SEMESTER 2 EXAMINATION 2012-2013

Energy & Matter

Duration: 120 MINS (2 hours)

This paper contains 9 questions.

Answers to Section A and Section B must be in separate answer books

Answer **all** questions in **Section A** and **only two** questions in **Section B**.

Section A carries 1/3 of the total marks for the exam paper and you should aim to spend about 40 mins on it.

Section B carries 2/3 of the total marks for the exam paper and you should aim to spend about 80 mins on it.

An outline marking scheme is shown in brackets to the right of each question.

A Sheet of Physical Constants is provided with this examination paper.

Only university approved calculators may be used.

A foreign language translation dictionary (paper version) is permitted provided it contains no notes, additions or annotations.

Section A

A1. The radius of the Sun is 7.0×10^5 km and its surface temperature is approximately 6000 K. Treating Mars as a black body in thermal equilibrium with the radiation from the Sun, estimate the mean surface temperature of Mars at its distance of closest approach to the Sun of 2.1×10^8 km. [4]

A2. The rate at which a certain chemical reaction proceeds increases from R_0 to $10\,R_0$ when the temperature is raised from $0°$C to $20°$C. Determine the rate (in terms of R_0) at $40°$C. [4]

A3. Thermodynamic systems which undergo changes may be subject to a number of conditions. Explain briefly what is meant if

(a) A change is described as isothermal; [1]

(b) A change is described as adiabatic; [1]

(c) A system is said to be in thermal equilibrium. [2]

A4. A cubic metre of air at 0°C and 1 atm is compressed reversibly to 10 atm.

(a) What is meant by a reversible compression? [1]

(b) What is the final temperature if it is compressed adiabatically? [3]

(You may assume that air behaves as an ideal gas.)

A5. Calculate the change in entropy of the Universe as a result of the following operations:

(a) A copper block of mass 0.8 kg and thermal capacity 80 J K^{-1} at 100°C is placed in a lake at 10°C. [2]

(b) The same block at 10°C is dropped from a height of 50 m into the lake. [2]

Section B

B1. (a) Explain how a $p - V$ diagram can be used to show graphically the cycle followed by the working gas in a heat engine. How is the work per cycle done by the engine represented on the indicator diagram? [4]

(b) A 4-stroke car engine may be modelled by a cycle of 4 processes:

1) gas is compressed adiabatically from initial volume V_1 to final volume V_2;

2) gas is heated at constant volume V_2;

3) gas expands adiabatically back to volume V_1;

4) gas is cooled at constant volume back to its initial temperature and pressure.

Sketch this cycle on a $p - V$ diagram, with arrows to show in which direction the changes occur. Label the stroke in which the fuel is ignited, and the power stroke in which the engine delivers work. [4]

(c) Prove that if the working substance in this engine follows the ideal gas law, the ratio of the net work W done by the engine per cycle to the heat Q_1 absorbed by the working substance on the ignition stroke is given by

$$\frac{W}{Q_1} = 1 - \left(\frac{V_2}{V_1}\right)^{\gamma - 1}$$

where γ is the specific heat capacity ratio of the gas. You may assume that in an adiabatic expansion or compression, the pressure p and volume V of an ideal gas obey the rule

$$pV^{\gamma} = \text{constant}.$$

[8]

(d) State what is meant by the compression ratio of a 4-stroke engine, and say how the size of the compression ratio influences the performance of the engine. [4]

TURN OVER

B2. (a) Summarise three of the basic assumptions of the kinetic theory of gases. [3]

(b) The molecular flux J and the pressure of an ideal gas p are given by $J = \frac{1}{4}n\langle v\rangle$ and $p = \frac{1}{3}nm\langle v^2\rangle$. Define all the terms in these expressions. [2]

Explain through logical argument why p is proportional to $nm\langle v^2\rangle$. [2]

(c) For a Maxwell-Boltzmann distribution of molecular speeds, how do $\langle v\rangle$ and $\langle v^2\rangle$ depend on the temperature T of the gas? [2]

(d) Two chambers containing the same gas are kept in equilibrium at temperatures T_1 and T_2, respectively. They are connected by a very small hole, so that particles can pass between the chambers without scattering, and the volumes of the two chambers are equal. Calculate p_1/p_2, the ratio of pressures in the two chambers, stating clearly any assumptions you make. [5]

(e) In no more than 3-4 sentences each, write brief notes on **2** of the following: [6]

(i) the Boltzmann factor;

(ii) the equipartition theorem;

(iii) the zeroth law of thermodynamics.

B3. (a) Explain why for n moles of gas the heat capacity at constant pressure (c_p) is always greater than the heat capacity at constant volume (c_v). [2]

(b) Starting from the first law of thermodynamics, show that $c_p - c_v = nR$ for n moles of ideal gas, where R is the universal gas constant. [4]

(c) Consider n moles of gas with entropy

$$S = \frac{n}{2}\left[\sigma + 5R\ln\frac{U}{n} + 2R\ln\frac{V}{n}\right],$$

where U is the internal energy, V is the volume and σ is a constant.

Re-write this expression for entropy in terms of specific heat capacity at constant volume. [3]

By considering specific heat capacity as a function of entropy, use this relationship to determine c_v and hence c_p for this gas in terms of the universal gas constant, R. [5]

(d) The air inside a house with open windows is initially in thermal equilibrium with the air outside the house at 273 K. Some time after the heating in the house is turned on, the temperature of the air inside the house reaches an equilibrium value of 300 K. The air outside the house remains at 273 K. You may assume that the air inside and outside the house behaves as an ideal gas.

In this new equilibrium, which two thermodynamic variables will be the same for the air inside and outside the house? [2]

Hence, using the entropy S given above, derive an expression which relates the molar internal energy inside and outside the house to the molar volume inside and outside the house. [2]

By converting molar energy to energy per unit volume, determine how the energy density of the air inside the house compares at the two temperatures. [2]

TURN OVER

B4. For one mole, the van der Waals' equation of state is

$$\left(p + \frac{a}{V^2}\right)(V - b) = RT.$$

(a) Discuss the physical significance of the parameters a and b and explain why the correction to the pressure is inversely proportional to the square of the volume. [4]

(b) A gas obeying the van der Waals' equation of state undergoes an isothermal expansion from volume V_1 to volume V_2.

Show that the change in the Helmholtz free energy, $F = U - TS$ is given by $-RT/(V - b)$. [5]

Show that the corresponding change in the internal energy U is given by $a(V_2 - V_1)/V_1 V_2$. [3]

(c) Sketch a $p - V$ curve for the gas at its critical temperature. Mark the critical point on the curve. [2]

(d) Using the properties of this curve at the critical point and van der Waals' equation of state as stated above, evaluate the dimensionless quantity pV/RT at the critical point. [6]

END OF PAPER