SEMESTER 1 EXAMINATION 2014-2015

GALAXIES

Duration: 120 MINS (2 hours)

This paper contains 8 questions.

Answer **all** questions in **Section A** and **only two** questions in **Section B**.

Section A carries 1/3 of the total marks for the exam paper and you should aim to spend about 40 mins on it.

Section B carries 2/3 of the total marks for the exam paper and you should aim to spend about 80 mins on it.

An outline marking scheme is shown in brackets to the right of each question.

A Sheet of Physical Constants is provided with this examination paper.

Only university approved calculators may be used.

A foreign language translation dictionary (paper version) is permitted provided it contains no notes, additions or annotations.

Section A

A1. Classify the following galaxies according to their Hubble types:

a) An elliptical galaxy with an ellipticity of $\epsilon = 0.3$.

b) A galaxy with a large bulge and tightly-wound, ordered and well-defined spiral arms.

c) A giant galaxy with massive star formation and no regular structure. [3]

Covered in lecture and problem sheets.

Answer: a) E3 (because n = 10 ϵ) [1], b) Sa [1], c) Irr II [1].

A2. A galaxy has a spherical shape and a density profile that can be described as $\rho(R) = \rho_0 R^{-2}$ out to a truncation radius R_{out}, beyond which the density drops to zero. Show that the kinetic energy of the galaxy can be expressed as $K = 6\pi\rho_0\sigma_r^2 R_{out}$, with σ_r the radial velocity dispersion. [5]

Answer: For an elliptical galaxy we may assume that random motion of the stars dominate over ordered motion, so we can assume

$K = \frac{3}{2}m\sigma_r^2$ [1].

To derive the total mass, we need to integrate over all mass elements dm:

$m = \int dm$

and because $m = \rho V$ we need to integrate over the volume of a sphere, split into thin shells and in steps of dR, thus

$dV = 4\pi R^2 dR$ [1].

So we have

$m = \int \rho dV = \int 4\pi R^2 \rho_0 R^{-2} dR = \int 4\pi\rho_0 dR$ and thus

$dm = 4\pi\rho_0 dR$

and

$m = 4\pi\rho_0 \, [R]$ [1].

Put this back into

$K = \frac{3}{2}\sigma_r^2 \int 4\pi\rho_0 dR = 6\pi\sigma_r^2\rho_0 \, [R]$ [1]

integrating from 0 to R_{out} we get

$K = 6\pi\rho_0\sigma_r^2 R_{out}$ [1].

Covered in lecture and problem sheets.

A3. A star cluster is located behind a layer of dust which contributes a B band extinction of $A_B = 3.7$ mag. The apparent B-band magnitude of the star cluster is 16.7 mag. Assuming that the star cluster is at a distance of 580 pc and contains ten equally bright stars, and further assuming that the cluster's luminosity is dominated by these ten bright stars, calculate the absolute B-band magnitude of these ten stars. [4]

Answer: First, let's calculate the cluster's absolute magnitude, using the distance modulus $m - M = 5 \log d - 5 + A$ and solve for M:
$M_{cluster} = m - 5 \log d + 5 - A = 16.7 - 5 \log 580 + 5 - 3.7 = 4.18$ mag [1]
The star cluster is made up of ten equally bright stars, i.e.
$F_{cluster} = 10 \times F_{star}$ [1]

Next, use $M_{cluster} - M_{star} = -2.5 \log \frac{F_{cluster}}{F_{star}} = -2.5 \log \frac{10 \times F_{star}}{F_{star}} = -2.5 \log 10 = -2.5$ [1]
and thus $M_{star} = M_{cluster} + 2.5 = 4.18 + 2.5 = 6.68$ mag [1].

Another possible way of solving this: $M_{star} = -2.5 \log F_{star}$
and $M_{cluster} = -2.5 \log F_{cluster} = -2.5 \log(10 \times F_{star})$
Note: we can use F instead of L as the stars are in the star cluster, so all are at the same distance.
We get $F_{star} = \frac{1}{10} 10^{-\frac{M_{cluster}}{2.5}}$ [1]
and thus $M_{star} = -2.5 \log(\frac{1}{10} 10^{-\frac{M_{cluster}}{2.5}}) = 6.68$ mag [1].

Covered in lecture and problem sheets.

