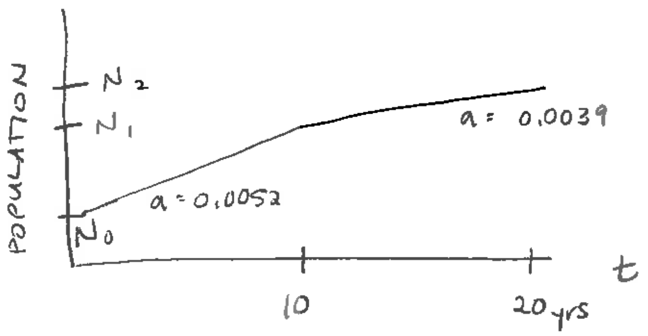
(A1)




[2 FOR GRAPH]

THE POPULATION

$$P(t) = e^{t/T}$$

WHERE

$$T = \frac{1}{\ln(1+r)}$$

IS THE TIME CONSTANT & r IS THE GROWTH RATE.

ON A PLOT OF $\log_{10} P$ AGAINST t,

THE SLOPE IS $\dfrac{(\log_{10} e)}{T} \approx \dfrac{0.434}{T}$

$T(1.2\%) = \dfrac{1}{\ln(1+0.012)} = 84 \Rightarrow$ SLOPE $= 0.0052$ [1]

$T(0.9\%) = \dfrac{1}{\ln(1+0.009)} = 112 \Rightarrow$ SLOPE $= 0.0039$ [1]

USING THE 'RULE OF 70',

THE DOUBLING TIME IS $t_d = \dfrac{70}{(r \text{ IN } \%)}$ [½]

SO $t_d(r = 1.2\%) = \dfrac{70}{1.2} = 58$ YEARS [½]

$t_d(r = 0.9\%) = \dfrac{70}{0.9} = 78$ YEARS [½]

AND THE POPULATION TAKES 20 YEARS LONGER TO DOUBLE [½]

(NEW PROBLEM)

(A3) THE SUESS EFFECTS PROVIDE OBSERVATIONAL EVIDENCE THAT THE OBSERVED INCREASE IN ATMOSPHERIC $CO_2$ CONCENTRATIONS IS DUE TO THE BURNING OF FOSSIL FUELS. [1/2]

1. FOSSIL FUELS ORIGINATE FROM PLANTS THAT DIED MILLIONS OF YEARS AGO, SO THEIR $^{14}C$ CONTENT HAS DECAYED RELATIVE TO ATMOSPHERIC LEVELS [1] AS $^{14}C$ HAS A HALF LIFE OF 5700 YEARS. THUS $CO_2$ COMING FROM FOSSIL FUEL BURNING IS DEPLETED IN $^{14}C$ & WILL CAUSE THE ATMOSPHERIC PERCENTAGE OF $^{14}C$ TO DECREASE. [1]

2. PHOTOSYNTHESIS LEAVES PLANTS DEPLETED IN $^{13}C$. [1] HENCE FOSSIL FUELS ARE ALSO DEPLETED IN $^{13}C$ & BURNING THEM SHOULD CAUSE A DECREASE IN ATMOSPHERIC $^{13}C$ RATIOS. [1]

BOTH EFFECTS HAVE BEEN CLEARLY OBSERVED. [1/2]

(BOOKWORK)

A3 ENERGY TO RAISE A VOLUME V OF WATER BY $\Delta T$ :

$$E = V C_V \Delta T \qquad [1/2]$$

POWER IS ENERGY PER UNIT TIME ;

$$P = \frac{E}{\Delta t} \qquad [1/2]$$

SO THE TIME IS

$$\Delta t = \frac{E}{P}$$

$$= \frac{V C_V \Delta T}{P}$$

$$= \frac{1.7\,\ell \cdot 1.2\,\frac{Wh}{\ell\,°C} \cdot (100 - 10)\,°C}{2000\ W}$$

$$= 0.092\ h \cdot 60\ \frac{min}{h}$$

$$= 5.5\ min \qquad [1]$$

(NEW PROBLEM, PARTLY SIMILAR TO PROBLEM IN LECTURE)

A4 $$P = \frac{1}{e}\left(\frac{m_{car}V^3}{2d} + \frac{C_d \rho_{air} A_{car} V^3}{2} + C_{rr} m_{car} g v\right)$$

1ST TERM: CHANGING SPEED (BRAKING & ACCELERATING) & DIRECTION [1]

2nd TERM: OVERCOMING AIR RESISTANCE [1]

