SEMESTER 1 EXAMINATION 2013-2014

ELECTRICITY AND MAGNETISM

Duration: 120 MINS (2 hours)

This paper contains 8 questions.

Answers to Section A and Section B must be in separate answer books

Answer **all** questions in **Section A** and **only two** questions in **Section B**.

Section A carries 1/3 of the total marks for the exam paper and you should aim to spend about 40 mins on it.

Section B carries 2/3 of the total marks for the exam paper and you should aim to spend about 80 mins on it.

An outline marking scheme is shown in brackets to the right of each question.

A Sheet of Physical Constants is provided with this examination paper.

Only university approved calculators may be used.

A foreign language translation dictionary (paper version) is permitted provided it contains no notes, additions or annotations.

Maxwell's Equations:

The Maxwell equations for electric field $\underline{E}$ and magnetic field $\underline{B}$ are given by

$$\int \underline{E}.d\underline{A} = \frac{Q_{\text{enc}}}{\epsilon_0}$$

$$\int \underline{B}.d\underline{A} = 0$$

where these two equations are area integrals over closed surfaces and Q_{enc} is the charge enclosed by the surface.

$$\int \underline{E}.d\underline{l} = -\frac{d\Phi_B}{dt}, \qquad \Phi_B = \int \underline{B}.d\underline{A}$$

$$\int \underline{B}.d\underline{l} = \mu_0 I + \mu_0\epsilon_0\frac{d\Phi_E}{dt}, \qquad \Phi_E = \int \underline{E}.d\underline{A}$$

where these two integrals are line integrals round closed loops, Φ_E and Φ_B are the electric and magnetic fluxes through the areas enclosed by the loops, and I is the current passing through the closed loop.

Section A

A1. Two charges of value +4C and +3C are separated by 1cm. What is the magnitude of the force they experience and in what direction is it? Note the constant $\frac{1}{4\pi\epsilon_0} = 9.0 \times 10^9$ Nm^{2}C^{-2}. [2]

A2. A disc of radius a has a surface charge density, σ, given by

$$\sigma = kr^3$$

where r is the radial distance from the centre of the disc and k a constant with units of Cm^{-5}. What is the total charge on the disc? [4]

A3. A long length of uniform metal wire is charged so that it has a linear charge density of λ Cm^{-1}. Compute the magnitude and direction of the electric field induced outside the wire at its mid-point using Gauss' law. [5]

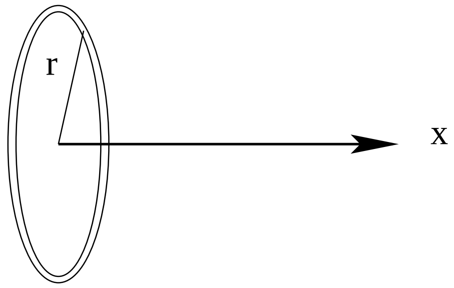
A4. Derive an expression for a cyclotron's frequency in terms of the mass, m, and charge, q of the accelerated particles, and the magnetic field strength of the cyclotron, B. [5]

A5. A capacitor of capacitance C is charged by a battery with emf V, which initially generates a current I_{in} in the circuit. Assume other elements of the circuit have resistance R. Derive an expression for the current in the circuit as a function of time. [4]

TURN OVER

Section B

B1. A thin hoop of radius r has net charge $+Q$ uniformly distributed on it.



The electric field strength on the line perpendicular to the hoop through its centre (shown as the x axis in the figure) is given by

$$\underline{E} = \frac{Q}{4\pi\epsilon_0(x^2 + r^2)^{3/2}} \ \underline{x}$$

(a) Use this result for the hoop to compute the electric field a perpendicular distance x from an infinite, charged plane with surface charge density σ Cm^{-2}. [10]

Hint: You may find it useful to compute the derivative with respect to r of $-(x^2 + r^2)^{-1/2}$ where x is independent of r.

(b) Now use Gauss' law to calculate the same electric field value you computed in (a). [5]

(d) The potential due to the infinite, charged plane along the x-axis is

$$\phi = -\frac{\sigma|x|}{2\epsilon_0} + c$$

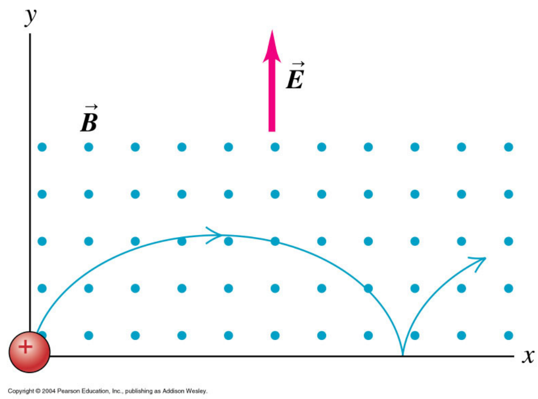
where c is a constant. Compute the electric field for a third time using this result. [3]

(e) What is the physical meaning of the potential? [2]

B2. (a) It is easy to create electric fields because charges are freely available sources for them. There are no magnetic monopoles in the Universe though. Explain, using the relevant Maxwell's equations, how magnetic fields can be generated. In addition explain, at the atomic level, what generates the magnetic field of a traditional metal magnet. [7]

(b) A charge moves through a region where there are non-zero $\underline{E}$ and $\underline{B}$ fields. It experiences the force $\underline{F} = q(\underline{E} + \underline{v} \times \underline{B})$. How is it possible that the charge does not accelerate? (You may neglect gravity.) [3]

(c) A particle with charge q and mass m is initially at rest at the origin as shown in the figure.There is a uniform electric field $\underline{E}$ in the $+y$-direction and a uniform magnetic field $\underline{B}$ directed out of the page. The path of the particle is a cycloid whose radius of curvature at the top of the cycloid is twice the y-coordinate at that level.



(i) Why does the $\underline{B}$ field do no work on the particle? [2]

(ii) Explain why the path has this general shape and why it is repetitive. [3]

(iii) Prove that the speed at any point is equal to

$$\sqrt{\frac{2qEy}{m}}$$ [2]

(iv) Applying Newton's second law at the top point and taking as given that the radius of curvature here equals $2y$, prove that the speed at this point is $2E/B$. [3]

TURN OVER

B3. (a) Use Ampere's law to show that the magnitude of the magnetic field inside an infinitely long solenoid, with n turns of wire per unit length and current I passing through it, is given by

$$B = \mu_0 n I$$

[5]

(b) State Faraday's Law in words and through an appropriate equation. [3]

(c) Explain why a long solenoid has a self inductance. Give an expression for the back emf, as the current, I, flowing through it changes, in terms of the coefficient of self inductance, L, which is the ratio of the magnetic flux of the solenoid, Φ_B, to the current, I. [4]

(d) Show that the energy stored in the magnetic field of the solenoid is given by

$$\frac{1}{2}LI^2$$

[5]

(e) Hence show the stored energy density in the magnetic field is given by

$$\frac{1}{2}\frac{B^2}{\mu_0}$$

[3]

END OF PAPER