UNIVERSITY OF SOUTHAMPTON PHYS1022W1

SEMESTER 1 EXAMINATION 2012/13

ELECTRICITY AND MAGNETISM

Duration: 120 MINS

VERY IMPORTANT NOTICE Section A answers MUST be in a separate blue answer book. If any blue answer booklets contain work for both Section A and B questions - the latter set of answers WILL NOT BE MARKED.

*Answer **all** questions in **Section A**,and two **and only two** questions from **Section B**.*

Section A carries 1/3 of the total marks for the exam paper and you should aim to spend about 40 mins on it. Section B carries 2/3 of the total marks for the exam paper and you should aim to spend about 80 mins on it.

An outline marking scheme is shown in brackets to the right of each question.

A sheet of Physical Constants will be provided with this exam paper.

Only University approved calculators may be used.

Maxwell's Equations:

The Maxwell equations for electric field $\underline{E}$ and magnetic field $\underline{B}$ are given by

$$\int \underline{E}.d\underline{A} = \frac{Q_{\rm enc}}{\epsilon_0}$$

$$\int \underline{B}.d\underline{A} = 0$$

where these two equations are area integrals over closed surfaces and $Q_{\rm enc}$ is the charge enclosed by the surface.

$$\int \underline{E}.d\underline{l} = -\frac{d\Phi_B}{dt}, \qquad \Phi_B = \int \underline{B}.d\underline{A}$$

$$\int \underline{B}.d\underline{l} = \mu_0 I + \mu_0\epsilon_0\frac{d\Phi_E}{dt}, \qquad \Phi_E = \int \underline{E}.d\underline{A}$$

where these two integrals are line integrals round closed loops, Φ_E and Φ_B are the electric and magnetic fluxes through the areas enclosed by the loops, and I is the current passing through the closed loop.

Section A

A1. A thin, insulating rod of length L has a linear charge density, λ, given by

$$\lambda = \alpha x^3$$

where α is a constant with units of Cm^{-4} and x is the distance from one end of the rod. What is the total charge on the rod? [3]

A2. State Gauss' law, then use it to determine the electric field generated by an infinitely large charged sheet with uniform charge density σ Cm^{-2}. [5]

A3. Two capacitors, of capacitance $C_1 = 4\ \mu$F and $C_2 = 12\ \mu$F, are connected in series across a 12 V battery. They are disconnected when fully charged and reconnected to each other, positive plate to positive plate and negative plate to negative plate. What is the potential difference across each capacitor after they are reconnected? [4]

A4. Two current carrying wires lie parallel, with the current in each in the same direction. Do the wires attract or repel? Explain your answer in detail. [4]

A5. State Lenz's law. [2]

The north pole of a magnet is moved towards a wire loop. Show, with a diagram and explanation, in which direction a current is induced in the loop. [2]

Section B

B1. (a) Define electric potential. [2]

(b) Calculate the work done by the electric field of a fixed charge +Q as a charge +q is brought radially towards it from infinity to a distance r away. Hence compute the potential due to the charge Q. [5]

(c) Explain why the work done on the charge q is independent of the path it takes from infinity to its final position. What does this result imply for the potential? [4]

(d) A conducting sphere of radius a is surrounded by a thin, concentric, conducting, spherical shell of radius b. There is charge +Q on the inner sphere and -Q on the outer shell. Use Gauss' law to compute the electric field between the sphere and the shell as a function of r, the radial distance from the centre of the sphere. [5]

(e) Compute the potential difference between the sphere and the shell. Sketch a graph of the potential against the distance from the centre of the sphere out to the shell. [4]

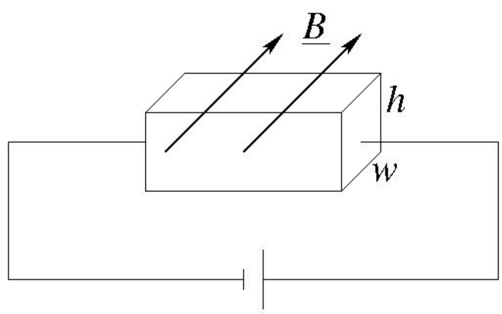
B2. A charge q moving with velocity $\underline{v}$ in the presence of electric field $\underline{E}$ and magnetic field $\underline{B}$ experiences the force

$$\underline{F} = q(\underline{E} + \underline{v} \times \underline{B})$$

(a) If the charge q experiences no acceleration, is it true that $\underline{E} = \underline{B} = \underline{0}$ at the point where the charge is? Explain your answer. (You may neglect gravity.) [4]

(b) A velocity selector uses a 60 mT magnetic field perpendicular to a 24 kNC^{-1} electric field. At what speed will charged particles pass through the selector undeflected? [3]

(c) A long, flat conducting bar of height h and width w, is connected to a battery as shown.



Show, assuming that the current is uniformily distributed, that the magnitude of the current through the sample is given by

$$I = vhwnq$$

where there are n charge carriers (each of charge $+q$) per unit volume moving with drift velocity v. [4]

(d) A magnetic field is now applied, parallel to the side of length w, as shown in the figure. Explain what force acts on the charge carriers and how an equilibrium is attained with a Hall Voltage across the material. [4]

(e) Derive an expression for the magnitude of the Hall Voltage. [5]

B3. (a) State Ampere's Law in words and through an appropriate equation. [3]

(b) Use Ampere's law to show that the magnitude of the magnetic field inside an infinitely long solenoid, with n turns of wire per unit length and current I passing through it, is given by

$$B = \mu_0 n I$$

[5]

(c) State Faraday's Law in words and through an appropriate equation. [3]

(d) Explain why, when the current through a solenoid is increased, there is a "back-emf" in the circuit. [4]

(e) The self-inductance of an infinitely long solenoid as in part (b) is defined as

$$L = \frac{N\Phi}{I}$$

where Φ is the magnetic flux through a single turn of a solenoid, with N turns. Derive an expression for L in terms of the radius, r, the length, l, and the number of turns of wire per unit length, n, of the solenoid. [5]

END OF PAPER