3rd TERM: OVERCOMING ROLLING RESISTANCE [1]

$e$ REFERS TO THE EFFICIENCY OF THE ENGINE: MOST (~75%) OF ENERGY IS CONVERTED TO HEAT RATHER THAN THE CAR'S MOTION [1]

(BOOKWORK)

A5

1. FISSION: HEAVY NUCLEI ARE SPLIT BY NEUTRON CAPTURE INTO SMALLER NUCLEI, RELEASING BINDING ENERGY [1/2]

ADVANTAGES: (NAME ONE) [1/2]

CURRENTLY TECHNOLOGICALLY FEASIBLE
BREEDER REACTORS CAN BE QUITE EFFICIENT
TOTAL WASTE MASS IS SMALL
(OTHERS POSSIBLE)

DISADVANTAGES: (NAME ONE) [1/2]

SAFETY CONCERNS
HAZARDOUS WASTE
UK MUST IMPORT URANIUM/OTHER FUEL
(OTHERS POSSIBLE)

2. FUSION: TWO LIGHT NUCLEI MERGE TO FORM A HEAVIER ONE, ALSO RELEASING BINDING ENERGY [1/2]

ADVANTAGES: (NAME ONE) [1/2]

HUGE POWER POTENTIAL
LARGE FUEL RESERVES (LITHIUM/DEUTERIUM)
MELTDOWN NOT POSSIBLE
HARMLESS WASTE (He)
(ETC.)

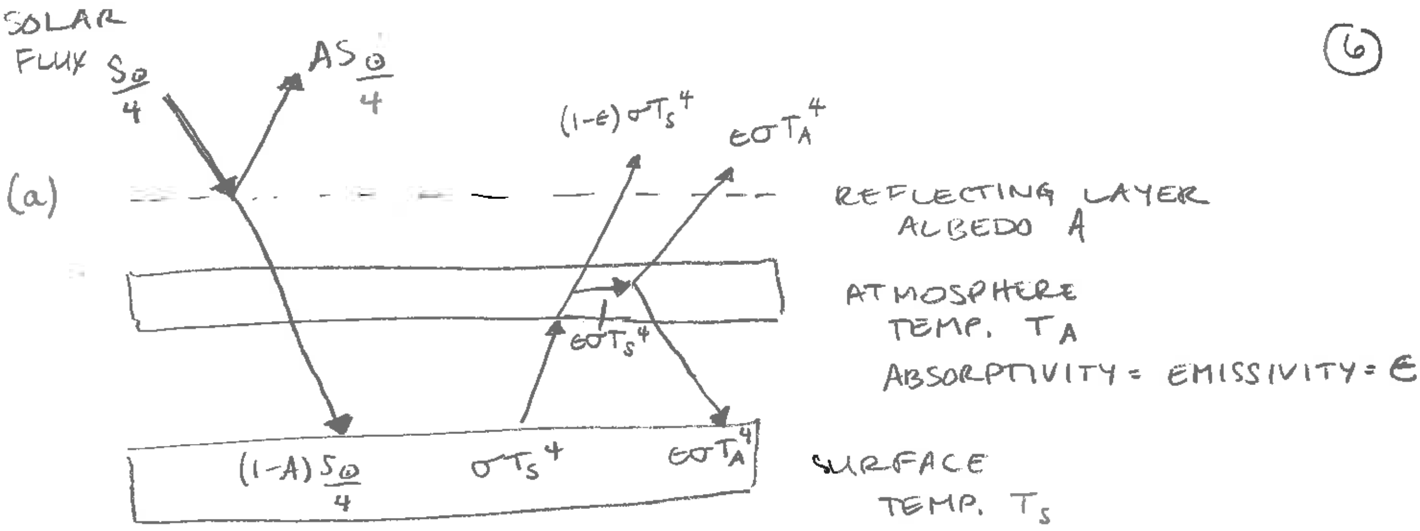
DISADVANTAGES: (NAME ONE) [1/2]

NOT YET COMMERCIALLY VIABLE
REQUIRES HIGH TEMPS
DIFFICULT TO CONFINE PLASMA
(ETC.)

