UNIVERSITY OF SOUTHAMPTON PHYS2023W1

SEMESTER 1 EXAMINATION 2012/13

WAVE PHYSICS

Duration: 120 MINS

VERY IMPORTANT NOTE

Section A answers MUST BE in a separate blue answer book. If any blue answer booklets contain work for both Section A and B questions - the latter set of answers WILL NOT BE MARKED.

Answer ***all*** *questions in* ***Section A*** *and two* ***and only two*** *questions in* ***Section B****.*

Section A carries 1/3 of the total marks for the exam paper and you should aim to spend about 40 mins on it. Section B carries 2/3 of the total marks for the exam paper and you should aim to spend about 80 mins on it.

A Sheet of Physical Constants will be provided with this examination paper.

An outline marking scheme is shown in brackets to the right of each question.

Only university approved calculators may be used.

Number of Pages 9

Section A

A1. A disturbance, $\Psi(x,t)$, on a string of length L, fixed at both ends, may be expanded as

$$\Psi(x,t) \;=\; \sum_{n=1}^{\infty} A_n \sin\left(\frac{n\pi x}{L}\right) \cos\left(\frac{n\pi c t}{L}\right)$$

(where c is the wave-velocity).

Explain why n must be integer. [1]

Show that the coefficients A_n are given by

$$A_n \;=\; \frac{2}{L}\int_0^L dx\,\Psi(x,0)\sin\left(\frac{n\pi x}{L}\right) \qquad [4]$$

$$\left[\int_0^L dx \sin\left(\frac{n\pi x}{L}\right)\sin\left(\frac{m\pi x}{L}\right) \;=\; \frac{L}{2}\delta_{mn}\right]$$

A2. The disturbance, $\Psi(x,t)$ on a stiff string obeys the dispersive wave-equation

$$\frac{\partial^2\Psi(x,t)}{\partial t^2} \;=\; c^2\frac{\partial^2\Psi(x,t)}{\partial x^2} - \alpha\frac{\partial^4\Psi(x,t)}{\partial x^4}$$

By considering a harmonic-wave solution with wavenumber k and angular frequency $\omega(k)$, show that the dispersion relation obeyed by the waves on this string is given by

$$\omega(k) \;=\; ck\sqrt{1+\alpha\frac{k^2}{c^2}} \qquad [5]$$

A3. A lens made of glass with refractive index 1.56 is coated with a thin layer of material whose refractive index is n, in order to eliminate reflection of light of wavelength $600\,\mathrm{nm}$ (in air). Calculate the refractive index n and the required thickness of the layer. [5]

[The characteristic impedance of an electromagnetic wave in a given medium is inversely proportional to the refractive index of the medium. The refractive index of air may be taken to be unity.]

A4. A transmission diffraction grating of width $5\,\mathrm{cm}$ has 1000 narrowly ruled lines (perpendicular to the width). Light of wavelength $550\,\mathrm{nm}$ is incident normally on the grating. What is the diffraction angle of the first principal diffraction maximum? [4]

How many subsidiary maxima are there between the first and second principal maxima? [1]

Section B

B1. (a). Derive the wave equation for transverse waves on a string whose mass per unit length is μ under tension T and show that the transverse wave velocity, c_T is given by

$$c_T = \sqrt{\frac{T}{\mu}} \qquad [6]$$

(b). Consider a uniform rod of cross-section A made of material of density ρ with Young's modulus Y. The force at a point x on the rod at time t is given by

$$F(x) = YA\frac{\partial\Psi(x,t)}{\partial x},$$

where $\Psi(x,t)$ is the longitudinal displacement of the material of the rod at point x and time t.

By considering an element of the rod of length dx, derive the wave equation for longitudinal waves on the rod and show that the longitudinal wave velocity, c_L

$$c_L = \sqrt{\frac{Y}{\rho}} \qquad [4]$$

(c). Explain why a violin produces a high-pitched sound when it is incorrectly bowed so as to excite longitudinal as well as transverse vibrations. [2]

(d). A vertical wire made out of steel of density $7.8 \times 10^3\,\mathrm{kg\,m^{-3}}$ and Young's modulus $2 \times 10^{11}\,\mathrm{Pa}$ is fixed at the upper end. The lower end is free, but is attached to a weight of mass $10\,\mathrm{kg}$.

The fundamental frequency of longitudinal waves on the wire is $1400\,\mathrm{Hz}$. What is the (stretched) length of the wire? [3]

(e). The fundamental frequency of transverse waves is 175 Hz. What is the cross-sectional area of the wire? [5]

B2. (a). A source emitting a harmonic sound wave with frequency ν is travelling with velocity u directly towards an observer. Show that the frequency, ν' of the sound wave measured by the observer is given by

$$\nu' = \frac{\nu}{1 - u/c},$$

where c is the velocity of sound in the air. [6]

(Hint: Consider the time difference between the emission of adjacent peaks by the source and the time difference between the receipt of these peaks by the observer.)

