SEMESTER 1 EXAMINATION 2013-2014

MOTION AND RELATIVITY

Duration: 120 MINS (2 hours)

This paper contains 9 questions.

Answers to Section A and Section B must be in separate answer books

Answer **all** questions in **Section A** and **only two** questions in **Section B**.

Section A carries 1/3 of the total marks for the exam paper and you should aim to spend about 40 mins on it.

Section B carries 2/3 of the total marks for the exam paper and you should aim to spend about 80 mins on it.

An outline marking scheme is shown in brackets to the right of each question.

A Sheet of Physical Constants is provided with this examination paper.

Only university approved calculators may be used.

A foreign language translation dictionary (paper version) is permitted provided it contains no notes, additions or annotations.

Section A

In order to gain full credit you *must* show your working throughout.

A1. A horizontal force of 100 N pushes a 10 kg block up a frictionless incline that makes an angle of 30° to the horizontal. List all forces acting on the block and sketch them in a diagram. What is the acceleration of the block up the incline? You may assume that the apparatus is located near the Earth's surface. [4]

A2. Mars has a mass of $6.4 \times 10^{23}\,\mathrm{kg}$ and a radius of 3400 km. If a projectile is launched vertically from the surface of Mars at a speed of 6 km/s, will it escape the gravitational pull? [4]

A3. A mass attached to a spring is slowly pulled 50 cm away from its equilibrium position and then released. When the mass reaches the equilibrium position, it is travelling at a speed of 2.3 m/s. What is the frequency of oscillation? [4]

A4. As a result of an incident cosmic ray, a muon is created in the upper atmosphere, travelling at a speed 0.9995c. If a muon at rest takes 2.2 μs to decay, how long would the decay take from the perspective of an observer on the Earth's surface? If the muon is created at a height of 15 km, can the muon reach the Earth's surface? [4]

A5. A rocket 20 m long approaches a barn 12 m long at a speed of 0.9c. Does the rocket ever fit entirely in the barn (a) from the perspective of an observer in the barn? (b) from the perspective of an observer in the rocket? [4]

Section B

B1. (a) The work done by a force F in displacing a particle of mass m from x_i to x_f is given by

$$W_{i\to f} = \int_{x_i}^{x_f} F dx \,.$$

(i) Using Newton's second law, which relates the force to the rate of change of momentum of the particle, show that

$$W_{i\to f} = \frac{1}{2}mv_f^2 - \frac{1}{2}mv_i^2 \,,$$

where v_f and v_i are the final and initial velocities respectively. [6]

(ii) State the work-energy theorem. [2]

(iii) How is the work done by a machine related to its power as a function of time? [2]

(b) A 1600 kg car is travelling at 10 m/s. The car then accelerates with the engine operating at a power of 80 kW for 3 seconds.

(i) How much work is done by the engine? [2]

(ii) Neglecting inefficiency of the engine and frictional or drag forces, calculate the final speed obtained. [4]

(iii) If the same car brakes from a speed of 10 m/s to a halt over a distance of 10 m, calculate the force applied by the brakes, assuming it to be constant. [4]

TURN OVER

B2. (i) A particle of mass m is attached to a spring which exerts a restoring force $-kx$ when the mass is displaced by x from its equilibrium position. Assuming there are no other forces acting on the particle, write down the differential equation describing the resulting simple harmonic motion as a function of time. [2]

Now suppose that in addition, a damping force $-bv$ is applied, where v is the velocity of the particle. Write down the differential equation describing the damped motion. Describe the behaviour of the system in each of the following cases: (a) underdamping, (b) critical damping, (c) overdamping. Describe what is meant by *weak damping*. [7]

(ii) Assume a car's suspension acts like a weakly damped mass-spring system with m = 1600 kg and spring constant, $k = 6.25 \times 10^4$ N/m. The car's shock absorbers provide a damping constant, b = 245 kg/s. After the car hits a pothole, calculate how many oscillations it will make before the amplitude drops to half its initial value? [7]

Note: You may find it helpful to use the fact that weakly damped simple harmonic motion can be described by the following equation,

$$x(t) = A_0 \exp\left\{\frac{-bt}{2m}\right\} \cos(\omega t),$$

where A_0 is the initial amplitude, b is the damping constant and m the mass of the object.

(iii) Give an approximate expression in terms of A_0, b, m, ω for the total mechanical energy of the system as a function of time. [2]

How many oscillations does the car above make before the energy of oscillation drops to half its initial value? [2]

B3. (a) (i) Define the relativistic energy E and relativistic momentum p of a particle in terms of its velocity v, its rest mass m and the speed of light c.

(ii) Hence show that

$$E^2 - p^2c^2 = m^2c^4 \, .$$

[7]

(iii) Explain how the particle's energy is related to its kinetic energy and rest energy. [2]

(b) A subatomic particle called a Kaon with symbol K^0 has a rest mass given by $m_K = 498\,\mathrm{MeV}/c^2$. It can decay into a muon-anti-muon pair,

$$K^0 \longrightarrow \mu^+ + \mu^- \, .$$

The muons μ^+, μ^- each have a rest mass of $m_\mu = 106\,\mathrm{MeV}/c^2$.

(i) Working in the rest frame of the Kaon, find the kinetic energy and relativistic momentum of one of the muons after the decay. [7]

(ii) What is the speed of the muons as a fraction of the speed of light? [4]

TURN OVER

B4. (i) The Special Theory of Relativity is based on two postulates. What are they? What is meant by an inertial frame? What is meant by an event? [6]

(ii) An inertial frame S' is moving with velocity v along the x-axis of an inertial frame S. Write down the Lorentz transformations which relate the coordinates x', y', z', t' of S' to the coordinates x, y, z, t of S (assuming the two coordinate frames coincide at $t = t' = 0$). [6]

(iii) Two civilisations are evolving on opposite sides of a galaxy whose diameter is 10^5 light years. At time $t = 0$ in the galaxy frame of reference, civilisation **A** launches its first interstellar spacecraft. Civilisation **B** launches its first spacecraft 60,000 years later. A being from a more advanced civilisation is passing through the galaxy at $0.95c$ on a line from **A** to **B**. Which civilisation does this being judge to have first achieved interstellar travel? [8]

END OF PAPER