PHYS1022 [redacted] 2014/15 SOLUTIONS

SECTION A

A1)

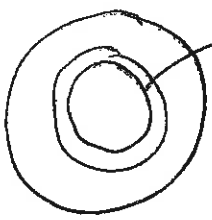
$$\vec{F}_3 = \sum_i \frac{q_i q_3}{4\pi\epsilon_0 r_{i3}^3} \vec{r}_{i3} \qquad [1]$$

$$= \frac{17\times10^{-12}}{4\times3.14\times8.85\times10^{-12}} \frac{1}{\cancel{\;}} \left[\frac{57}{5^{3/2}}\begin{pmatrix}2\\1\end{pmatrix} - \frac{23}{2^3}\begin{pmatrix}0\\2\end{pmatrix}\right] \qquad [1]$$

$$= \begin{pmatrix}1.6\\-0.65\end{pmatrix} C \qquad [1]$$

(seen similar in class & problems)

A2)



$4\pi r^2 dr$ shell

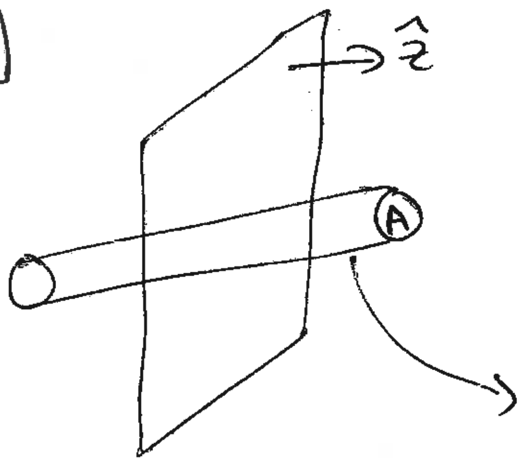
$$Q = \int_0^a 4\pi r^2 \rho \, dr \qquad [2]$$

$$= 4\pi k \int_0^a r^4 \, dr \qquad [1]$$

$$= \frac{4\pi k a^5}{5} \qquad [1]$$

(seen similar in class & problems)

A3)



By symmetry near sheet

$$\vec{E} = E(z)\,\hat{z} \qquad [1]$$

Gauss' Law

$$\int_{\text{gaussian surface}} \vec{E}\cdot d\vec{A} = \frac{Q_{enc}}{\epsilon_0} \qquad [1]$$

$$2A\,|\vec{E}| = \sigma A/\epsilon_0 \qquad [1]$$

perp.

(seen in class)

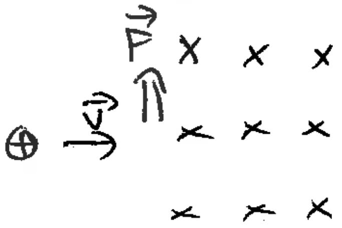
$$\vec{E} = \frac{\sigma}{2\epsilon_0}\,\hat{z}$$

$\vec{F} = q\vec{E} \quad \Rightarrow \quad \sigma = \frac{|\vec{F}|}{q} 2\epsilon_0$ [1]

$= \frac{2.8\times10^{-12}}{1.6\times10^{-19}} \cdot 2 \cdot 8.85\times10^{-12}$

$= 3.1\times10^{-4}\ \mathrm{C\,m^{-2}}$ [1]

A4) Use $\vec{F} = q\vec{v}\times\vec{B}$ [1]



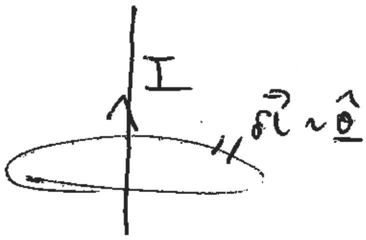
$\vec{F}$ upwards [1]

$\vec{F}$ to left [1]

$|\vec{F}| = 1.6\times10^{-19} \cdot 2\times10^{7} \cdot 0.7 = 2.2\times10^{-12}\ \mathrm{N}$ [1]

(discussed key points in class)

A5) Ampere's Law: $\oint \vec{B}\cdot d\vec{l} = \mu_0 I_{enc}$ [1]



