1} \frac{dV}{V} = +RT \ln [V_2/V_1] \text{ [1]}$$

- (d) By noting that the compression is isothermal and the gas is ideal, write down an expression for the heat flow into the gas during this compression. [2]

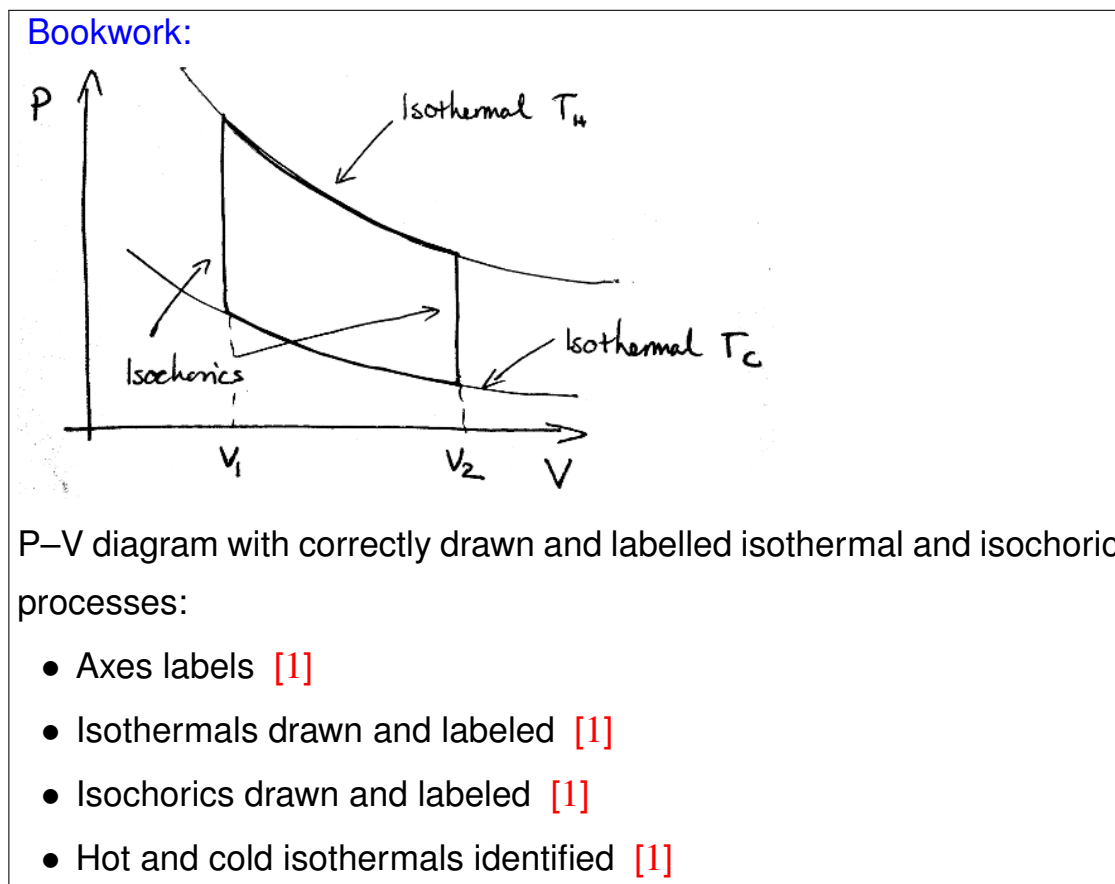
Bookwork:

$$\text{Isothermal } dU = dQ + dW = 0 \text{ [1]}$$

$$\Rightarrow \text{Heat flow in } Q = +RT \ln [V_2/V_1] \text{ [1]}$$

TURN OVER

- (e) A Stirling cycle uses isothermal and isochoric processes between volumes V_1 and V_2 and using reservoirs at temperatures T_H and T_C . Sketch and label a P–V diagram for this cycle [4]



- (f) Explain why the heat entering during isochoric heating is exactly equal to the heat leaving during isochoric cooling. [2]

Bookwork:

No work is done, so $dU = dQ$ [1]

For ideal gas, $\Delta U = c_V \Delta T$ and isochoric processes are between the same isotherms [1]

- (g) Define efficiency in terms of heat and work, and use the results in this B question to derive an expression for the efficiency of a Stirling engine. [3]

Seen similar:

$$\text{Efficiency } \eta = \frac{W}{Q_H} \text{ [1]}$$

$$Q_H = RT_H \ln [V_2/V_1] \text{ [}\frac{1}{2}\text{]}$$

$$W = RT_H \ln [V_2/V_1] - RT_C \ln [V_2/V_1] \text{ [}\frac{1}{2}\text{]}$$

$$\Rightarrow \eta = \frac{T_H - T_C}{T_H} = 1 - \frac{T_C}{T_H} \text{ [1]}$$

END OF PAPER