Solutions to PHYS3002

A1. (bookwork) Write down 4-momenta of e^+ and e^-

$$P_{e^+} = (m_e c, \vec{0})$$
$$P_{e^-} = (E/c, \vec{P})$$ [1]

using $E = \sqrt{\vec{P}^2 c^2 + m_e^2 c^4}$ [1]

we find

$$(P_{e^+} + P_{e^-})^2 = (E/c + m_e c)^2 - |\vec{P}|^2$$

$$= \frac{E^2}{c^2} + 2E\cdot m_e + m_e^2 c^2 - \frac{E^2}{c^2} + m_e^2 c^2$$ [1]

$$= 2E\cdot m_e + 2m_e^2 c^2 = 2m_e(E + m_e c^2)$$

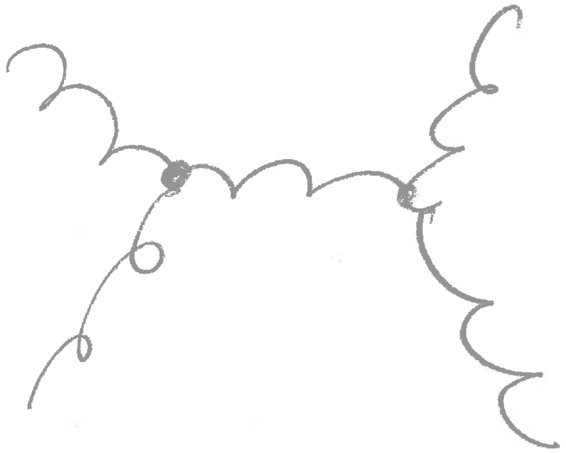
Putting numbers in we get

$$m_{e^+e^-} = \frac{\sqrt{(P_{e^+} + P_{e^-})^2}}{c} = \sqrt{2\cdot 0.5(3.5 + 0.5)}$$

$$= \underline{2\ \mathrm{MeV}/c^2}$$ [1]

A2. (new, based on problem sheets)

Diagrams for $gg \to gg$



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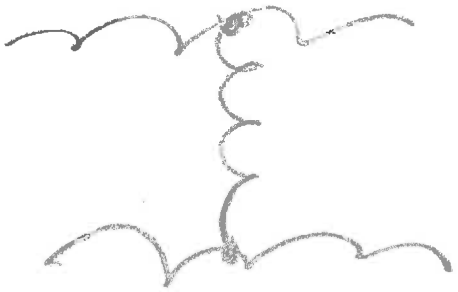

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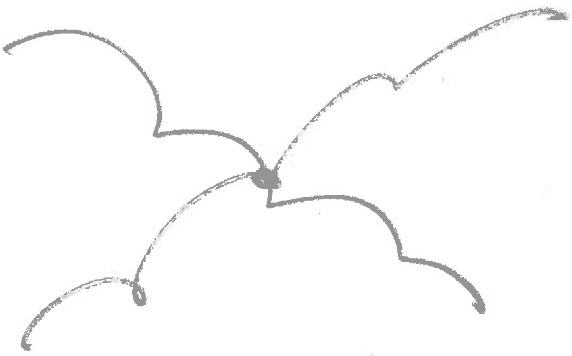


[1]

A3. (bookwork)

The Ratios R_A, R_B are related to τ_A and τ_B as

$$R_A = e^{-T/\tau_A}$$
$$R_B = e^{-T/\tau_B} \qquad [1]$$

so one has

$$\tau_{A,B} = -\frac{T}{\log R_{A,B}} \qquad [1]$$

and therefore

$$\frac{\tilde{\tau}_A}{\tilde{\tau}_B} = \frac{\log R_B}{\log R_A} \qquad [1]$$

A4. (new, based on problem sheets)

the cross section for

$$pp \to H \to ZZ^* \to e^+e^-e^+e^-$$

process is given by

$$\sigma(pp \to e^+e^-e^+e^-) = \sigma(pp \to H \to ZZ^*) \times Br^2(Z \to e^+e^-) \quad [1]$$

$$= \sigma(pp \to H) \times Br(H \to ZZ^*) \times Br^2(Z \to e^+e^-) \quad [1]$$

$$= 30 \times (0.03)^2 = 8.1 \times 10^{-4}\, pb$$

$$= 0.81\, fb \quad [1]$$

Number of events, luminosity and crossection are related by this formula. [1]

$$N = L \times \sigma$$

while accuracy is given by $\varepsilon = \frac{1}{\sqrt{N}}$

$$\Rightarrow N = \frac{1}{\varepsilon^2} = L \times \sigma \quad [1]$$

Putting numbers we get

$$L = \frac{1}{\varepsilon^2 \sigma} = \frac{1}{(0.2)^3 \cdot 0.81} = 30.9\, fb^{-1} \quad [1]$$

