SEMESTER 1 EXAMINATION 2013-2014

WAVE PHYSICS

Duration: 120 MINS (2 hours)

This paper contains 9 questions.

Answers to Section A and Section B must be in separate answer books

Answer **all** questions in **Section A** and **only two** questions in **Section B**.

Section A carries 1/3 of the total marks for the exam paper and you should aim to spend about 40 mins on it.

Section B carries 2/3 of the total marks for the exam paper and you should aim to spend about 80 mins on it.

An outline marking scheme is shown in brackets to the right of each question.

A Sheet of Physical Constants is provided with this examination paper.

Only university approved calculators may be used.

A foreign language translation dictionary (paper version) is permitted provided it contains no notes, additions or annotations.

Section A

A1. A clarinet may be considered to be a tube which is closed at one end and open at the other, whereas a flute may be considered to be a tube which is open at both ends. Explain why it is that when one over-blows a clarinet, thereby suppressing the fundamental frequency, the frequency triples, whereas in the case of a flute the frequency doubles. [4]

A2. Explain why it is that when a string instrument is incorrectly bowed, thereby exciting longitudinal as well as transverse waves on the string, a high-pitched sound (a "squeak") is produced. [3]

A3. For a wave on a transmission line with capacitance C per unit length and inductance L per unit length (and negligible resistance) the voltage, $V(x,t)$ and current $I(x,t)$ are related by

$$\sqrt{C}\, V(x,t) \;=\; \sqrt{L}\, I(x,t).$$

Show that at a boundary between two such transmission lines with capacitance per unit length, C_1 and C_2 respectively and inductance per unit length L_1 and L_2 respectively, the (amplitude) reflection coefficient $\mathcal{R}$ is given by

$$|\mathcal{R}| \;=\; \left| \frac{\sqrt{L_2 C_1} - \sqrt{L_1 C_2}}{\sqrt{L_2 C_1} + \sqrt{L_1 C_2}} \right| .$$

[5]

A4. The wave velocity v of water waves depends on the depth d of the water as

$$v \propto \sqrt{d}.$$

Explain why the amplitude of waves on the sea increases as the wave approaches the shore. [3]

A5. Explain why it is that if a lens with one plane surface and one convex hemi-spherical surface, is placed curved side downwards on a mirror and illuminated from above with monochromatic light, alternate light and dark circular fringes are observed. [3]

Explain why there is a dark spot at the centre. [2]

TURN OVER

Section B

B1. (a). Derive the wave equation for transverse waves on a string whose mass per unit length is μ under tension T and derive the expression for the velocity, c, of the transverse waves, in terms of T and μ. [6]

(b). A string of mass μ per unit length has length L and is fixed at both ends under tension T.

Show that the general expression for the transverse disturbance, $\Psi(x,t)$, (with appropriate boundary conditions) for such a string, assumed to be momentarily stationary at time $t = 0$, can be written as

$$\Psi(x,t) \ = \ \sum_{n=1}^{\infty} A_n \sin\left(\frac{n\pi x}{L}\right)\cos(\omega_n t),$$

explaining why n must be integer. [5]

Write down an expression for ω_n in terms of n, μ, L and T. [1]

(c). Show that if the disturbance at time $t = 0$ is given by the function $\phi(x)$, the coefficients A_n are given by

$$A_n \ = \ \frac{2}{L}\int_0^L \phi(x)\sin\left(\frac{n\pi x}{L}\right)\, dx. \qquad [\ 5\ ]$$

$$\left[\text{Note that } \int_0^L \sin\left(\frac{n\pi x}{L}\right)\sin\left(\frac{m\pi x}{L}\right)\, dx \ = \ \frac{L}{2}\delta_{mn}.\right]$$

(d). For the case where $\phi(x)$ is given by

$$\phi(x) \ = \ x(L-x),$$

write down the first 3 coefficients A_n, $(n = 1,\ 2,\ 3)$. [3]

$$\left[\int_0^L x(L-x)\sin\left(\frac{n\pi x}{L}\right)\, dx \ = \ \frac{2L^3\,(1-(-1)^n)}{n^3\pi^3}.\right]$$

