SEMESTER 1 EXAMINATION 2014-2015
MOTION AND RELATIVITY
Duration: 120 MINS (2 hours)

This paper contains 9 questions.

## Answers to Section A and Section B must be in separate answer books

Answer all questions in Section A and only two questions in Section B.

Section A carries $1 / 3$ of the total marks for the exam paper and you should aim to spend about 40 mins on it.

Section B carries $2 / 3$ of the total marks for the exam paper and you should aim to spend about 80 mins on it.

An outline marking scheme is shown in brackets to the right of each question.

A Sheet of Physical Constants is provided with this examination paper.
Only university approved calculators may be used.

A foreign language translation dictionary (paper version) is permitted provided it contains no notes, additions or annotations.

## Section A

## In order to gain full credit you must show your working throughout.

A1. An 800 g mass on a spring is oscillating at 1.4 Hz . The total energy of the oscillation is 0.81 J . What is the amplitude of oscillation?

A2. A skydiver falls through the air towards the ground with a constant speed of $192 \mathrm{~km} / \mathrm{hr}$ (the terminal velocity). If his mass is 70 kg , calculate the power of the drag force.

A3. An electron has a rest energy of 0.511 MeV and a total energy of 1.50 MeV . Find (a) its kinetic energy in MeV , and (b) its speed as a fraction of the speed of light.

A4. Give the expression for the magnitude of the gravitational force acting on an object of mass $m$ as a function of its distance $r$ away from from the centre of a planet of mass $M$. Derive an expression for the gravitational potential energy $U(r)$ of the object, assuming that the potential energy vanishes at infinite separation.

A5. Excited Hydrogen atoms emit light in an identifiable pattern called the Balmer series. In the light observed from a galaxy, the first Balmer emission appears at a wavelength of $7.18 \times 10^{-7} \mathrm{~m}$. The same emission is observed from Hydrogen atoms on Earth at a wavelength of $6.56 \times 10^{-7} \mathrm{~m}$. Using the relativistic Doppler formula, determine the velocity of recession of the galaxy.

## Section B

B1. (a) In the context of a collision between two bodies, explain what is meant by the terms elastic, inelastic and totally inelastic.
(b) Two billiard balls sit on a horizontal surface next to a smooth incline of height $h$ as shown in the figure below. Ball 1 with mass $m_{1}$ approaches Ball 2 (initially stationary) with an initial velocity $u$ in the positive $x$ direction. It then makes an elastic collision with Ball 2 (mass $m_{2}$ ).
(i) What is the speed of Ball 1 immediately after the collision and in which direction is it moving?
(ii) What is the kinetic energy of Ball 2 immediately after the collision?
(iii) What is the maximum height that Ball 2 reaches up the incline after the collision, assuming it does not get to the top?
(c) If the collision had been totally inelastic, what initial velocity would be required for both balls to reach the top of the incline after the collision?


B2. (a) A cannon fires a cannonball at an initial speed $v_{0}$ and elevation angle $\theta$. Neglecting any effects due to air resistance and starting from Newton's second law, derive expressions for the vertical and horizontal distances travelled as a function of time.
(b) Derive an expression for the time of flight of the cannonball.
(c) A 130 m high hill is located halfway between the cannon and its target.
(i) If the cannonball is fired at an elevation angle of $\pi / 6$ and just clears the hill to strike the target, calculate the initial velocity and the distance from the cannon to the target.
(ii) The cannon is then moved to the top of the hill and fires a cannonball horizontally with the same initial speed. What is the horizontal distance travelled when the cannonball hits the ground?

B3. (a) A beam of electrons is accelerated reach a speed of $0.995 c$ relative to the laboratory. One of the electrons collides with a target producing a muon moving in the same direction as the electron beam with a speed of $0.9 c$ relative to the laboratory. The rest mass of the muon is $105.7 \mathrm{MeV} / c^{2}$.
(i) What is the muon's momentum in the laboratory frame?
(ii) What is the muon's momentum in the frame of the electron beam?
(b) In proton-proton scattering, particles called pions $\left(\pi^{+}, \pi^{-}, \pi^{0}\right)$ can be created. A possible scenario is:
$p+p \rightarrow p+p+\pi^{0}$.
The mass of the neutral pion $\left(\pi^{0}\right)$ is $135 \mathrm{MeV} / c^{2}$ and the mass of the proton is $938 \mathrm{MeV} / \mathrm{c}^{2}$.
(i) Working In the centre of mass frame, calculate how much energy each incoming proton needs to create a $\pi^{0}$ and the corresponding incoming velocities (Hint: assume that the least possible energy is that which is necessary to produce the final particles at rest).
(ii) Now consider a frame in which one of the protons is at rest. Calculate, in this frame, the minimum kinetic energy of the other proton needed to create the $\pi^{0}$.

B4. (a) The Special Theory of Relativity is based on two postulates. What are they? What is meant by an inertial frame?
(b) A lunar lander is moving horizontally with velocity $v$ near the surface of the moon. We denote the inertial frame of an astronaut standing on the surface by $S$ and that of the lander by $S^{\prime}$. If the lander is moving along the $x$ axis of the frame $S$, write down the Lorentz transformations that relate the coordinates $x^{\prime}, y^{\prime}, z^{\prime}$ and time $t^{\prime}$ in $S^{\prime}$ to coordinates $x, y, z$ and time $t$ as measured in $S$ (assuming that the two coordinate frames coincide at $t=t^{\prime}=0$ ).
(c) Consider two events that are simultaneous in $S^{\prime}$, and separated by a distance $\Delta x$ along the $x$ axis in $S$. Find the time separation of the events in $S$.
(d) Two signal posts $\mathbf{A}$ and $\mathbf{B}$, separated by a distance of 1 km are tuned so that they almost send signals at the same time, however careful measurements by the astronaut on the surface determine that the signal $\mathbf{B}$ is sent just $10^{-12} \mathbf{s}$ later than signal $\mathbf{A}$. The pilot of the lander, travelling at constant velocity on a straight line between the signal posts, determines that the signals are sent at the same time. In which direction was the pilot flying and at what speed?

Is it possible in principle for a pilot in some other lander to find that the signal from $\mathbf{B}$ is sent first?

## END OF PAPER