(BOOKWORK)

B1

(a)




[1] FOR LAYERS

[2] FOR ARROWS

[2] FOR FLUXES

(BOOKWORK)

(b) IN THERMAL EQUILIBRIUM, [½] INCOMING FLUX / POWER / RADIATION MUST BE EQUAL TO OUTGOING FLUX / POWER / RADIATION [½]

- OR -

$$\epsilon_{abs} = \epsilon_{emiss}$$

OTHERWISE THERE WOULD BE A NET ENERGY GAIN OR LOSS, LEADING TO A TEMP. CHANGE, HENCE NOT THERMAL EQUILIBRIUM [1]

(BOOKWORK)

(c) SURFACE :

$$(1-A)\frac{S_\odot}{4} + \epsilon\sigma T_A^4 = \sigma T_s^4 \qquad [2]$$

ATMOSPHERE :

$$\epsilon\sigma T_s^4 = \underbrace{\epsilon\sigma T_A^4}_{\text{UPWARD}} + \underbrace{\epsilon\sigma T_A^4}_{\text{DOWNWARD}} \qquad [2]$$

$$= 2\epsilon\sigma T_A^4$$

(BOOKWORK)

B1 cont.

(d) RADIATIVE FORCING IS DEFINED AS THE EQUIVALENT FLUX $\Delta F$ REQUIRED TO PRODUCE THE SAME CHANGE IN SURFACE TEMP. $\Delta T_S$ AS A CHANGE IN ANY OTHER PARAMETER. [2]

(BOOKWORK)

TWO APPROACHES

i) A CHANGE IN THE ALBEDO $\Delta A$ MEANS THAT THE REFLECTED LIGHT BECOMES $(A + \Delta A)\frac{S_\odot}{4}$. [1]

HENCE THE LIGHT REACHING THE SURFACE IS

$$(1 - A - \Delta A)\frac{S_\odot}{4}. \quad [2]$$

THIS CORRESPONDS TO AN EXTRA FLUX

$$\Delta F = -\Delta A \frac{S_\odot}{4} \quad [2]$$

— OR —

ii) USE EQUATIONS FROM (c)

$$① \quad (1 - A - \Delta A)\frac{S_\odot}{4} + \epsilon\sigma(T_A + \Delta T_A)^4 = \sigma(T_S + \Delta T_S)^4 \quad [1/2]$$

$$② \quad \epsilon\sigma(T_S + \Delta T_S)^4 = 2\epsilon\sigma(T_A + \Delta T_A)^4 \quad [1/2]$$

$$② \rightarrow \quad (T_A + \Delta T_A) = 2^{-1/4}(T_S + \Delta T_S) \quad [1/2]$$

PUTTING THIS BACK INTO ①,

$$(1 - A - \Delta A)\frac{S_\odot}{4} = \sigma\left(1 - \frac{\epsilon}{2}\right)(T_S + \Delta T_S)^4 \quad [1/2]$$

USE BINOMIAL APPROXIMATION:

$$(1 - A - \Delta A)\frac{S_\odot}{4} = \sigma\left(1 - \frac{\epsilon}{2}\right)T_s^4\left(1 + 4\frac{\Delta T_s}{T_s}\right) \qquad [½]$$

REARRANGE:

$$\Delta T_s = -T_s + \frac{[1-A]S_\odot/4}{4\sigma(1-\frac{\epsilon}{2})T_s^3} - \frac{\Delta A S_\odot/4}{4\sigma(1-\frac{\epsilon}{2})T_s^3} \qquad [½]$$

BUT FROM THE UNPERTURBED CASE WE KNOW

$$\frac{[1-A]S_\odot/4}{4\sigma(1-\frac{\epsilon}{2})} = T_s^4 \qquad [½]$$

$$\Delta T_s = -\frac{\Delta A S_\odot/4}{4\sigma(1-\frac{\epsilon}{2})T_s^3}$$

$$= -\frac{\Delta A S_\odot}{16\sigma(1-\frac{\epsilon}{2})T_s^3} \qquad [½]$$

RECALL THAT FOR AN ADDITIONAL FLUX $\Delta F$

$$\Delta T_s = \frac{\Delta F}{4\sigma(1-\frac{\epsilon}{2})T_s^3} \qquad [½]$$

SO

$$\frac{\Delta F}{\cancel{4}\cancel{\sigma}\cancel{(1-\frac{\epsilon}{2})}\cancel{T_s^3}} = \frac{-\Delta A S_\odot}{\underset{4}{\cancel{16}}\cancel{\sigma}\cancel{(1-\frac{\epsilon}{2})}\cancel{T_s^3}}$$

AND

$$\Delta F = -\frac{\Delta A S_\odot}{4} \qquad [½]$$

(SIMILAR DONE IN LECTURE)

(e) $\Delta A = -\Delta F \left(\frac{4}{S_0}\right)$

$= (2.5 \, W/m^2)\left(\frac{4}{1400 \, W/m^2}\right)$

$= 0.007$ [1]