By symmetry & RH rule

$\vec{B} = B(r)\,\hat{\underline{\theta}}$ [1]

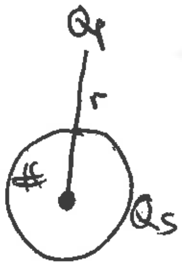
Choose Amperian loop circle of radius r

$\Rightarrow \quad |\vec{B}| \cdot 2\pi r = \mu_0 I_{enc}$ [1]

$\vec{B} = \frac{\mu_0 I}{2\pi r}\,\hat{\underline{\theta}}$ [1]

(seen in class)

B1) (a)



$\vec{E}$ given by Gauss' Law

$$\int_{\substack{\text{Gaussian}\\ \text{surface}}} \vec{E}\cdot d\vec{A} = \frac{Q_{enc}}{\epsilon_0} \qquad [1]$$

↑ choose concentric sphere of radius r [1]

$$\Rightarrow \quad \vec{E} = \frac{Q_s}{4\pi\epsilon_0 r^2}\,\hat{r} \qquad [1]$$

$$\vec{F} = q\vec{E} = \frac{Q_p Q_s}{4\pi\epsilon_0 r^2}\,\hat{r} \qquad [1]$$

Provided Q_s remains within Gaussian surface of radius r $\vec{F}$ remains unchanged [1]

(discussed similar in class)

Inside shell $\vec{E}=0$ so no energy change for $r < R$ [1]

(b)
$$\text{energy} = \int_R^L \mathcal{E}\; 4\pi r^2\, dr \qquad [2]$$

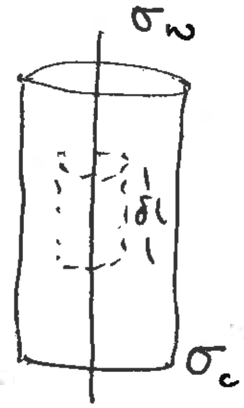
$$= \frac{1}{2}\, 4\pi\epsilon_0 \int_R^L \frac{Q_s^2}{(4\pi\epsilon_0 r^2)^2}\, r^2\, dr$$

$$= \frac{1}{2}\,\frac{Q_s^2}{4\pi\epsilon_0}\int_R^L \frac{dr}{r^2} \qquad [1]$$

$$= \frac{Q_s^2}{8\pi\epsilon_0}\left[\frac{1}{R} - \frac{1}{L}\right] \qquad [1]$$

(new example of volume integral)

(c)



By symmetry $\vec{E} = E(r)\hat{\underline{r}}$ [1]

$$\int_{\substack{\text{Gaussian}\\ \text{surface}}} \vec{E} \cdot d\vec{A} = \frac{Q_{enc}}{\epsilon_0}$$ [1]

↑ choose co-axial cylinder of radius r [1]

Between wire & cylinder:

$\vec{E}$ ⊥ to end cap $d\vec{A}$, ∥ to curved $d\vec{A}$s [1]

$$|\vec{E}|\, 2\pi r\, \delta l = \frac{\sigma_w\, \delta l}{\epsilon_0}$$ [1]

$$\vec{E} = \frac{\sigma_w}{2\pi\epsilon_0 r}\,\hat{\underline{r}}$$ [1] (in lecture derivation)

Outside cylinder similarly $\vec{E} = \dfrac{(\sigma_w + \sigma_c)}{2\pi\epsilon_0 r}\,\hat{\underline{r}}$ [2]

(i) $$|\vec{E}| = \frac{4.7\times10^{-9}}{2\times3.14\times8.85\times10^{-12}\times0.005} = 17{,}000\ \mathrm{NC^{-1}}$$ [1]

(ii) $$|\vec{E}| = \frac{1.5\times10^{-9}}{2\times3.14\times8.85\times10^{-12}\times0.015} = 1{,}800\ \mathrm{NC^{-1}}$$ [1]

(unseen problem)

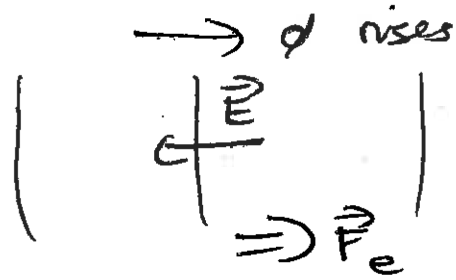
12] (a) Potential difference is <u>energy</u> [1] an <u>external agent</u> [½] must provide to <u>move between 2 pts</u> [½] a <u>test charge</u> [½] <u>per unit charge</u> [½]

