SEMESTER 2 EXAMINATION 2014-2015

NUCLEI AND PARTICLES

Duration: 120 MINS (2 hours)

This paper contains 10 questions.

Answers to Section A must be in separate answer books from Section B and Section C

Answer **all** questions in **Section A** and **one** question in **each** of **Section B** and **Section C**.

Each section carries 1/3 of the total marks for the exam paper and you should aim to spend about 40 mins on each.

An outline marking scheme is shown in brackets to the right of each question.

A Sheet of Physical Constants is provided with this examination paper.

Only university approved calculators may be used.

A foreign language word to word® translation dictionary (paper version) is permitted provided it contains no notes, additions or annotations.

Section A

A1. Find the invariant mass of an electron-positron pair, e^+e^-, for a stationary positron and a moving electron with total energy 3.5 MeV ($m_e = 0.5\ \mathrm{MeV/c^2}$). [4]

A2. Draw the complete set of Feynman diagrams for $gg \to gg$ gluon-gluon scattering process to the lowest order of perturbation theory. [3]

A3. The ratio of the final to initial number of nuclei in a sample of the radioactive isotope A is R_A after a certain time period, T, while for isotope B, this ratio is R_B. Find the ratio of the mean life-times τ_A/τ_B for these isotopes. [3]

A4. The cross section of Higgs boson production at the Large Hadron collider with $\sqrt{s} = 13$ TeV is $\sigma(pp \to H) = 30$ pb, the branching ratio for Higgs boson decay to ZZ is $Br(H \to ZZ) = 3\%$, and the branching rato of Z-boson decay to an electron-positron pair is $Br(Z \to e^+e^-) = 3\%$. What is the minimal integrated luminosity, L, required to measure the cross section of the process $pp \to H \to ZZ \to e^+e^-$ with an accuracy $\epsilon = 20\%$? [6]

A5. Consider the shell model, where the first two shells are $1s$ and $1p$. The $1p$ shell is split into $j = 1/2$ and $j = 3/2$ sublevels. Explain why this splitting takes place and indicate which sublevel has the higher energy and why. [2]

A6. The atomic mass of nuclide ${}^A_Z X$ is M_X. Express its binding energy in terms of Z, A, m_p(proton mass), m_n(neutron mass), m_e(electron mass) and M_X. [2]

Section B

B1. (a) State two ways a nucleus A can undergo β-decay decay to nucleus B. What interactions are responsible for β-decay? What are the quantum numbers of the elementary particles appearing in the final state of β-decaying nuclei? [4]

(b) According to the liquid drop model the nuclear binding energy may be approximated by the semi-empirical formula

$$B(A,Z) \;=\; a_V A \;-a_S\, A^{2/3} \;-a_C\frac{Z^2}{A^{1/3}} \;-a_A\frac{(Z-N)^2}{A} \;+\frac{\left((-1)^Z+(-1)^N\right)}{2}\frac{a_P}{A^{1/2}},$$

where A is the atomic mass number, Z is the atomic number and $N = A - Z$. From fitting to the measured nuclear binding energies, the values of the parameters are $a_V = 15.56$ MeV, $a_S = 17.23$ MeV, $a_C = 0.697$ MeV, $a_A = 23.285$ MeV, $a_P = 12.0$ MeV.

Using this formula, find the most stable isobar with respect to β-decay with an atomic mass number $A = 69$. [6]

(c) Use this semi-empirical formula to estimate the maximum energy of the electrons emitted in the β-decay $\;^{69}_{27}Co \rightarrow ^{69}_{28} Ni + e^- + \bar{\nu}$. [6]

(d) Discuss why the data on the energy spectrum of electrons (or positrons) emitted in β-decay led to the postulate of the existence of the neutrino, and explain why the neutrino has to have spin one-half. [4]

TURN OVER

B2. The differential cross section of the scattering of a relativistic electron (with energy $E \gg m_e c^2$) off a nucleus (considered as a point-like particle) with atomic number, Z, at angle θ is given by the Mott formula:

$$\frac{d\sigma}{d\Omega} = \left(\frac{Z^2\alpha^2\hbar^2c^2}{16E^2\sin^4(\theta/2)}\right)(1-\sin^2(\theta/2)).$$