(NEW PROBLEM)

RADIATIVE FORCING FOR $CO_2$ - DOUBLING IS $3.7 \, W/m^2$ [1]

(BOOKWORK)

**B2**.

(a) Estimate how much power, in kWh/day, is used on average per passenger on plane flights, assuming one long-distance round trip per year. You may assume a range of 8000 miles with a fuel capacity of 240,000 litres, and a passenger capacity of 400 persons for a B747 jet. You may further assume an energy density of 10 kWh/litre of fuel. **[2]**

Answer: The amount of energy used per passenger on one round trip is the amount of fuel used per round trip $\times$ the energy per unit fuel, divided by the number of passengers [1].
So we have: $P = \frac{2\times 240000\text{l}\times 10\text{kWh}}{400\times 365\text{pld}} = 33$ kWh/d per person [1].

*Bookwork*

(b) Show that the power required to keep an airplane up in the air and moving forward can be described as

$$P = \frac{1}{\epsilon}\left(\frac{m_{plane}^2 g^2}{2\rho_{air}A_{tube}v} + \frac{\rho_{air}A_{tube}v^3}{2}\right)$$

where $\epsilon$ is the engine efficiency, $m_{plane}$ the mass of the plane, $\rho_{air}$ the air density, $v$ the speed of the plane, and $A_{tube}$ the cross sectional area of air tube created by the plane as it moves through the air. **[13]**

Answer: In order to stay up, the plane needs to overcome gravity. It does so by "throwing air down" (which is achieved by the tilt of the wings). The plane moves through the air and pushes down on the air in the tube. The air tube, in return, pushes back and "lifts" the plane. So the energy required to keep the plane up is just the energy of the downward motion of the air particles in the tube: $E_{kin} = \frac{1}{2}m_{air}u^2$ [1].
Next, we need to find an expression for the air mass and the downward speed $u$ of the air particles.
The mass of the air tube can be expressed as $m_{air} = \rho_{air}V$ and $V = A_{tube} \times l_{tube}$ [1/2] the volume of the air tube, where $l_{tube} = v \times t$ [1/2] the length of the air tube and $v$ is the forward speed of the plane (which defines the length of the air tube the plane passes through) [1].
So we have $m_{air} = \rho_{air}A_{tube}vt$ [1]
To figure out the downward speed, we have to look at the forces acting on the plane:
$F = m_{plane}g$ is the gravity acting on the plane [1]
$F = m_{air}a$ is the force of the plane acting on the tube, where $a = u/t$ the acceleration of the air particles [1].
Both forces must balance each other, so we have:

$m_{plane}g = m_{air}a$. Solving for $a$ and $u$, we get: $a = \frac{m_{plane}g}{m_{air}} = \frac{u}{t}$
$u = \frac{m_{plane}gt}{m_{air}} = \frac{m_{plane}gt}{\rho_{air}A_{tube}vt} = \frac{m_{plane}g}{\rho_{air}A_{tube}v}$ [1]
Put together, we have: $E_{kin} = \frac{1}{2}m_{air}u^2 = \frac{1}{2}\rho_{air}A_{tube}vt\frac{m_{plane}^2g^2}{\rho_{air}^2A_{tube}^2v^2} = \frac{m_{plane}^2g^2t}{2\rho_{air}A_{tube}v}$ [1]
And since $P = E/t$, $P = \frac{m_{plane}^2g^2}{2\rho_{air}A_{tube}v}$ is the power requirement for staying up [1].

Next, we need the power requirement for moving forward. To do this, the plane needs to overcome the air resistance. Again, we look at the kinetic energy of the air particles in the tube which are disturbed by the plane moving through the air tube: $E_{kin} = \frac{1}{2}m_{air}v^2$ where $v$ is the speed of the disturbed air particles, corresponding to the speed of the plane [1]. This gives us $E_{kin} = \frac{1}{2}\rho_{air}A_{tube}vtv^2 = \frac{\rho_{air}A_{tube}v^3t}{2}$ [1], and because $P = E/t$ we get $P = \frac{\rho_{air}A_{tube}v^3}{2}$ the power requirement to overcome air resistance and move forward [1].
Finally, since no engine is 100% effective, we need to include the engine efficiency. [1]
So in total we get a power requirement of $P_{tot} = \frac{1}{\epsilon}(\frac{m_{plane}^2g^2}{2\rho_{air}A_{tube}v} + \frac{\rho_{air}A_{tube}v^3}{2})$.