(bookwork from lectures)

(b) $\vec{E} = -\vec{\nabla}\phi$ [1]

$\vec{F} = q\vec{E}$ [1]

If ϕ rises to right:



[1]

[1]

Electron will move towards higher potential [1]

(discussed in lectures)

(c) $\Delta U = q\,\Delta\phi$ [2]

$|\Delta U| = 1{\cdot}6\times10^{-19} \times 1{\cdot}2\times10^{9}$

$=$ [redacted] $\times 10^{-10}$ J [1] (problem sheet problem)

1.9

(d)

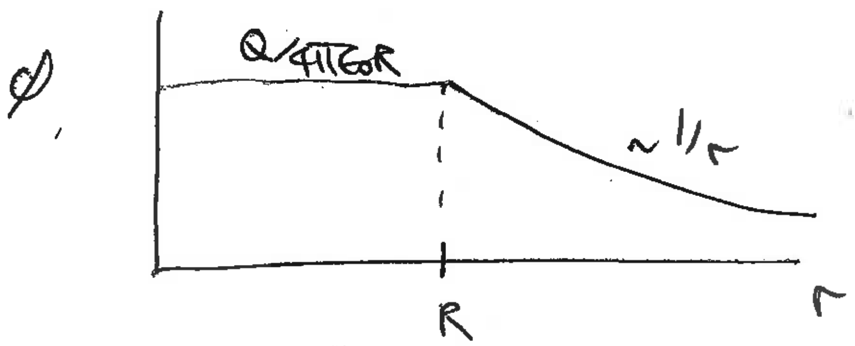
Inside a conductor $\vec{E} = 0$

$\Rightarrow \phi =$ constant [2]

Outside sphere $\vec{E} = \dfrac{Q}{4\pi\epsilon_0 r^2} \hat{\underline{r}}$ [1]

$$\vec{E} = -\vec{\nabla}\phi = -\frac{\partial\phi}{\partial r}\hat{\underline{r}}$$

$$\phi = Q/4\pi\epsilon_0 r$$ [2]



[2]

(seen similar problems)

(e)

Assume charge settles so equal amount on each sphere (only) → 14nC on each. [1]

$$\phi = \frac{14\times10^{-9}}{4 \cdot 3.14 \cdot 8.85\times10^{-12} \cdot 0.01} = 12.6\times10^{3}\ \mathrm{JC^{-1}}$$

[1]

(neglected second sphere)

(new problem)

B3

(a) Faraday's Law: an emf is generated in a closed circuit when the magnetic flux through the circuit changes [2]

$$\text{emf} = -\frac{d\Phi_B}{dt} \quad [1] \qquad \Phi_B = \int_{loop} \vec{B} \cdot d\vec{A} \quad [1]$$

Lenz's Law: (the -ve sign): the induced emf is such as to generate a $\vec{B}$ field that opposes the change in flux [2]

(bookwork)

(b)

(i) $\vec{B}$ increases $\Rightarrow$ $\vec{B}_{ind}$ in opposite direction to oppose flux change [1]

By RH rule I is anti-clockwise [1]

(ii) r decreases $\Rightarrow$ reduction in flux so $\vec{B}_{ind}$ in same direction as original to reinforce [1]

By RH rule I is clockwise [1]

(seen similar examples)

(c) $V = \frac{\Delta \phi}{\Delta t}$ [1]

$= \frac{\pi d^2/4 \, B}{\Delta t}$ [1]

$P = VI$ [1]

$I = V/R$ [1]

$P = V^2/R$ [1]

$U = P\Delta t = \frac{\pi^2 r^4 B^2}{R \Delta t}$ [1]

$= 28 \text{ mJ}$ [1]

$U = Cm\Delta T$ [2]

$\Delta T = \frac{U}{Cm} = \frac{0.028}{129 \times 0.015} = 0.014°C$ [1]

(new problem)