SEMESTER 2 EXAMINATION 2014-2015

CLASSICAL MECHANICS

Duration: 120 MINS (2 hours)

This paper contains 9 questions.

Answers to Section A and Section B must be in separate answer books

Answer **all** questions in **Section A** and **only two** questions in **Section B**.

Section A carries 1/3 of the total marks for the exam paper and you should aim to spend about 40 mins on it.

Section B carries 2/3 of the total marks for the exam paper and you should aim to spend about 80 mins on it.

An outline marking scheme is shown in brackets to the right of each question.

A Sheet of Physical Constants is provided with this examination paper.

Only university approved calculators may be used.

A foreign language word to word® translation dictionary (paper version) is permitted provided it contains no notes, additions or annotations.

Section A

A1. Consider the gravitational attraction of a thin uniform spherical shell of mass m. State the vector form of the gravitational acceleration for the cases where the test particle is inside, and outside the shell, in terms of its position $\mathbf{r}$ relative to the centre of the sphere. [4]

A2. A spinning top, making a constant angle α to the vertical, is undergoing slow, steady precession due to gravity. How does the rate of precession ω_p depend on α? Justify your answer by explaining how the torque depends on α and how the rate of change of angular momentum depends on ω_p and α. [4]

A3. Explain why a planet's orbit lies in a fixed plane. [4]

A4. A small heavy ball thrown from some point in Southampton, has position $\mathbf{r}$ relative to this point and satisfies an equation of motion of the form

$$\ddot{\mathbf{r}} = -g\mathbf{R}/R - 2\boldsymbol{\omega} \times \dot{\mathbf{r}} - \boldsymbol{\omega} \times (\boldsymbol{\omega} \times \mathbf{R})$$

where g is the acceleration due to gravity. Define $\mathbf{R}$ and $\boldsymbol{\omega}$ in this equation. Furthermore, describe briefly each of the three terms on the right hand side of it. [4]

A5. Consider a planet orbiting the sun with some non-zero eccentricity e. Sketch the orbit, labelling the semimajor and semiminor axes, the position of the Sun in terms of e and the semimajor axis, the points on the orbit where the planet's radial velocity momentarily vanishes, and finally the point on the orbit where the planet's angular velocity is at a maximum. [4]

Section B

B1. A rocket at rest in deep space has a casing of mass C and a mass M of fuel. It then subsequently burns fuel in such a way that the escaping gases have a constant speed u relative to the rocket. Show that when the rocket has a mass m of remaining fuel, the rocket's speed is given by

$$u \ln \left(\frac{C+M}{C+m} \right) .$$ [8]

The rocket then rotates to point in the opposite direction, and the remainder m of fuel is completely burned with the thrust retarding the rocket. Show that the final velocity of the rocket is

$$u \ln \left(\frac{C(C+M)}{(C+m)^2} \right) .$$ [7]

In terms of C and M, what mass of fuel must be left after the first burn, if at the end of the second burn, the rocket is again at rest? [5]

TURN OVER

B2. Two identical particles are connected by a massless inextensible string of length ℓ. One of the particles moves on a smooth horizontal table. The second hangs vertically below a hole in the table through which the string passes.

In terms of plane polar coordinates r and θ, defined on the table top and centred on the hole, and by applying what you should know from planetary motion, write down expressions for the kinetic energy and angular momentum of the particle on the table. [6]

Write down expressions for the two conserved quantities in this situation using plane polar coordinates. [5]

The particle on the table is initially at distance $\ell/2$ from the hole, moving with speed v perpendicular to the string. Show that the particle below the table will be pulled all the way up to the hole if $v^2 \geq 4g\ell/3$ (where g is the acceleration due to gravity). [9]

B3. An infinite number of identical beads, each of mass $m = 1\,\mathrm{g}$, are attached to a light elastic string which is infinitely long. When undisturbed the beads and string lie in a straight line on a smooth horizontal surface with each bead separated from its neighbours by distance $a = 2\,\mathrm{cm}$ and with the string stretched to tension $T = 2 \times 10^{-3}\,\mathrm{N}$. Label each bead by an integer n giving its position in the sequence, so that bead n lies a distance na along the line when the system is undisturbed.

Show that the line of beads can support small transverse oscillations with the displacement of the nth bead given by

$$u_n = Ae^{i\omega t}e^{in\theta},$$

where ω is a normal frequency and θ a phase related by

$$\omega^2 = \frac{2T}{ma}(1 - \cos\theta).$$ [10]

The bead labelled $n = 0$ is now clamped in place and the bead at position $n = 5$ is forced to oscillate transversely with displacement $u_5 = h\cos(\omega t)$ where $h = 0.1\,\mathrm{cm}$ and $\omega = 10\,\mathrm{rad\,s^{-1}}$. Find the displacement of the bead labelled by $n = 2$ as a function of time. [10]

TURN OVER

B4. (a) Show that the moment of inertia of a uniform solid sphere of mass m and radius a about a diameter is

$$\frac{2}{5}ma^2.$$

[9]

(b) A solid sphere of mass m and radius a rotates freely about a vertical diameter. A small insect of mass $2m/5$, initially at one pole, walks down the sphere. Let θ be the angle between the vertical and the radius from the centre to the insect. If the sphere is initially rotating with angular velocity ω, show that the angular velocity when the insect is at θ, is given by

$$\omega(\theta) = \frac{\omega}{1 + \sin^2\theta}.$$

[6]

The insect walks with constant speed along a great circle of the sphere, reaching the other pole after time T. Find the angle that the sphere turned through, during this time. [5]

You may find the following integral useful:

$$\int_0^\pi \frac{d\theta}{1 + \sin^2\theta} = \frac{\pi}{\sqrt{2}}.$$

END OF PAPER