A 5. (bookwork)

The 1p shell is split into $j = \frac{1}{2}$ and $j = \frac{3}{2}$ [1]
because of spin-orbital interaction

The level $j = \frac{3}{2}$ is below the $j = \frac{1}{2}$ one because the spin-orbital interaction has a „negative" contribution to the nuclear potential, contrary to the case in Atomic physics [1]

A6. (bookwork) The binding energy is the difference between the sum of the masses of free particles [1] and the mass of the atom they form:

$$B_E = \left|\left(Z \cdot m_p + (A-Z)m_n + Z m_e\right) - M_X\right| c^2$$

[1]

B1. (new, bookwork)

a) β-decay of nucleus A to nucleus B is given by

$$A \to B + e^- + \bar{\nu}_e \quad (\beta^-) \qquad [1]$$

OR $$A \to B + e^+ + \nu_e \quad (\beta^+)$$

Weak interactions are responsible for β-decay [1]

Quantum numbers of e^+, e^-: - charge $\pm$ [1]

spin $= \frac{1}{2}$

Quantum number of $\bar{\nu}_e, \nu_e$: zero charge

spin $= \frac{1}{2}$ [1]

B1 b)

To find the most stable isobar with respect to β- decay we need to minimaze an atomic mass [1]

$$M_A = (Z m_p + (A-Z) m_n + Z \cdot m_e) - BE/c^2 \quad [1]$$

minimisation M_A with respect to Z gives

$$\frac{\partial M_A}{\partial Z} = 0 \Rightarrow (m_p - m_n + m_e)c^2 + \frac{a_c \cdot 2Z}{A^{1/3}} - \frac{2a_A}{A} \cdot 2(A-2Z) = 0 \quad [1]$$

(The last term of the semi-empirical formula vanishes for odd A) [1]

$$\Rightarrow Z = \frac{4a_A - (m_p - m_n + m_e)c^2}{8a_A + 2a_c A^{2/3}} \quad [1]$$

Substituting numbers

$$\Rightarrow Z \approx 31 \quad [1]$$

B1

c) Since A is the same for daughter and parent and since A is odd we need to compare Coulomb and asymmetry terms [1]

The maximum energy of the electron is equal to a difference of binding energies of Daughter and parent nuclei [1]

$$E_1 = BE_D - B_P = \frac{a_c}{A^{1/3}}\left(Z_P^2 - Z_D^2\right) + \frac{a_A}{A}\left((Z_P - N_P)^2 - (Z_D - N_D)^2\right)$$ [1]

plus an energy released in conversion of neutron into proton

$$E_2 = (m_n - m_p - m_e)c^2$$ [1]

Putting numbers, one gets

$$E_1 = \frac{0.697}{69^{1/3}}\left(27^2 - 28^2\right) + \frac{23.285}{69}\left(15^2 - 13^2\right) \simeq 9.6\,\text{MeV}$$ [1]

$$E_2 = 939.57 - 938.27 - 0.511 = 0.79\ \text{MeV}$$

so $E = E_1 + E_2 \simeq 10.4\,\text{MeV}$ [1]

B1 d)

If the electron were the only particle emitted in β-decay, then the electron would always have kinetic energy = Q value (c^2 × the mass difference of the parent and daughter nuclei). [1]

