B = Bookwork ie. lect notes, discussed in lects, Problem sheets etc.
(B) = Partly bookwork.

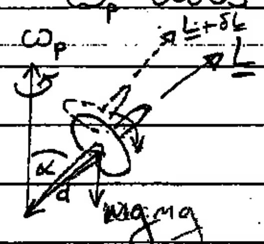
A1

Inside the sphere $g(r) = 0$ $[\frac{1}{2}]_B$

Outside $g(\underline{r}) = -\frac{GM\hat{\underline{r}}}{r^2}$ $[\frac{1}{2}]_B$ $[\frac{1}{2}]_B$ $[\frac{1}{2}]_B$ $[\frac{1}{2}]_B$ $[\frac{1}{2}]_B$ $[\frac{1}{2}]_B$

where G is Newton's gravitational constant and $\hat{\underline{r}}$ is the unit vector $\underline{r}/r$ in the direction r
$[\frac{1}{2}]_B$

A2 ω_p does not depend on α $[1]_B$



Torque $\tau = $ " $\underline{r} \times \underline{F}$ "
thus $\tau = dMg \sin\alpha$ $[1]_B$

[illegible]

$[1]_B$ for diag and/or def^{ns} d

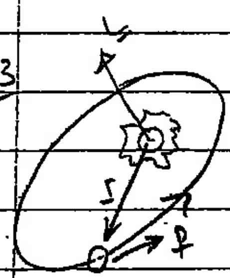
In a small time δt

$\delta L = |\delta \underline{L}| = L \sin\alpha \, \omega_p \delta t$

$\frac{dL}{dt} = \omega_p L \sin\alpha$

$\tau = dL/dt \Rightarrow \omega_p = \frac{dMg}{L}$ $[1]_B$

A3 $[\frac{1}{2}]_B$



with the Sun at the origin of coordinates, angular momentum $\underline{L} = \underline{r} \times \underline{p}$ $[1]_B$ for the planet is conserved $[1]_B$ This is because gravitational force is central and therefore the total torque $= \underline{r} \times \underline{F} = 0$ $[\frac{1}{2}]_B$ Consequently $\underline{L}$ forms a fixed direction in space and $\underline{r}$ and $\underline{p}$ are confined to the plane orthogonal to $\underline{L}$ $[1]_B$

A4 $\underline{R}$ is the position of the point in Southampton relative to the centre of the Earth, using axes which rotate with the Earth. [1]B

[1]B ω is the vector angular velocity of the Earth (and thus points north and has magnitude ---)

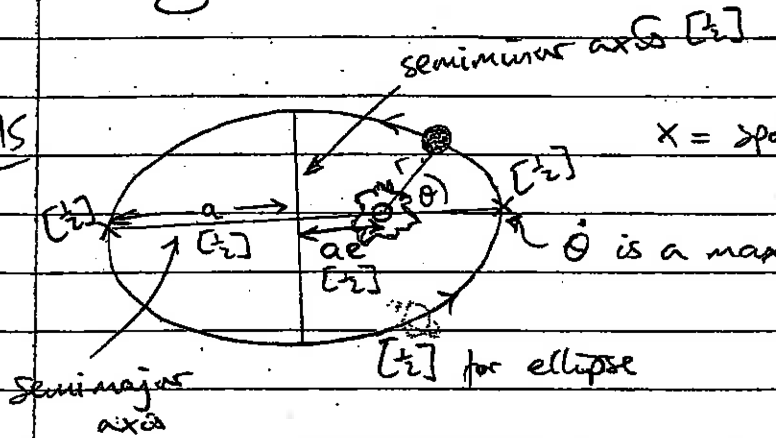
[1]B $-2\omega \times \dot{r}\, m$, where m is the mass of the ball, is the coriolis force. It is proportional to the balls velocity $\dot{r}$ and acts perpendicular to it.

For mostly complete answers

$m\,\omega \times (\omega \times R)$ is the centrifugal force due to the Earth spinning. In total $g\frac{R}{R} + \omega \times (\underline{\omega} \times \underline{R}) \equiv g^*$

[1]B an ~~gravity~~ apparent gravity pointing slightly more towards the equator and with slightly smaller magnitude than g.

