SEMESTER 2 EXAMINATION 2013-2014

CLASSICAL MECHANICS

Duration: 120 MINS (2 hours)

This paper contains 9 questions.

Answers to Section A and Section B must be in separate answer books

Answer **all** questions in **Section A** and **only two** questions in **Section B**.

Section A carries 1/3 of the total marks for the exam paper and you should aim to spend about 40 mins on it.

Section B carries 2/3 of the total marks for the exam paper and you should aim to spend about 80 mins on it.

An outline marking scheme is shown in brackets to the right of each question.

A Sheet of Physical Constants is provided with this examination paper.

Only university approved calculators may be used.

A foreign language translation dictionary (paper version) is permitted provided it contains no notes, additions or annotations.

Section A

A1. A spinning top making a constant angle α to the vertical is undergoing slow steady precession due to gravity. How does the rate of precession ω_p depend on α? Justify your answer by explaining how the torque depends on α and how the rate of change of angular momentum depends on ω_p and α. [4]

A2. Consider the gravitational attraction of a thin uniform spherical shell of mass m. State the vector form of the gravitational acceleration for the cases where the test particle is inside, and outside the shell, in terms of its position $\mathbf{r}$ relative to the centre of the sphere. [4]

A3. Consider a planet orbiting the sun with some non-zero eccentricity e. Sketch the orbit, labelling the semimajor and semiminor axes, the position of the Sun in terms of e and the semimajor axis, the points on the orbit where the planet's radial velocity momentarily vanishes, and finally the point on the orbit where the planet's angular velocity is at a maximum. [4]

A4. A small heavy ball thrown from some point in Southampton, has position $\mathbf{r}$ relative to this point and satisfies an equation of motion of the form

$$\ddot{\mathbf{r}} = -g\mathbf{R}/R - 2\boldsymbol{\omega} \times \dot{\mathbf{r}} - \boldsymbol{\omega} \times (\boldsymbol{\omega} \times \mathbf{R})$$

where g is the acceleration due to gravity. Explain the meaning of the terms $\mathbf{R}$ and $\boldsymbol{\omega}$ in this equation. Explain, briefly, how the last term in the equation above results in apparent effective gravity and, qualitatively, how the latter differs from true gravity. [4]

A5. Express the principle of Time Translation Invariance in the context of simple or damped harmonic motion. [4]

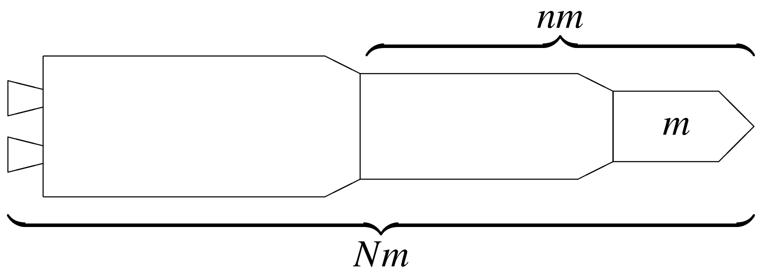
TURN OVER

Section B

B1. (a) A rocket in deep space burns fuel and reduces its mass from m_i to m_f. If the escaping gases have speed u relative to the rocket, show that the rocket's speed increases by

$$u \ln\left(\frac{m_i}{m_f}\right).$$ [7]

(b) A payload (for example a satellite) of mass m is mounted on a two stage rocket. The *total* mass of both rocket stages, fully fuelled, plus the payload, is Nm. The mass of the fully fuelled second stage plus payload, after first stage burnout and separation is nm. For both stages the ratio of burnout mass (casing) to initial mass (casing plus fuel) is r and the exhaust speed is u.



(i) Show that the speed gained from the first stage burn, starting from rest, is

$$v_1 = u \ln\left(\frac{N}{rN + n(1-r)}\right).$$ [4]

(ii) Show that the additional speed v_2, gained from the second burn, after separation of the burned out first stage is

$$v_2 = u \ln\left(\frac{n}{rn + 1 - r}\right).$$ [3]

(iii) If N and r are constants, verify that the resulting payload velocity, v_1+v_2, is maximised when $n = \sqrt{N}$. Show also that v_1 and v_2 are equal for this value of n. [6]

B2. (a) Define the moment of inertia of an object about a *fixed* rotation axis. [2]

Show that the moment of inertia of a uniform solid sphere of mass m and radius a about a diameter is

$$\frac{2}{5}ma^2.$$ [6]

(b) A billiard ball is struck by a cue and moves off with speed u, sliding (without rotation) along the table top.

Firstly, sketch a diagram and obtain the equations of linear and angular motion. [8]

Then, if the coefficient of friction between the ball and the table is μ, show that the time which elapses before the ball rolls rather than skids is

$$\frac{2u}{7\mu g}.$$ [4]

TURN OVER

B3. (a) A satellite orbits the Earth, subject only to the Earth's gravitational attraction. Explain why the vector angular momentum is conserved and why this means that the orbit lies in a plane. What other quantity or quantities is/are conserved? [6]

(b) A satellite is moving in a circular orbit of radius $2r_e$ around the Earth, where r_e is the Earth's radius. The satellite's direction of motion is instantaneously changed through an angle α towards the Earth, without any change in speed, so that in its new orbit, the satellite just grazes the Earth's surface.

Firstly, sketch the diagram of motion and relate the satellite velocities along the two orbits. [10]

Then, find the angle α. [4]

[Assume the Earth to be spherically symmetric. Ignore any complications due to reduced mass or the effects of the Earth's atmosphere.]

B4. (a) Explain what is meant by a *normal mode* for an oscillating system. [2]

(b) A bead of mass m slides freely on a smooth circular ring of radius R and mass m. The ring is free to move in its own plane about a fixed axis through a point on its circumference and the system makes *small* oscillations under gravity.

Find the eigenvalue matrix equation for the angular frequencies of the two normal modes of the system. [8]

Find the values of such angular frequencies and the associated displacements. [8]

Describe the motions of the ring and the bead in each mode. [2]

END OF PAPER