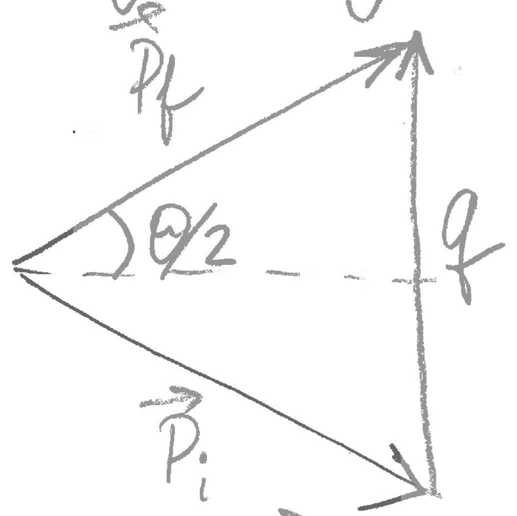
What is observed is a spectrum of electron energies FROM ZERO up to a maximum equal to the Q value. [1]

Therefore it follows that there must be another particle involved in the β-decay which carries the remaining energy. This particle is the neutrino. [1]

In β-decay neutron is converted into proton emitting an electron and anti-neutrino. Since neutron and proton all have spin $\frac{1}{2}$ it is only possible to conserve angular momentum if neutrino also has spin $\frac{1}{2}$ [1]

B2

(a) Connection between scattering angle Θ, and the magnitude q of the momentum transferred is given by this picture [1]



where $\vec{p}_i$ and $\vec{p}_f$ are initial and final momenta respectively.

For relativistic electron with energy E, E/c is its momentum, so for q [1] we can write a relation

$$q = 2\frac{E}{c}\sin\left(\frac{\Theta}{2}\right)$$ [1]

using Figure above

[12]

(b) The assumption that nucleus can be treated as a point-like charge [1] will break down if the incident electron has sufficient energy to [1] probe the nucleus to within the nucleus radius

(c) At such electron energies the de Broglie wave length of the electron is of the order ~~or~~ smaller than the size of the nucleus, [1] So the wave-like properties of the electron must be taken into consideration with the nucleus acting as a diffractive [1] object.

d) The differential cross section for such diffractive scattering is given by

$$\frac{d\sigma}{d\Omega} = \frac{d\sigma}{d\Omega_{mott}} F^2(q) \qquad [1]$$

where $F(q)$ is the form-factor, which accounts for the diffraction. [1]
The FORM FACTOR is the Fourier transform of the charge distribution of the nucleus in space. [1]
Therefore, measurent of the differential scattering cross section and comparing it with the Mott formula can be used to deduce the charge distribution within the nucleus. [1]

e) FOR $q = 43.6$ MeV and $E = 1$ GeV the scattering angle is

$$\Theta = 2\sin^{-1}\left(\frac{qc}{2E}\right) = 2\sin^{-1}\left(\frac{43.6}{2000}\right) \simeq 2.5^\circ \quad [2]$$

the Mott Formula gives

$$\frac{d\sigma}{d\Omega}_{Mott} = \frac{Z^2\alpha^2\hbar^2c^2}{16E^2\sin^4(\theta/2)}\left(1-\sin^2(\theta/2)\right) =$$

$$= \left(\frac{20}{137}\,197\right)^2 \frac{1}{16\times(10^3)^2\sin^4(1.25^\circ)}\left(1-\sin^2(1.25^\circ)\right) \quad [2]$$

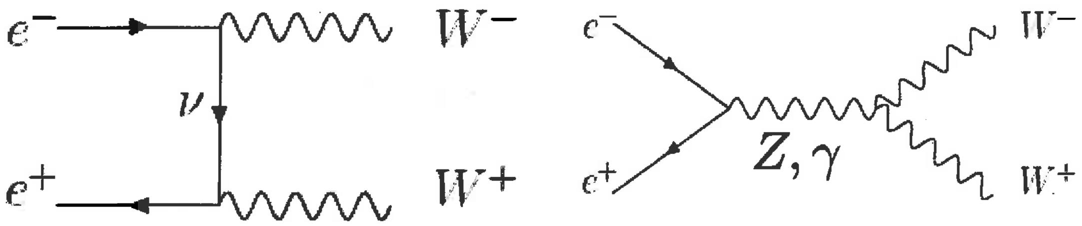
$$= 228\ fm^2 = 2280\ mb \quad [2]$$

FOR $\Theta = 2.5^\circ$ one reads cross setion $= 1000$ mb [1]

$$\Rightarrow F(q) = \sqrt{\frac{1000}{2280}} = \underline{0.66} \quad [2]$$

C1 Solution (Bookwork + Seen similar on the problem sheets, c) is a new problem)

a) *Draw the Feynman Diagrams for $e^+e^- \rightarrow W^+W^-$ annihilation.*



[2]

b) *Find the minimal energy of the e^+ (E) which would be colliding with e^- with the energy* **quater as low** *as that ($E/4$) and producing W^+W^- pair (M_W = 80GeV/c²). For simplicity, you may neglect the mass of the electron as compared to its energy.*