A4. The brightest star in the star cluster is a main sequence star with a mass of 7 $M_\odot$. The initial mass function (IMF) of main sequence stars follows the Salpeter IMF

$$n(M) = kM^{-2.35}$$

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where $k = 213$ is a constant of normalization, and M is the stellar mass in solar masses. Assuming that the lowest-mass stars have masses of 0.5 $M_\odot$, what is the total mass of the star cluster? Assuming a mass-to-light ratio of $M/L = 1.6$ in B-band, what is the total B-band luminosity of the star cluster? [4]

Answer: The initial mass function gives the number of stars per mass interval dM. As such, the IMF gives the number density of stars in a given mass interval dM. To derive the total mass of the star cluster, we need to integrate over the mass density× mass per mass interval:

$M_{tot} = \int Mn(M)dM$ [1]

$M_{tot} = \int_{0.5}^{7} MkM^{-2.35}dM = k\int_{0.5}^{7} M^{-1.35}dM = \frac{k}{-0.35}[M^{-0.35}]_{0.5}^{7} = \frac{213}{-0.35} \times (7^{-0.35} - 0.5^{-0.35}) = 468\ M_\odot$ [1]

The total B-band luminosity is then simply derived using $M/L = 1.6$ and thus

$L_{B,tot} = \frac{1}{1.6}M_{tot}$ [1]

and so $L_{B,tot} = \frac{1}{1.6}468 = 292.5\ L_\odot$ [1]

Covered in lecture and problem sheets.

A5. Describe how the interstellar medium, and thus a galaxy, can become enriched in metals. [4]

Answer: Giant stars have strong stellar winds and thus put enriched material into the interstellar medium (ISM) [1].

Depending on the initial mass of the star, it will evolve to become a supernova (mass $> 8\ M_\odot$). During the supernova, the star is disrupted and even heavier elements are released to the ISM [1].

Less massive stars (mass $< 8\ M_\odot$) evolve to red giants which later shed their outer envelope, further enriching the ISM, and become planetary nebula with a white dwarf in its core [1].

The new generation of stars will form within a further enriched ISM, which is expected to favour more efficient fragmentation and stronger star formation [1].

Covered in lecture and problem sheets.

Section B

B1. (a) Classify the following different types of active galaxies (AGN) based on their characteristics: (i) A luminous nucleus ($L_{bol} > 10^{38}$ W, with L_{bol} the bolometric luminosity) with powerful optical, IR, UV and X-ray emission, and both broad and narrow emission lines. (ii) A bright radio point-source with a highly polarised optical continuum and no spectral lines. (iii) A radio galaxy with a bright optical nucleus and broad emission lines. [3]

Answer: (i) Quasar [1], (ii) BL Lac [1] (0.5 marks if answer is Blazar), (iii) BLRG (broad line radio galaxy) [1]

Covered in lecture and problem sheets

(b) Describe the basic structure of an AGN [6]

Answer: The basic structure of an AGN can be described as follows:
- a supermassive BH in its centre [1]
- surrounded by an accretion disk [1]
- dense gas clouds surrounding the disk, extending out to a few light weeks [1] and responsible for the broad line emission [0.5]
- further out at a distance of a few pc, a torus of molecular gas and dust surrounds the system [1]
- still further out, at distance of pc to 100s of pc from the centre, are low-density gas clouds [1] responsible for the narrow emission lines [0.5]

Covered in lecture and problem sheets

(c) An AGN has an X-ray luminosity of $L_X = 4.26 \times 10^{35}$ W. Assuming the X-ray radiation is about 30% of the bolometric radiation, at which fraction of the Eddington luminosity $L_{Edd} = 1.3 \times 10^{31} \frac{M_{BH}}{M_\odot}$ W does the black hole radiate? [3]

Answer: Work out the total, i.e. bolometric luminosity: $L_{bol} = L_X/0.3 =$ [1] $\frac{4.26\times10^{35}}{0.3} = 1.42 \times 10^{36}$ W [1].

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Then the black hole radiates at $L_{bol}/L_{Edd} = \frac{1.42\times10^{36}}{1.3\times10^{31}\times1.37\times10^{6}} = 0.08 = 8\%$ of the Eddington luminosity [1].

