SEMESTER 2 EXAMINATION 2014-2015

ENERGY AND MATTER

Duration: 120 MINS (2 hours)

This paper contains 8 questions.

Answers to Section A and Section B must be in separate answer books

Answer **all** questions in **Section A** and **only two** questions in **Section B**.

Section A carries 1/3 of the total marks for the exam paper and you should aim to spend about 40 mins on it.

Section B carries 2/3 of the total marks for the exam paper and you should aim to spend about 80 mins on it.

An outline marking scheme is shown in brackets to the right of each question.

A Sheet of Physical Constants is provided with this examination paper.

Only university approved calculators may be used.

A foreign language word to word® translation dictionary (paper version) is permitted provided it contains no notes, additions or annotations.

Useful formulae

Conversions

$0\,\mathrm{K} = -273.15\,^\circ\mathrm{C}$

$1\,\mathrm{atm} = 101\,325\,\mathrm{Pa} = 760\,\mathrm{Torr}$

$1\,\mathrm{Pa} = 10^{-5}\,\mathrm{bar} = 9.8686 \times 10^{-6}\,\mathrm{atm} = 7.5 \times 10^{-3}\,\mathrm{Torr}$

Thermodynamic potentials

Internal Energy: U

Enthalpy: $H = U + PV$

Helmhotz Free Energy: $F = U - TS$

Gibbs Free Energy: $G = U - TS + PV$

Availability: $A = U - T_0 S + P_0 V$

Atomic weights

Hydrogen: 1

Helium: 4

Nitrogen: 14

Section A

A1. The International Space Station has an internal volume $V = 916\,\text{m}^3$ and is pressurised to $P = 101.3\,\text{kPa}$ at a temperature of $20°\text{C}$. What is the total mass and RMS velocity of the air contained (assuming air is molecular nitrogen N_2 which behaves as an ideal gas)? [6]

A2. An ideal gas at pressure P_1, volume V_1 and temperature T_1 undergoes a free (Joule) expansion to volume V_2. Find the new temperature T_2 and pressure P_2. Define the terms 'adiabatic' and 'isochoric' and state whether either of these can be used to describe the Joule expansion. [4]

A3. A reversible heat engine is used as a heat pump to run a domestic freezer. It consumes $20\,\text{W}$ of electrical power and maintains the cold reservoir at $-10°\text{C}$ in a room at $+20°\text{C}$. Draw a heat engine diagram depicting the flow of heat and work, find the efficiency of this engine, and calculate the rate at which it extracts heat from the cold reservoir. [5]

A4. A cup of water at T_{initial} cools to room temperature T_{room}. Derive an expression for the change in entropy of 1) the water, and 2) the room. Comment on the sign of the total change in entropy. [5]

Section B

B1. (a) Write down the equation of state for an ideal gas of N particles. Derive an expression for the number density of particles n in terms of pressure and temperature. [3]

(b) State the equipartition theorem and explain carefully what is meant by a degree of freedom. [4]

(c) List the degrees of freedom for a diatomic molecule. [3]

(d) Explain why the molar heat capacity at constant volume of the diatomic molecule H_2 is $3R/2$ below $100\,K$ and $5R/2$ above $200\,K$. [2]

(e) $1\,mol$ of H_2 in a container of constant volume increases in temperature from $50\,K$ to $300\,K$. Using the heat capacities discussed above, calculate the heat supplied by the environment to the gas. [3]

(f) Calculate the RMS velocities of H_2 at $300\,K$ and at $50\,K$. [3]

(g) Find the ratio of the rate of effusion at the two temperatures given in part (f), noting that the volume remains constant. [2]

B2. (a) Using **two** heat engine diagrams, describe the two processes which are forbidden according to the Clausius and Kelvin formulations of the Second Law of Thermodynamics. State these two formulations in words.   [4]

   (b) (i) Draw a P–V diagram for the Carnot cycle.   [1]

       (ii) Label and describe each of the four stages in this cycle.   [2]

      (iii) Indicate which feature of the diagram represents the per cycle work.   [1]

      (iv) Draw a T–S diagram for the Carnot cycle and indicate the correspondence between stages in the P–V diagram.   [2]

   (c) (i) Show that the work necessary to compress one mole of ideal gas **adiabatically** from volume V_1 and temperature T_1 to volume V_2 is

$$W = \frac{RT_1}{1-\gamma} \left[\left(\frac{V_2}{V_1} \right)^{1-\gamma} - 1 \right].$$

   [5]

      (ii) Find an expression for the work necessary to perform the same compression **isothermally** and explain whether the work is larger in the adiabatic or the isothermal case.   [3]

     (iii) For a Joule expansion, describe **qualitatively**

       i. the change in entropy

      ii. the change in temperature.

   [2]

TURN OVER

B3. (a) A blackbody spectrum is observed to be maximum at a wavelength of 800 nm and the total power output is measured to be 1 kW. What is the temperature and area of the emitter? [4]

(b) The luminosity of the Sun is $L = 3.85 \times 10^{26}$ W. Treating Pluto (radius $r = 1.2 \times 10^6$ m) as a blackbody in equilibrium with the Sun, estimate its temperature at closest approach to the Sun ($d = 4.4 \times 10^{12}$ m). [6]

(c) Explain what it means for entropy to be a function of state and write down the expression the change in entropy for a system at temperature T when a small amount of heat is added in a reversible process. [2]

(d) State the Fundamental Equation of Thermodynamics and show that the differential change in entropy of an ideal gas is $dS = c_V \frac{dT}{T} + R \frac{dV}{V}$ [3]

(e) Heat is delivered to a heat engine operating between $T_H = 500$ K and $T_C = 300$ K via a cylindrical steel rod (thermal conductivity $\kappa = 400$ W m^{-1} K^{-1}) of area $A = 0.1$ m^2 and length $L = 1$ m. What is the maximum rate at which the engine can do **work**? [5]

B4. Throughout this question you may assume the gas is ideal.

(a) Starting with the availability A, find the appropriate thermodynamic poten-
tial for a process which is open to and in good thermal contact with the
environment, and identify the name given to this potential from the list at
the start of the examination paper. [4]

(b) The molar heat capacity of a gas **at constant pressure** is observed to be
$c_p = 5R/2$, independent of temperature. Find the molar heat capacity at
constant volume for this gas, and state the type of gas this is likely to be. [2]

(c) One mole of monatomic gas undergoes Joule expansion from volume V_1
to $V_2 > V_1$. It is then compressed isothermally back to V_1. Show that the
work done during this compression is $W = RT \ln [V_2/V_1]$. [3]

(d) By noting that the compression is isothermal and the gas is ideal, write
down an expression for the heat flow into the gas during this compression. [2]

(e) A Stirling cycle uses isothermal and isochoric processes between volumes
V_1 and V_2 and using reservoirs at temperatures T_H and T_C. Sketch and
label a P–V diagram for this cycle [4]

(f) Explain why the heat entering during isochoric heating is exactly equal to
the heat leaving during isochoric cooling. [2]

(g) Define efficiency in terms of heat and work, and use the results in this B
question to derive an expression for the efficiency of a Stirling engine. [3]

END OF PAPER