B2. The wave equation for a (one-dimensional) wave disturbance, $\Psi(x,t)$, in a particular dispersive medium is given by

$$\frac{\partial^2\Psi(x,t)}{\partial x^2} - \alpha^2\Psi(x,t) = \frac{1}{c^2}\frac{\partial^2\Psi(x,t)}{\partial t^2}.$$

(a). Show that the dispersion relation between the angular frequency, ω, and the wavenumber, k, is given by

$$\omega^2 = c^2k^2 + c^2\alpha^2.$$ [5]

(b). Explain how such a medium can be used as a high-pass filter, and write down an expression in terms of c and α for the critical angular frequency, ω_c, below which the wave is attenuated. [4]

(c). For an angular frequency, ω, above the critical angular frequency, ω_c, write down expressions (in terms of c, α and ω) for the phase velocity and group velocity of the wave. [4]

(d). For the case of an angular frequency, ω, below the critical angular frequency, ω_c, find an expression for the attenuation length (i.e. the distance over which the amplitude is reduced by a factor of e). [3]

(e). Find an expression for the distance Δx over which the wave intensity is attenuated by one decibel. [4]

TURN OVER

B3. (a). The number of standing waves, $N_\omega d\omega$, per unit volume with angular frequency between ω and $\omega + d\omega$ is given by

$$N_\omega d\omega = \frac{1}{2\pi^2 c^3}\omega^2 d\omega,$$

where c is the wave velocity.

How many modes of electromagnetic radiation are there in a rectangular box cavity with sides 2.5 cm, 3.5 cm and 4.5 cm, with wavelength λ, in the visible light range

$$390\ \mathrm{nm} \le \lambda \le 700\ \mathrm{nm}?$$ [6]

(b). The wave-vector $\mathbf{k}$ for a standing wave in a rectangular box cavity with sides a, b, d, whose disturbance vanishes on the walls of the cavity, is

$$\mathbf{k} = \left(\frac{n_x\pi}{a}, \frac{n_y\pi}{b}, \frac{n_z\pi}{d}\right),$$

where n_x, n_y and n_z are positive integers.

What is the longest wavelength of a standing wave in the cavity of part (a)?

[5]

(c). A tenor likes to sing in his shower. The shower is an enclosed rectangular box for which the longitudinal displacement of the sound-wave vanishes on the walls, floor and ceiling. The shower has a square base of side 0.810 m. The lowest note which resonates (the lowest frequency standing wave) is the D above middle C with a frequency of 294 Hz. What is the height of the shower? [5]

(Take the velocity of sound to be 330 $\mathrm{m\ s^{-1}}$.)

(d). What is the frequency of the next lowest note that resonates ? [4]

B4. (a). A (transmission) diffraction grating has N very narrow lines each separated by a distance a. It is uniformly illuminated at normal incidence with monochromatic light of wavenumber k.

Show that the intensity, $I(\theta)$, of light viewed at an angle θ to the normal is given by

$$I(\theta) \;=\; \frac{I(0)}{N^2}\frac{\sin^2\left(\frac{1}{2}Nka\sin\theta\right)}{\sin^2\left(\frac{1}{2}ka\sin\theta\right)}.$$ [6]

$$\left[\ \text{Note that:}\ \sum_{j=0}^{N-1} e^{ij\delta} \;=\; e^{i\delta(N-1)/2}\frac{\sin\left(\frac{1}{2}N\delta\right)}{\sin\left(\frac{1}{2}\delta\right)}\ .\right]$$

(b). Write down an expression for the values of θ for which there are principal maxima. [2]

(c). Write down an expression for the values of θ for which there are subsidiary minima. [2]

(d). How many subsidiary minima are there between two adjacent principal maxima? [1]

(e). Such a grating has 10^4 rulings with a spacing of 10^{-5} m. It is illuminated at normal incidence with light of wavelength 550 nm.
Find the angle of diffraction for the 3^{rd} principal maximum.
[The central maximum is defined as the zeroth principal maximum.] [2]

(f). Find the angular separation between the 3^{rd} principal maximum and the adjacent subsidiary minimum.
[You may assume the small angle approximation $\theta \approx \sin\theta$.] [4]

TURN OVER

(g). Using the Rayleigh criterion, namely that two spectral lines can just be resolved if the maximum for one spectral line coincides with the first adjacent minimum for the other, determine the wavelength of light that can just be resolved from the 550 nm line, using the third principal maximum. [3]

END OF PAPER