Covered in lecture and problem sheets

(d) There is a growing super-massive Black Hole (SMBH) with an initial mass of $M_{\rm BH} = 10^4\,\mathrm{M}_\odot$. It is accreting at its Eddington limit, with a radiative efficiency of $\epsilon = 0.1$. What would its expected mass be after $T = 0.4$ Gigayears assuming all the accretion parameters (radiative efficiency and Eddington ratio) are kept fixed? (Hint: a SMBH grows exponentially as $M_{\rm BH} \propto \exp(T/t_{ef})$, with $t_{ef} = \epsilon\times t_E$ the e-folding timescale, and $t_E = 4\times10^8$ yr) [4]

Answer: The final SMBH mass is $M_{\rm BH} = M_{\rm BH,seed} \times \exp(T/t_{ef})$. [1] Substituting, one gets $M_{\rm BH} = 10^4 \times \exp(0.4/t_{ef})$ [1], with $t_{ef} = 4 \times 10^7$ yr=0.04 Gyr. [1] The final SMBH mass is thus $M_{\rm BH} \sim 2.2 \times 10^8\,\mathrm{M}_\odot$. [1]

Covered in lecture and problem sheets

(e) The highest redshift SMBH, observed as a quasar, has been detected at a redshift $z \sim 7$. It has a mass of $M_{\rm BH} \sim 2 \times 10^9\,\mathrm{M}_\odot$. What do you think its seed mass was if it had accreted for the whole time since the Big Bang at its Eddington limit? (Simply approximate the cosmic time since the Big Bang as $t(z) = 13.5 \times (1 + z)^{-1.5}$, where z is the cosmological redshift.) [4]

Answer: The time since the Big Bang is $T = t(7) \approx 0.6$ Gyr. [1]
The final SMBH mass is then $M_{\rm BH} = M_{\rm seed} \times \exp T/t_{ef}$, [1]
with $t_{ef} \sim \epsilon t_E/\lambda = 4 \times 10^7$ yr=0.04 Gyr, [0.5]
this yields $\exp T/t_{ef} = 15$ [0.5].
The seed mass is then $M_{\rm seed} = M_{\rm BH}/3.3 \gtrsim 10^6 \sim 600\,\mathrm{M}_\odot$ [1].

Covered in lecture and problem sheets

B2. (a) Assume that we live in a universe populated by galaxies with a luminosity function

$$\frac{\partial n}{\partial L} = \begin{cases} \frac{\Phi_*}{L_*}\left(\frac{L}{L_*}\right)^{\alpha-1} & \text{for } L_{\rm min} \le L \le L_*, \\ 0 & \text{otherwise,} \end{cases}$$

with n is the number density of galaxies with luminosity in the range L and $L + dL$. Here Φ_*, α, $L_{\rm min}$ and L_* are constants. Show that the total density of galaxies in this universe is given by

$$n_{\rm tot} = \frac{\Phi_*}{-\alpha}\left[\left(\frac{L_{\rm min}}{L_*}\right)^{\alpha} - 1\right].$$

[2]

Answer: The total density is given by integration of the luminosity function over all possible luminosities. Given the limits above we have

$$n_{\rm tot} = \int_{L_{\rm min}}^{L_*} \frac{\partial n}{\partial L}\,{\rm d}L\ [1]\ = \Phi_* L_*^{-\alpha}\int_{L_{\rm min}}^{L_*} L^{\alpha-1}\,{\rm d}L = \frac{\Phi_*}{-\alpha}\left[\left(\frac{L_{\rm min}}{L_*}\right)^{\alpha} - 1\right]\ [1]\,.$$

Covered in lecture and problem sheets

(b) The characterization of the spatial distribution of galaxies is usually quantified via the two-point correlation function $\xi(r)$, defined as the excess number of galaxy pairs of a given separation r relative to that expected from a random distribution. The two-point correlation function of a sample of field galaxies is a power-law of the form

$$\xi(r) \simeq (r/r_0)^{-\gamma}$$

with $\gamma \sim 1.8$ and $r_0 \sim 7$ Mpc. The correlation of a similar sample of galaxies in cluster environments can be represented by the same function but with $\gamma \sim 1.8$ and $r_0 \sim 30$ Mpc. Which are more clustered at a scale of 5 Mpc? What do you conclude? What about scales of 1000 Mpc? What do you conclude from both functions being so small at the latter scale? [4]

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Answer: The characteristic correlation length is higher for denser environments such as clusters [1]. Thus galaxies in such environments should usually appear more correlated [1], inhabiting higher density peaks which correspond to more massive host dark matter haloes [1]. At 1000 Mpc all structures tend to follow the general homogeneity of the Universe [1].

