## SEMESTER 1 EXAMINATION 2012/13

MOTION AND RELATIVITY
Duration: 120 MINS

## VERY IMPORTANT NOTE

Section A answers MUST BE in a separate blue answer book. If any blue answer booklets contain work for both Section $A$ and $B$ questions - the latter set of answers WILL NOT BE MARKED.

Answer all questions in Section A and two and only two questions in Section B.

Section A carries $1 / 3$ of the total marks for the exam paper and you should aim to spend about 40 mins on it. Section B carries $2 / 3$ of the total marks for the exam paper and you should aim to spend about 80 mins on it.

A Sheet of Physical Constants will be provided with this examination paper. An outline marking scheme is shown in brackets to the right of each question. Only university approved calculators may be used.

## Section A

A1. A jet aircraft flies at $960 \mathrm{~km} / \mathrm{h}$ relative to the air. It needs to travel 1290 km northwards. At cruising altitude a wind is blowing eastward at $190 \mathrm{~km} / \mathrm{h}$. In what direction should the plane fly? How long will the trip take?

A2. A raging flood has washed away a section of the road, creating a wide gully 1.7 m deep. A car moving at $31 \mathrm{~m} / \mathrm{s}$ goes straight over the edge. Considering the car as a point mass, how far from the edge of the gully does it land?

A3. A rocket is launched vertically upwards at $3.1 \mathrm{~km} / \mathrm{s}$. How high above the Earth's surface does it go? Hint : you cannot assume that the acceleration due to gravity is a constant throughout the flight.

A4. At the Stanford Linear Accelerator Center subatomic particles are accelerated to high energies over a straight path whose length, in Earth's frame, is 3.2 km . For an electron travelling at $99.99995 \%$ of the speed of light how long does the trip take as measured in (a) the Earth's frame, and (b) the electron's frame? (c) What is the length of the accelerator in the electron's frame?

A5. An electron has a rest energy of 0.511 MeV and a total energy of 2.50 MeV . Find
(a) its kinetic energy, and (b) its speed.

## Section B

B1. Explain the meaning and differences between inertial mass and gravitational mass. Gravity is described as a central force - what does this imply?
(i) An astronaut whose height is 1.7 m floats feet down in an orbiting space station. at a distance of $6.77 \times 10^{6} \mathrm{~m}$ from the centre of the Earth (note that "down" means feet pointing towards the centre of the earth). What is the difference in gravitational acceleration between her feet and the top of her head? Hint: think about whether your calculator has enough significant digits to handle the difference directly.
(ii) If the same astronaut is now feet down at the same orbital distance from the centre of a black hole of mass $1.99 \times 10^{31} \mathrm{~kg}$ what will now be the difference in the gravitational acceleration between her head and feet?
(iii) Consider a pulsar with a mass equal to that of the Sun, but a radius of only 12 km , which rotates with a period of 0.041 s . At its equator, by what percentage does the free-fall acceleration differ from the gravitational acceleration?

B2. The work done by a force $F$ in displacing a particle of mass $m$ from $x_{i}$ to $x_{f}$ is given by

$$
W_{i \rightarrow f}=\int_{x_{i}}^{x_{f}} F \mathrm{~d} x
$$

Using Newton's second law, which relates the force to the rate of change of momentum of the particle, show that

$$
W_{i \rightarrow f}=\frac{1}{2} m v_{f}^{2}-\frac{1}{2} m v_{i}^{2}
$$

where $v_{f}$ and $v_{i}$ are the final and initial velocities respectively of the particle of mass $m$.

State the work-energy theorem.
How is the work done by a machine related to its power as a function of time?

A 1600 kg car accelerates from 60 to $80 \mathrm{~km} / \mathrm{h}$.

- How much work is done on the car?
- If the car then brakes to a stop, how much work is done on it?
- An identical car was travelling at $80 \mathrm{~km} / \mathrm{h}$ and crashed into the rear end of a stationary car. Fortunately, the driver was wearing a seat-belt. Using reasonable values for the mass of the driver and the stopping distance (i.e. the distance moved after impact) estimate the force (assuming it to be constant) exerted on the driver by the seat belt.

B3. The Special Theory of Relativity is based on two postulates. What are they? What is meant by an inertial frame? What is meant by an event?

A starship is approaching a planet-moon system on a straight line that will take it first past the moon and then past the planet. As it approaches, it detects a microwave burst from the moon and, 1.10 s later, an explosion on the planet. The planet is $4.00 \times 10^{8} \mathrm{~m}$ from the moon as measured in the ship's reference frame.
(i) The speed of the ship relative to the planet and its moon is 0.980 c. What is the distance between the burst and the explosion, and the time interval between them, both measured in the planet-moon inertial frame?
(ii) What can you say about the sequence of events as seen in the planet-moon system compared to the sequence seen from the starship?
(iii) Is it possible that the burst caused the explosion, or the explosion caused the burst, or neither?

B4. (i) Electrons in a particle accelerator reach a speed of 0.999 c relative to the laboratory. One collision of an electron with a target produces a muon that moves forward with a speed of 0.95 c relative to the laboratory. The muon's mass is $1.90 \times 10^{-28} \mathrm{~kg}$.

What is the muon's momentum in the laboratory frame and in the frame of the electron beam?
(ii) It is often observed in proton-proton ( $\mathrm{p}-\mathrm{p}$ ) scattering at relativistic energies that more particles come out than went in: in particular, particles called pions $\left(\pi^{+}, \pi^{-}, \pi^{0}\right)$ can be created. A possible scenario is:
$\mathrm{p}+\mathrm{p} \rightarrow \mathrm{p}+\mathrm{p}+\pi^{0}$.
The neutral-pion mass is $135 \mathrm{MeV} / \mathrm{c}^{2}$ and the mass of the proton is $938 \mathrm{MeV} / \mathrm{c}^{2}$. Calculate the incoming velocities of the protons in the centre of mass frame needed to create a $\pi^{0}$ (Hint: assume that the least possible energy is that which is necessary to produce the final particles at rest).
(iii) In the laboratory frame one of the initial protons is at rest. Calculate, in this frame, the minimum kinetic energy of the other initial proton needed to create the $\pi^{0}$.