A5

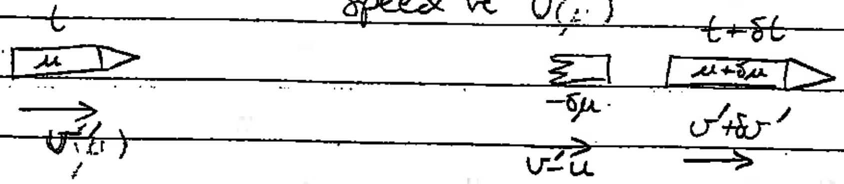


x = spots where $\dot{r} = 0$

$\dot{\theta}$ is a maximum here. [1]

B5

B1) Let the total mass of the rocket at time t be μ, & its speed be $v'(t)$ [1]$_B$



Conservation of momentum of isolated system [1]$_B$ $\Rightarrow$

$$\mu v' = (\mu+\delta\mu)(v'+\delta v') - \delta\mu\,(v'-u) \quad [2]_B$$

$$\Rightarrow \quad 0 = \mu\,\delta v' + \delta\mu\, u \quad [1]_B \quad (\text{to 1st order } [1]_B)$$

∴ Velocity when m of fuel remains

$$v = v_i - u\int_{C+M}^{C+m} \frac{d\mu}{\mu} \quad [1]_B \quad = u \ln\left(\frac{C+M}{C+m}\right)$$

v_i: initial velocity $= 0$ [1]$_B$

Initial velocity for 2nd stage $= v$ [1]$_B$ & initial mass $= C+m$ [1]$_B$

Final " " " " $= v_f$ [1]$_B$ & final mass $= C$ [1]$_B$

marks given if clearly implication

opposite thrust [1]$_B$

$$\therefore \quad v_f = v - u\ln\left(\frac{C+m}{C}\right) \quad [1]$$

$$= u\ln\left(\frac{C+M}{C+m}\right) + u\ln\left(\frac{C}{C+m}\right) \quad [1]_B$$

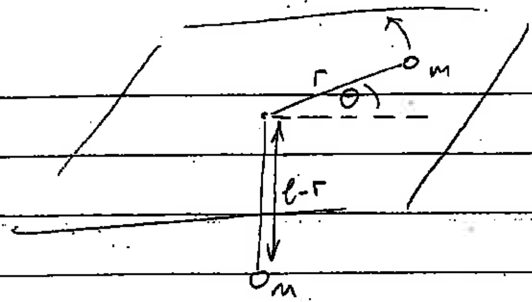
$$= u\ln\left(\frac{C(C+M)}{(C+m)^2}\right)$$

$$v_f = 0 \Rightarrow \frac{C(C+M)}{(C+m)^2} = 1 \quad [1]$$

i.e. $C+m = \sqrt{C(C+M)}$ [1] ($-$sign $\Rightarrow$ $-$ve m! so discard [1])

or $m = \sqrt{C(C+M)} - C$ [2]

B2



A novel question that adapts what the students should know from ~~planetary motion~~ [1]

Let the mass of the particles be m.

K.E. of particle on the table $= \underbrace{\frac{1}{2} m \dot{r}^2}_{[2]} + \underbrace{\frac{1}{2} m r^2 \dot{\theta}^2}_{[2]}$

Angular momentum of particle on the table $= m r^2 \dot{\theta}$] [2]

Conserved:

[1] Angular momentum $= m r^2 \dot{\theta}$ [1] — (1)

[1] Total energy (up to additive constant) $= \underset{[1]}{m\dot{r}^2} + \frac{1}{2} m r^2 \dot{\theta}^2 + \underbrace{mrg}_{[2]}$ — (2)

Initial angular momentum $= m \frac{l}{2} v$ — (3) [1]

Initial total energy $= \frac{1}{2} m v^2 + m \frac{l}{2} g$ — (4) [2]

Equating (1) & (3) $\Rightarrow$ $r^2 \dot{\theta} = \frac{1}{2} l v$ [1]

& substituting for $\dot{\theta}$ in (2) equated to (4) $\Rightarrow$] [2]

$$\dot{r}^2 + \frac{1}{8} \frac{l^2 v^2}{r^2} + rg = \frac{v^2}{2} + \frac{l}{2} g \quad [1]$$