Covered in lecture and problem sheets

(c) The cluster has a redshift $z = 0.5$ and acts as a gravitational lens for a background galaxy at redshift $z = 1.5$. The image of the lensed galaxy is distorted into an Einstein ring with measured angular diameter $\theta = 53$ arcseconds. The lensing equation gives the physical radius of the Einstein ring as

$$R_{\mathrm{E}} = \sqrt{\frac{D_{\mathrm{ls}} D_{\mathrm{l}}}{D_{\mathrm{ls}} + D_{\mathrm{l}}}} \sqrt{\frac{2GM}{c^2}},$$

where M is the total mass of the gravitational lens, $G \approx 4.3{\times}10^{-6}\,\mathrm{kpc}\,\mathrm{M}_{\odot}^{-1}\,(\mathrm{km/s})^2$ is the gravitational constant, and c is the speed of light. D_{l} is the distance of the gravitational lens from us and D_{ls} is the distance between the lens and the lensed object. What is the total mass of the cluster in solar masses? (*Hint: Assume we live in an Einstein-de Sitter universe where the distance from us to an object at redshift z is given by $D = 2\left(1 - 1/\sqrt{1+z}\right) c/H_0$, with $H_0 = 70\,km\,s^{-1}\,Mpc^{-1}$.*) [5]

Answer: Using the hint, we find the distance to the cluster as $D_{\mathrm{c}} = 1573$ Mpc [0.5] and the distance to the lensed galaxy as $D_{\mathrm{g}} = 3150$ Mpc [0.5]. The angular radius of the Einstein ring is $\theta_{\mathrm{E}} = 53/2$ arcseconds [0.5]. The physical radius of the Einstein ring is given by $R_{\mathrm{E}} = \theta_{\mathrm{E}} D_{\mathrm{c}} = 179$ kpc [1]. With these values we find that $D_{\mathrm{l}} = D_{\mathrm{c}}$ [0.5] and $D_{\mathrm{ls}} = D_{\mathrm{g}} - D_{\mathrm{c}} = 1577$ Mpc [0.5]. Solving the lensing equation for the cluster mass gives

$$M = \frac{R_{\mathrm{E}}^2 c^2}{2G} \frac{D_{\mathrm{ls}} + D_{\mathrm{l}}}{D_{\mathrm{ls}} D_{\mathrm{l}}} \approx 4.3 \times 10^{14}\,\mathrm{M}_{\odot}.[2]$$

Covered in lecture and problem sheets

(d) More than half of the galaxies in the Universe can be found in larger scale structures called groups and clusters. List the different properties of galaxy groups and galaxy clusters in terms of their sizes, galaxy numbers, velocities and types. Describe the distribution of galaxies within a group or cluster, including a probable explanation of the distribution of galaxy types. [6]

Answer: Galaxy groups are of a few Mpc in diameter and contain less than 100 galaxies, including a few giants, predominantly spirals, orbited by many dwarf galaxies [1]. The galaxies have peculiar velocities of ~ 100 km sec^{-2}. Galaxy clusters are similar in diameter (a few Mpc), but contain 100s to 1000s of galaxies. Peculiar velocities can be up to 1000 km sec^{-2} [1]. In the cluster centre, many giant ellipticals and S0s are found, whereas towards the outskirt the number of spirals increases [1]. The full reasons behind such variations in radial distribution with Hubble type are still somewhat debated. However, it is clear that an evolutionary effect has been at work, in the sense that ellipticals are older and formed at the centre of the highest density peaks [1]. Interactions mainly with the ICL may also strip gas from spirals and thus might explain the higher numbers of S0 galaxies in the overdense environments [1]. Cluster environments can also additional impact especially on smaller galaxies, such as ram-pressure stripping, strangulation, etc... [1].

Covered in lecture and problem sheets

(e) List three pieces of evidence for the existence of dark matter. [3]

Answer: Any three of the following will receive [1] mark:

- Spiral galaxy rotation curves (although *not* unique explanation!)
- Mass measurements of elliptical galaxies from velocity dispersion
- Globular cluster motions in galaxy halos, which can be used to determine the virial mass in the halo. Globular clusters move too fast to be bound to the parent galaxy without any dark matter in the halo

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(*but* MOND predictions may still apply here as well...).

