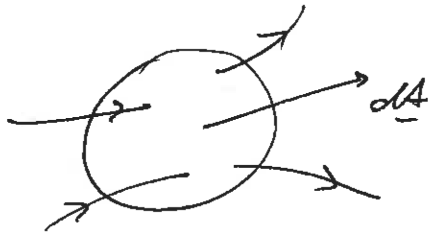
31. 3. 2015

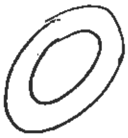
• PHYS 2001 W1 (ELECTROMAGNETISM): SOLUTIONS

$\boxed{A1}$ $d\Phi_E = \underline{E}(x,y,z)\cdot d\underline{A}$

FLUX IS PROPORTIONAL TO THE NUMBER OF LINES PASSING THROUGH dA



[2]



$$\Phi_E = \iint_{SURFACE} \underline{E}(\underline{r})\cdot d\underline{A} =$$

$$= \iint 5r\; dA = \int 5r\cdot 2\pi r\cdot dr =$$

$$= \left[2\cdot\frac{5}{3}r^3\right]_3^5 \pi = \frac{980}{3}\pi\,\frac{N\,mm^2}{C}$$

[3]

$\boxed{A2}$ $\Phi_E = \frac{Q}{\varepsilon_0}$ GAUSS' LAW

THE FLUX THROUGH A CLOSED SURFACE IS EQUAL TO $\frac{1}{\varepsilon_0}$ TIMES THE ELECTRIC CHARGE ENCLOSED WITHIN THE CLOSED SURFACE

[3]

$\boxed{A3}$ ρ SUCH THAT $Q = \iiint_V \rho(x,y,z)\,dV$

$\underline{J} = \rho\underline{v}$ WHERE $\underline{v}$ = VELOCITY VECTOR

[1]

$v = \frac{\underline{J}}{\rho} = \frac{10C}{mm^2 s} \frac{1}{2C} mm^3 = 5 \frac{mm}{s}$ [1]

$\underline{\nabla} \times \underline{B}(x,y,z) = \mu_0 \underline{J}(x,y,z)$ AMPERE'S LAW

A VOLUME CURRENT DENSITY $\underline{J}$ AT A POINT (x,y,z)
ACTS AS A SOURCE OF MAGNETIC FIELD ~~ORDER~~
WHICH CURLS ABOUT THAT POINT

[2]

A4

$l = 20\,mm \qquad I = 2mA$

$$\oint_C \underline{B}(\underline{r}) \cdot d\underline{l} = \mu_0 I$$

$$B \cdot 20mm = \mu_0\, 2mA$$

$$B = 0.1 \frac{mA}{mm} \mu_0 =$$

$$= 0{,}1 \cdot 4\pi \cdot 10^{-7} \frac{Tm}{A} \frac{mA}{mm} =$$

$$= 0.4\, \pi \cdot 10^{-7}\, T$$

[3]

$\oint_C$ IS INDEPENT ON THE PATH $\Rightarrow$ SAME RESULT FOR
A CIRCULAR WIRE

[1]

A5 a) SPEED OF LIGHT $c \sim 3 \cdot 10^8\, m/s$ [0,5]

b) ORTHOGONAL PLANET [1]

$\delta_E = \delta_B$ [0.5]

c) $\underline{S} = \frac{1}{\mu_0} \underline{E} \times \underline{B}$ $\quad \underline{S}$ IS ORTHOGONAL TO $\underline{E}$ AND $\underline{B}$

AVERAGE VALUE OF $S = I = \frac{1}{2} E_0^2 \frac{1}{\mu_0 c}$ [2]

B1.

a) $\nabla \times \underline{B} = \mu \varepsilon \frac{\partial \underline{E}}{\partial t}$

A TIME VARYING ELECTRIC FIELD ACTS AS A SOURCE OF MAGNETIC FIELD THAT CURLS AROUND IT

(Bookwork) [4]

b) $\underline{S} = \frac{1}{\mu} \underline{E} \times \underline{B}$

DIRECTION : DIRECTION OF ENERGY FLOW

MAGNITUDE : ENERGY FLOW PER UNIT AREA PER UNIT TIME

(Bookwork) [4]

c) $E_R = \left| \frac{n-m}{n+m} \right| E_I$

$\delta_R = \delta_I \qquad n > m$

$\delta_R = \delta_I + \pi \qquad m > n$

[4]

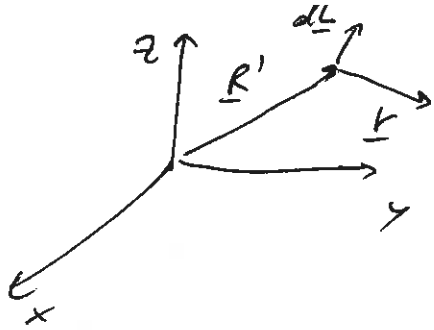
d) $E_T = \frac{2n}{n+m} E_I \qquad \delta_T = \delta_I$ [4]

e) $R = \frac{I_R}{I_I} = \left(\frac{E_R}{E_I}\right)^2 = \left(\frac{n-m}{n+m}\right)^2$

$T = \frac{I_T}{I_I} = \frac{v_2 \varepsilon_2}{v_1 \varepsilon_1}\left(\frac{E_T}{E_I}\right)^2 = \frac{4mn}{(n+m)^2}$ [4]