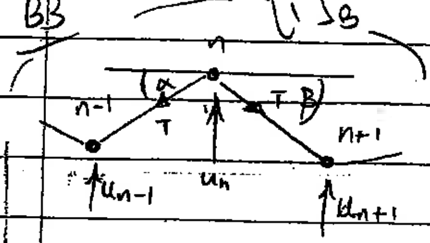
If the hanging particle is to reach the hole its kinetic energy must still be non-negative [1]

i.e. $0 \leq \dot{r}^2_{final} = \frac{v^2}{2} + \frac{l}{2} g - \left(\frac{1}{8} v^2 + l g \right)$ [1]

$$= \frac{3v^2}{8} - \frac{l}{2} g$$

i.e. $\underline{\underline{v^2 \geq \frac{4}{3} l g}}$

BB [1]$_B$ [1]$_B$



NII: $m\ddot{u}_n = -T\sin\alpha - T\sin\beta$ [1]$_B$

$\sin\alpha \simeq \frac{(u_n - u_{n-1})}{a}$ $\sin\beta \simeq \frac{(u_n - u_{n+1})}{a}$ [1]$_B$

for small displacements. [1]$_B$

so $m\ddot{u}_n = \frac{T}{a}(u_{n-1} - 2u_n + u_{n+1})$ [2]$_B$

Substitute normal mode $u_n = Ae^{i\omega t}e^{in\theta}$ [1]$_B$

[1]$_B$ [1]$_B$

$$\Rightarrow -\omega^2 m = \frac{T}{a}\left(e^{-i\theta} - 2 + e^{i\theta}\right) = \frac{2T}{a}(\cos\theta - 1)$$

$$\text{ie.}\quad \underline{\underline{\omega^2 = \frac{2T}{ma}(1-\cos\theta)}}$$

$u_0 = 0$ implied [1] (B)

$u_5 = h\cos(\omega t)$ (given)

Since the interactions between beads are nearest neighbour only, it's enough to find linear combination of normal modes of ∞ system satisfying the above. [1] (B) [1]

[1]

$u_5 \Rightarrow \cos(\omega t)$ & $u_0 \Rightarrow 0 \Rightarrow u_n = A\cos\omega t\sin(n\theta)$ [1]

Dispersion rel^n $\Rightarrow$ $\cos\theta = 1 - \frac{ma\omega^2}{2T} = 1 - \frac{10^{-3}\times 2\times 10^{-2}\times 100}{2\times 2\times 10^{-3}}$

$= \frac{1}{2}$ [1]

And $u_5 \Rightarrow$ $A = h\big/\sin(5\theta)$ [1]

$\therefore$ without loss of generality, take sol^n $\theta = \frac{\pi}{3}$ & then [1]

[1]

$$u_2 = \frac{h}{\sin\left(\frac{5\pi}{3}\right)}\cos\omega t\sin\left(\frac{2\pi}{3}\right) = -h\cos(\omega t)$$ [1]

$$= \underline{\underline{-0.1\text{cm}\cos(10t)}}$$

B4

(a) Consider MoI about 3 orthogonal axes with origin at centre.

$$I_x = \int_{vol} (y^2+z^2)\,dm \quad [1]_B$$

$$I_y = \int (x^2+z^2)\,dm \quad [1]$$

$$I_z = \int (x^2+y^2)\,dm \quad [1]$$

$[3]_B$

Symmetry $\Rightarrow$ $I = I_x = I_y = I_z$ $[1]_B$

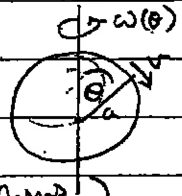
So $3I = I_x + I_y + I_z = 2\int (x^2+y^2+z^2)\,dm$ $[1]_B$

$$= 2\, 4\pi\rho \int_0^a r^4\,dr \quad [2]_B$$

$$= \frac{8\pi\rho a^5}{5} \quad [1]_B$$

Now $m = \frac{4\pi}{3}\rho a^3$ $[1]_B$ $\therefore$ $\underline{\underline{I = \frac{2}{5}ma^2}}$

(b)



[1]
(and/or diagram)

By conservation of angular momentum [1]

$$\underset{[1]}{I\omega} = \Big(\underset{[1]}{I} + \underbrace{\tfrac{2}{5}ma^2\sin^2\theta}_{[1]}\Big)\,\omega(\theta)$$

$$\Rightarrow \quad \underline{\underline{\omega(\theta) = \frac{\omega}{1+\sin^2\theta}}} \quad [1]$$

Insect walks with constant speed reaching $\theta = \pi$ [1] at time $t = T$. [1]

Thus $\theta(t) = \pi\, t/T$ [1]

Total angle turned by sphere $= \underset{[1]}{\int_0^T \omega(t)\,dt} = \frac{T}{\pi}\int_0^\pi \omega(\theta)\,d\theta = \frac{T\omega}{\pi}\frac{\pi}{\sqrt{2}} = \underbrace{\frac{\omega T}{\sqrt{2}}}_{[1]}$