(b). In what way is this expression modified if the direction of motion of the source makes an an angle θ with the (instantaneous) line between source and observer. [2]

(c). An ambulance whose siren emits a harmonic sound wave of frequency $440\,\mathrm{Hz}$ is travelling in a straight line at a speed of $33\,\mathrm{m\,s^{-1}}$ past an observer. The distance of closest approach is $19\,\mathrm{m}$. What is the observed frequency of the siren emitted:

(1) One second before the ambulance passes the observer ?

(2) One second after the ambulance passes the observer ? [4]

(Take the velocity of sound in air to be $330\,\mathrm{m\,s^{-1}}$)

(d). For an observer moving with velocity u towards a source emitting a harmonic sound wave with frequency ν, the frequency, ν' measured by the observer is given by

$$\nu' = \nu\left(1 + \frac{u}{c}\right)$$

Explain why it is that the Doppler effect is different from the case of a moving source and stationary observer, whereas in the case of light (travelling in a vacuum), it makes no difference whether it is the source or observer that is moving (as required by the first postulate of Special Relativity). [3]

(e). An ambulance is travelling with velocity $33\,\mathrm{m\,s^{-1}}$ directly towards a reflecting wall and emits a harmonic sound wave of frequency $440\,\mathrm{Hz}$. What is the frequency of the sound reflected from the wall as measured by the ambulance driver? [5]

[The wall reflects sound by emitting sound at the same frequency as it receives it, but in the opposite direction.]

B3. (a). A transmission line has a capacitance C per unit length, an inductance L per unit length, and resistance R per unit length. The voltage $V(x,t)$ across the transmission line and the current, $I(x,t)$, along the transmission line are related by

$$\begin{aligned} C\frac{\partial V(x,t)}{\partial t} &= -\frac{\partial I(x,t)}{\partial x} \\ L\frac{\partial I(x,t)}{\partial t} + RI(x,t) &= -\frac{\partial V(x,t)}{\partial x}. \end{aligned}$$

Show that the wave equation for the current is

$$\frac{\partial^2 I(x,t)}{\partial t^2} + \frac{R}{L}\frac{\partial I(x,t)}{\partial t} = \frac{1}{CL}\frac{\partial^2 I(x,t)}{\partial x^2}$$ [3]

(b). By considering a solution for the current of the form

$$I(x,t) = I_0 e^{i(Kx-\omega t)}$$

and inserting into the wave-equation, show that

$$K = \omega\sqrt{LC}\sqrt{1 + i\frac{R}{\omega L}}$$ [4]

Explain the significance of the imaginary part of K [1]

(c). The characteristic impedance of the transmission line is the ratio of the voltage to the current. Show that the (complex) characteristic impedance for a transmission line with resistance is given by

$$Z = \sqrt{\frac{L}{C}}\sqrt{1 + i\frac{R}{\omega L}},$$

for a wave with angular frequency ω. [3]

(d). Z is complex. Explain what this complexity means in terms of the relative phase of the voltage and current waves. [1]

(e). A transmission line has $C = 2 \times 10^{-11} \mathrm{F\,m^{-1}}$, $L = 8 \times 10^{-7} \mathrm{H\,m^{-1}}$, and $R = 0.1 \Omega\,\mathrm{m^{-1}}$ and carries a wave with angular frequency $10^6\,\mathrm{s^{-1}}$.

What is the phase difference between the voltage and current waves? [2]

[Since $R \ll L\omega$, you may use the approximation

$$\sqrt{1 + i\frac{R}{\omega L}} \approx 1 + i\frac{R}{2\omega L}$$

]

(f). What is the attenuation length (i.e the distance along the transmission line over which the amplitude of the wave decreases by a factor of e)? [3]

(g). Hence find the distance along the transmission line over which the power of the wave is attenuated by 1 dB. [3]

B4. (a). What is the difference between “gravity waves” and “ripples” for water waves at the surface? [2]

(b). The angular frequency, $\omega(k)$, of waves with wavenumber k at the surface of water with depth d is given by

$$\omega(k)^2 \; = \; \left(g + \frac{\sigma k^2}{\rho}\right) k \; \tanh(kd),$$

where σ is the surface-tension of the water, ρ its density and g is the acceleration due to gravity.

Write down an expression for the critical wavelength λ_c where gravity waves take over from ripples. [2]

Do the ripples dominate for shorter or longer wavelengths? [1]

(c). Explain why it is that for waves of wavenumber k in deep water with depth, $d \gg 1/k$, the wave velocity is almost independent of depth, whereas in shallow water ($d \ll 1/k$) the wave-velocity is proportional to $\sqrt{d}$. [4]

$$\left[\tanh(x) \; \approx \; 1, \text{ for } x \gg 1, \quad \text{and} \quad \tanh(x) \; \approx \; x, \text{ for } x \ll 1\right]$$

(d). Explain why it is that in shallow water there is very little dispersion for gravity waves, but there is dispersion for ripples. [6]

(e). Write down an expression for the group velocity for ripples of wavenumber k in shallow water of depth d (in terms of k, σ, ρ and d.) [2]

Is this dispersion normal or anomalous (state your reason) [1]

(f). Explain why it is that as a sea-wave approaches the shore (the depth becomes smaller) the amplitude of the wave increases. [2]

END OF PAPER