$R+T = 0.85\left[\left(\frac{n-m}{n+m}\right)^2 + \frac{4nm}{(n+m)^2}\right]$

$\boxed{B2}$ a)



$$d\underline{B} = \frac{\mu_0}{4\pi} \frac{I\, d\underline{L} \times \underline{r}}{r^2}$$ [5]

b) $dL = 10\,mm$ $I = 2A$ $r = 30\,mm$

$$B = \frac{\mu_0}{4\pi} \frac{2A \; 30\,mm \; \sin\theta}{30^2 \; mm^2}$$

$\theta = 0 \quad \rightarrow B = 0$

$$\theta = 90 \qquad B = \frac{1}{15} \frac{\mu_0}{4\pi} \frac{A}{mm} = 4\pi\, 10^{-7} \frac{Tm}{A} \frac{A}{60\pi\, mm} =$$

$$= \frac{1}{15} 10^{-4}\, T$$

[5]

c) $R = 5\,mm$ $r = 150\,mm$ $J = \frac{2A}{mm^3}$

$$B(\underline{r}) = \frac{\mu_0}{4\pi} \iiint_V \frac{\underline{J}(\underline{R}')\, dV \times \underline{r}}{r^2} =$$

$$= \frac{\mu_0}{4\pi} J \frac{4}{3}\pi \frac{R^3\, r \sin\phi}{r^2} =$$

$$= \frac{\mu_0}{4\pi} \frac{4}{3}\pi \frac{2A}{mm^3} \frac{5^3\, mm^3}{150\, mm} \sin\phi =$$

$$= 4\pi\, 10^{-7} \frac{Tm}{A} \frac{2}{3} A \frac{125}{150} \frac{1}{mm} \sin\phi = \frac{10^5}{450} T \sin\phi$$

[5]

d) $\text{div}\,\underline{J} = -\frac{\partial \rho}{\partial t} = -\rho_0$

CONTINUITY EQUATION

CHARGED FLUID THAT FLOWS IN MUST ALSO FLOW OUT [3]

if $\rho = \rho_0 \rightarrow \frac{\partial \rho}{\partial t} = \frac{\partial \rho_0}{\partial t} = 0 \rightarrow \text{div}\,\underline{J} = 0$

[2]

B3

a) FARADAY OBSERVED CURRENT FLOWING THROUGH A LOOP WHEN A MAGNET WAS INSERTED IN IT

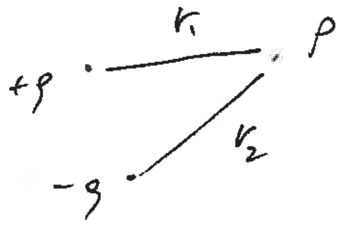
$$\oint_C \underline{E}(x,y,z)\cdot d\underline{L} = -\frac{\partial}{\partial t}\iint_{\text{open surface}} \underline{B}(x,y,z)\cdot d\underline{A}$$

FARADAY'S LAW

INCREASE OF MAGNETIC FLUX INDUCES A CURRENT THAT CREATES A MAGNETIC FIELD THAT OPPOSES THE INCOMING ONE (NATURE ABHORS A CHANGE IN FLUX)

[4]

b)

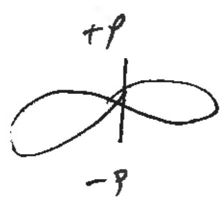


$$V = \frac{1}{4\pi\varepsilon_0}\left(\frac{q}{r_1} - \frac{q}{r_2}\right) =$$

$$= \frac{q}{4\pi\varepsilon_0}\left(\frac{1}{2} - \frac{1}{3}\right) = \frac{q}{24\pi\varepsilon_0}$$

[3]

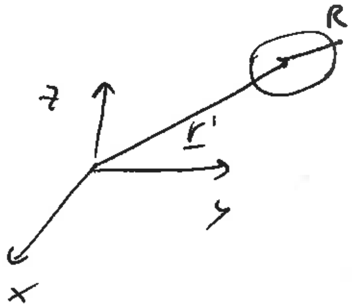
c)



IT'S A TORUS

[3]

d)



$$\underline{B}(\underline{r}) = \frac{\mu_0}{4\pi}\int \frac{I\, d\underline{L} \times \hat{\underline{r}}'}{r'^2} = \frac{\mu_0\, 2A\, 2\pi\, 1m}{4\pi\, 4m^2}$$

$$= \frac{\mu_0}{4}\frac{A}{m}$$

$\underline{B} \perp d\underline{L}$, $\underline{B} \perp \underline{r}'$

$\Rightarrow \underline{E} \perp \underline{B} \rightarrow \underline{E} \parallel d\underline{L}$, $\underline{E} \parallel \underline{r}'$

[5]

e)

$B = 5\,T$

$I = 2A$

$R = 1\,m \rightarrow dL = 2\pi\,m$

$$|\underline{F}| = |I d\underline{L} \times \underline{B}| = 2A \cdot 2\pi m \cdot 5T \cdot \sin 0 = 0$$ [3]

if $B \perp$ PLANE OF WIRE $\quad \sin 90 = 1 \rightarrow |\underline{F}| = 20\pi\ A\,m\,T =$

$= 20\pi\ N$

(2)