- Mass measurements of galaxy clusters from the peculiar motions of galaxies
- Mass measurements from X-ray emission from clusters
- Mass measurements from gravitational lensing in galaxy clusters
- Simulations of structure formation in the Universe require dark matter to explain the observed structure.

Covered in lecture and problem sheets

B3. (a) The total flux observed from an elliptical galaxy is $F = 5.5 \times 10^{-15}\,\mathrm{Wm}^{-2}$, and the galaxy's radial velocity is measured to be 1370 km s^{-1}. Calculate the luminosity in solar luminosities and state whether the galaxy is a giant or a dwarf. You may assume that 1 pc = 3.086×10^{16} m, and $H_0 = 73$ km s^{-1} Mpc^{-1}. [4]

Answer: Use the Hubble law to calculate the distance: $v_r = H_0 D$, so $D = v_r/H_0 = 1370/73 = 18.77$ Mpc [1]
Next calculate the luminosity: L = 4 π d^2 F = 4 π $(18.77 \times 10^6 \times 3.086 \times 10^{16})^2 5.5 \times 10^{-15} = 2.3 \times 10^{34}$ W [1].
Expressed in solar luminosities: L = $2.3 \times 10^{34}/3.85 \times 10^{26} = 6 \times 10^7$ L$_\odot$ [1].
The galaxy has a luminosity L $< 10^9$ L$_\odot$ and is classified as a dwarf galaxy. [1]

Covered in lecture and problem sheets

(b) Explain what is meant with “cosmic downsizing”. [5]

Answer: Cosmic downsizing denotes the shift of star forming activity from larger galaxies to smaller galaxies over cosmic time [1]. This is because the densest regions in the Universe, home of giant ellipticals in galaxy clusters, have collapsed first under gravity [1]. This explains why ellipticals are “old, red and dead”: their stars formed a long time ago [1]. The lower density regions, however, are still collapsing, and the galaxies inside them have a higher probability to still be star-forming [1]. AGN also undergo cosmic downsizing [1].

Covered in lecture and problem sheets

(c) Explain why the distances to the most distant galaxies cannot be measured from shifted spectral lines, and describe the method that can be used instead. [4]

Answer: The most distant galaxies are too faint to obtain spectra, so

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we cannot measure redshifts from spectral lines [1]. Instead, we make use of the Lyman break, which is a strong absorption feature due to the absorption of atomic hydrogen and appears at UV wavelengths in the rest frame, and leads to a drop in UV flux [1]. In distant galaxies, the Lyman break is shifted to much longer wavelengths. [1] We can then use photometry and look for Lyman break dropouts, that is galaxies that drop out at longer wavelengths which correspond to the red shifted Lyman break [1].

Covered in lecture and problem sheets

(d) For each of the following three astronomical objects, detail one possible method to determine the distance to the object.
(i) A nearby star, less than 100 pc away.
(ii) A galaxy, less than 1 Mpc away.
(iii) A galaxy at a redshift larger than $z > 4$. [7]

Answer:

(i) Parallax measurements [1]: The star seems to move (against distant background objects) due to the motion of Earth around the Sun [0.5], measure it's position difference within half a year gives the parallax according to $d = 1/p$ where d is in pc and p is in arcsec [0.5].

(ii) Use either Cepheids (or possibly other variables like RR Lyrae, but the relationship is best calibrated for Cepheid stars) period-luminosity relationship [1]: The Cepheids apparent magnitude and period is measured [1], and compared to the calibrated period-luminosity function, so that the absolute magnitude can be calculated.
Or: Use spectral redshift measurements [1]: Features in the galaxy's spectrum will be shifted towards the red due to its motion with the Hubble flow [1], according to $v_r = H_0 D$.

(iii) Use the Lyman α break [1]: The Lyman break is a strong absorption feature in galaxy spectra [0.5] that occurs in the UV-band in the rest-frame

[0.5]. Due to the large distance and hence strong redshift of the galaxy in question, the Lyman break is shifted to redder wavebands [0.5]. Such distant galaxies can thus be detected at longer wavebands, but drop out at in wavebands that correspond to their redshifted Lyman break [0.5].

Covered in lecture and problem sheets

END OF PAPER