*Bookwork*

(c) The plane-based energy cost can be expressed as $EC = \frac{1}{\epsilon}m_{plane}g(c_df_A)^{1/2}$ where $(c_df_A)^{1/2}$ is the drag-to-lift ratio. Based on this equation, how can the energy consumption of planes be improved? Are any of these approaches realistic? **[5]**

Answer: To lower our energy consumption, i.e. to lower our energy cost, the mass of the plane should be small [1], the drag-to-lift ratio should be small [1], and the engine efficiency should be high [1]. As it is, planes are already highly efficient [1], so it is probably not realistic to think that we can make much of an improvement [1].

[Answers that indirectly address the first three criteria may be accepted as well, e.g. the use of reinforced carbon-fibre to reduce weight]

*Bookwork*

**B3**

(a) Solar energy is a pervasive form of energy that has many effects on Earth. List four modes of solar energy that can be used for power production. You may include direct (i.e. first order) and indirect (second order) modes of power production. **[4]**

Answer: Any four of the items listed below are awarded one mark each:

- Solar Thermal
- Solar Photovoltaic
- Biomass
- Wind Power
- Wave Power
- Hydroelectricity
- Fossil Fuels

*Bookwork*

(b) On average, the solar power reaching the surface of the Earth (after albedo reflection and atmospheric absorption) is 170 W $m^{-2}$. This is, however, derived assuming that the sun is directly overhead. The UK is at a latitude of 52° North, and Earth's spin axis has a tilt of 23°. How much solar power does the UK receive in mid-summer, in mid-winter, and on average? Hint: in summer, the spin axis is tilted towards the Sun, in winter, it is tilted away from the Sun. **[8]**

Answer: If the Sun is not directly overhead but at an angle $l$, the Sun "sees" a projected area of $A_{projected} = A \times \cos l$ [1]. In mid-summer, the Sun is $l = 52° - 23° = 29°$ away from being overhead [1]. That means that for a unit area of 1 $m^2$, the Sun actually only "sees" $\cos l = \cos 29° = 0.87 = 87\%$ of the 1 $m^2$. The solar power is reduced accordingly to 148 $\approx$ 150 W $m^{-2}$ [1]. In mid-winter, the Sun is $l = 52° + 23° = 75°$ away from being overhead [1]. That means that for a unit area of 1 $m^2$, the Sun actually only "sees" $\cos l = \cos 75° = 0.26 = 26\%$ of the 1 $m^2$. The solar power is reduced accordingly to 44 W $m^{-2}$ [1]. So, on average, the Sun is just 52° away from being overhead [1], reducing the incoming solar power by $\cos 52° = 0.62 = 62\%$ [1] to 105 W $m^{-2}$ [1]. (allow for rounding differences)

*Bookwork*

(c) Assuming an average incoming solar power of 110 W $m^{-2}$ for south-facing roofs, and further assuming an efficiency of 50% for solar thermal panels, how much power can be produced in the UK using solar thermal? You may further assume an average 10 $m^2$ of south-facing roof area per person to put the solar thermal panels on, and a population of 60 million people in the UK. **[2]**

Answer: The power produced is the available area times power per area divided by the number of people, including the efficiency and converting to our standard units of kWh/dp, we get: $\frac{0.5 \times 10\mathrm{m}^2 \times 60 \times 10^6 \times 110\mathrm{W} \times 24\mathrm{h}}{60 \times 10^6 \mathrm{pdm}^2}$ [1] $= \frac{10\mathrm{m}^2 \times 110\mathrm{W} \times 24\mathrm{h}}{2\mathrm{pdm}^2} = 13.2$ kWh/dp [1].

*Bookwork.*

(d) Explain in no more than 100 words the basic physics behind the photovoltaic effect and why there is a limit to the efficiency of photovoltaic cells. **[6]**

Answer: PV cells are semiconductors containing atoms and electrons in a bound lattice [1]. If enough energy (i.e. from sun light) is given to an electron in the lattice, it is freed and can move freely, producing a current [1]. The energy needed to free an electron is the critical $E_{crit}$, so that any photon with $E_{crit}$ is converted to electricity with a $\epsilon = 100\%$ efficiency [1]. However, photons with energies below $E_{crit}$ are not converted to electricity and are wasted ($\epsilon = 0\%$) [1]. For photons with energies larger than $E_{crit}$, the excess energy is also wasted ($\epsilon = E/E_{crit} - 1$) [1]. This leads to the so called Shockleigh-Queisser limit [1], and limits the efficiency of standard PV cells to 30%.

*Bookwork*