The 4-momentum of the electron is $P_{e^-} = (E/c, \vec{P})$ (with $|\vec{P}| = E/c$ once we neglect m_e), and the 4-momentum of the incoming positron is $P_{e^+} = (E/(4c), -\vec{P}/4)$.

[2]

The invariant mass squared of the e^+e^- pair is the $(P_{e^-} + P_{e^+})^2 = (2M_W)^2c^2 = 4M_W^2c^2$ for minimal requested E. [2]

Evaluating $(P_{e^-} + P_{e^+})^2$ we find

$$(P_{e^-} + P_{e^+})^2 = \frac{25}{16}E^2/c^2 - \frac{9}{16}E^2/c^2 = E^2/c^2$$

[2]

Therefore $E = 2M_Wc^2 = 160$ GeV [2]

c) *Write down electric charges of all fundamental fermions.*

fermions	e^- , μ^- , τ^-	ν_e , $\nu_m u$, τ^-	u, c, t	d, s, b
charge	-1	0	+2/3	-1/3
	[1]	[1]	[1]	[1]

d) *Calculate*

$$R = \frac{\sigma(e^+e^- \to \text{hadrons})}{\sigma(e^+e^- \to \mu^+\mu^-)}$$

for center-of-mass energy just below the threshold of "charm" production and for center-of-mass energy just below the threshold of b-quark production. Explain your results. The weak diagram can be ignored.

e)Explain how the experimentally measured value of R can be used as evidence for the existence of quark colours.

In case of hadrons one needs to calculate a cross section for quarks in the final state, i.e. cross section for $e^+e^- \to q\bar{q}$ process and therefore take into account a colour factor for quarks which is equal to 3 as well as EM charges Q_i of quarks which come squared into formula for the cross section. [2]

Therefore, summing over all quarks produced, we have

$$R = \frac{\sigma(e^+e^- \to \text{hadrons})}{\sigma(e^+e^- \to \mu^+\mu^-)} = \sum_i \frac{\sigma(e^+e^- \to q_i\bar{q}_i \to \text{hadrons})}{\sigma(e^+e^- \to \mu^+\mu^-)} = 3\sum_i Q_i^2$$

[1]

Just below "charm" threshold u, d and s quarks can be produces, so

$$R = 3\sum_i Q_i^2 = 3 \times \left(\frac{4}{9} + \frac{1}{9} + \frac{1}{9}\right) = 2.$$

[1]

Just below b-quark threshold u, d , s and c quarks can be produces, so

$$R = 3\sum_i Q_i^2 = 3 \times \left(\frac{4}{9} + \frac{1}{9} + \frac{1}{9} + \frac{4}{9}\right) = \frac{10}{3}.$$

[1]

The number of quark colours N_c can be directly measured from the R value which is proportional to N_c given that that Q_i charges of quarks are known (measured) from different experiments. [1]

C2 Solution(Seen in the problem sheets and in the notes)

C2: *K-mesons and π-mesons have negative parity and zero intrinsic spin. ρ mesons have negative parity and intrinsic spin one.*

(a) *Explain why the observation of the weak-interaction decay modes*

$$K^+ \rightarrow \pi^+ + \pi^0$$

is evidence that weak interactions violate parity, whereas the decay

$$\rho^+ \rightarrow \pi^+ + \pi^0$$

can proceed via the strong interactions.

Since K^+ has zero spin the final state pions must be in an orbital nagular momentum $l = 0$ state which means that the parity of the final two-pion state is positive, whereas the initial K^+ state has negative parity. Therefore parity is violated in his decay. [1]

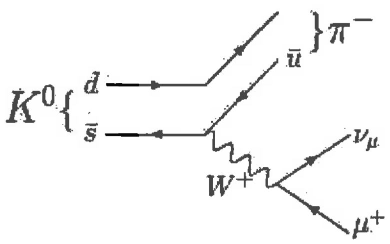
On the other hand for the decay of the spin-1 ρ-meson the final state pions must have orbital angular momentum $l = 1$ and therefore the two-pion state has parity $(-1)^l$=-1, which is the same as that of the initial state so the decay can proceed through parity conserving strong interactions. [2]

(b) *Draw the relevant Feynman graphs show that a semi-leptonic decay mode of the K^0 (quark content $\bar{s}\,d$):*

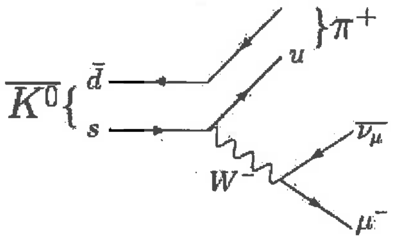
$$K^0 \rightarrow \pi^- + \mu^+ + \nu_\mu$$

and a semi-leptonic decay mode of the $\overline{K}^0$:

$$\overline{K}^0 \rightarrow \pi^+ + \mu^- + \bar{\nu}_\mu$$



[2]



[2]

(c) *What is meant by CP?*
The combined operations of parity reversal and charge conjugation which interchanges particles and their antiparticles [2]

(d) *From the fact that the weak interactions are (to a very good approximation) CP invariant, explain why K^0 and $\overline{K}^0$ are not mass eigenstates, whereas the superposition states*

$$K_L = \frac{1}{\sqrt{2}}\left(K^0 + \overline{K}^0\right)$$

$$K_S = \frac{1}{\sqrt{2}}\left(K^0 - \overline{K}^0\right)$$

are mass eigenstates.

$$CP|K^0\rangle = -|\overline{K}^0\rangle$$
$$CP|\overline{K}^0\rangle = -|K^0\rangle$$

[2]

(e) There K^0 and $\overline{K}^0$ are *not* eigenstates of CP and hence not eigenstates of the Hamiltonian. which means that they are not energy and hence mass eigenstates.

On the other hand

$$CP|K_L\rangle = \frac{1}{\sqrt{2}}\left(CP|K^0\rangle + CP|\overline{K}^0\rangle\right) = -\left(|K^0\rangle + |\overline{K}^0\rangle\right) = -|K_L\rangle$$

and

$$CP|K_S\rangle = \frac{1}{\sqrt{2}}\left(CP|K^0\rangle - CP|\overline{K}^0\rangle\right) = +\left(|K^0\rangle - |\overline{K}^0\rangle\right) = +|K_s\rangle$$

so that K_L and K_S are eigenstates of CP with eigenvalues -1 and +1 respectively. [2]

(f) *Explain why K_S can decay into two pions whereas K_L can only decay into three pions.*
A neutral two-pion state is an eigenstate of CP with eigenvalue +1, so by CP conservation the state K_S can decay into two pions, whereas a neutral three-pion state is an eigenstate of CP with eigenvalue -1, so by CP conservation the state K_L can decay into three pions. [2]

(g) *The mean lifetime of K_S, τ_S is much shorter than the mean lifetime of K_L. A K^0 is produced at time $t = 0$ and after a time t, which is much larger than τ_S, it decays semi-leptonically. Explain why in such a case the decay product π^+, μ^-, $\bar{\nu}_\mu$ is just as likely as π^-, μ^+, ν_μ.* The inital state is a K^0 which is a superposition of the states K_L and K_S

$$|K^0\rangle = \frac{1}{\sqrt{2}}\left(|K_L\rangle + |K_S\rangle\right)$$ [2]

After a time much larger than τ_S, the K_S will have decayed away and we are left with pure K_L which can be written

$$|K_L\rangle = \frac{1}{\sqrt{2}}\left(|K^0\rangle + |\overline{K}^0\rangle\right),$$ [2]

so that there is a 0.5 probability that the state is a K^0 decaying semi-leptonically into $\mu^+\pi^-\nu_\mu$ and 0.5 probability that the state is a $\overline{K}^0$ decaying semi-leptonically into $\mu^-\pi^+\bar{\nu}_\mu$. [1]