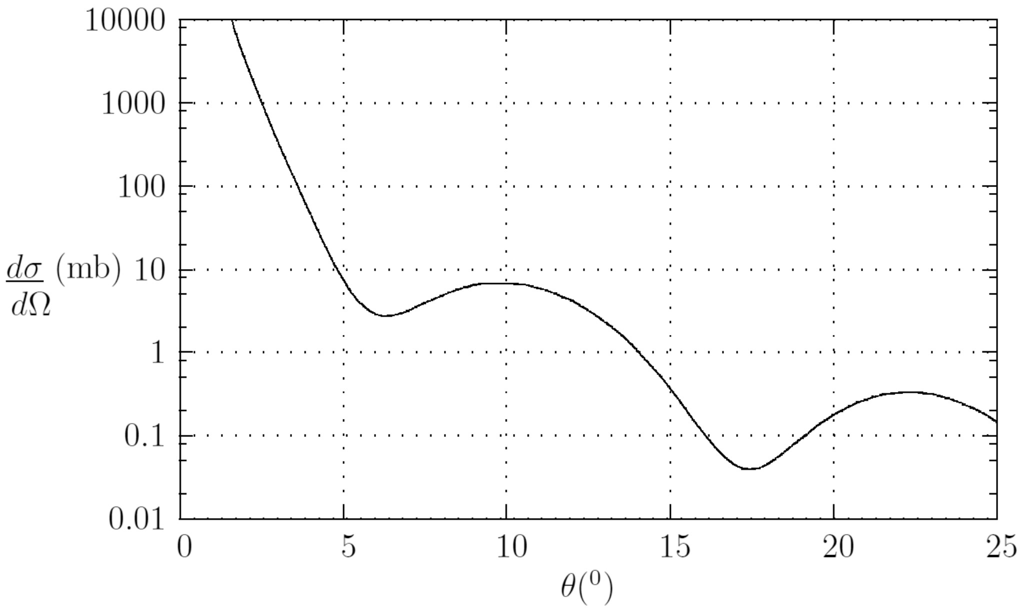
(a) What is the relation between the scattering angle, θ, and the magnitude, q, of the momentum transferred from the electron to the target nucleus in the approximation of a heavy nucleus ? [3]

(b) Explain why this formula is expected to break down for incident electrons of a sufficiently large energy. [2]

(c) Explain qualitatively why diffraction occurs in very high energy scattering off a nucleus. [2]

(d) Explain what is meant by the form-factor, $F(q)$, and indicate how the form-factor can be used to obtain information about the charge distribution inside the nucleus. [4]

(e) The figure below shows the differential cross-section (in millibarns) with respect to solid angle, Ω, of an electron with energy 1 GeV, scattering off a calcium target ($Z = 20$), as a function of the scattering angle (in degrees).



Estimate the value of the form factor for a momentum transfer of $q = 43.6$ MeV/c.

[9]

Section C

C1. (a) Draw the Feynman diagrams for the annihilation process $e^+e^- \rightarrow \mathrm{W}^+\mathrm{W}^-$. [2]

(b) Find the minimal value of the energy E of an e^+ scattering off an e^- with energy $E/4$ (i.e. a quarter of the e^+ energy) that is required in order to produce a W^+W^- pair ($M_W = 80$ GeV/c^2). For simplicity, you may neglect the mass of the electron in comparison with its energy. [8]

(c) Write down the electric charges of all the fundamental fermions. [4]

(d) Calculate

$$R = \frac{\sigma(e^+e^- \rightarrow \text{hadrons})}{\sigma(e^+e^- \rightarrow \mu^+\mu^-)}$$

for a centre-of-mass energy just below the threshold for “charm” production and for centre-of-mass energy just below the threshold for b-quark production. Explain your results. The contribution from weak interactions can be ignored. [5]

(e) Explain how the experimentally measured value of R can be used as evidence for the existence of quark colours. [1]

TURN OVER

C2. (a) K-mesons and π-mesons have negative parity and zero intrinsic spin, while ρ-mesons have negative parity and intrinsic spin one.
Explain why the observation of the weak-interaction decay

$$K^+ \to \pi^+ + \pi^0$$

provides evidence that weak interactions violate parity, and explain why the decay

$$\rho^+ \to \pi^+ + \pi^0$$

can proceed via the strong interaction. [3]

(b) Draw the relevant Feynman diagrams for the semi-leptonic decay of the K^0 (quark content $\overline{s}d$) via $K^0 \to \pi^- + \mu^+ + \nu_\mu$
and the semi-leptonic decay of $\overline{K}^0$ via $\overline{K}^0 \to \pi^+ + \mu^- + \overline{\nu}_\mu$. [4]

(c) What is meant by CP? [2]

(d) From the fact that the weak interactions are (to a very good approximation) CP invariant, explain why $|K^0\rangle$ and $|\overline{K}^0\rangle$ are *not* mass eigenstates, whereas the superposition states $|K_L\rangle$ and $|K_S\rangle$ are. [2]

(e) Show that the superposition

$$|K_L\rangle = \frac{1}{\sqrt{2}}\left(|K^0\rangle + |\overline{K}^0\rangle\right)$$

is CP-odd whereas the superposition

$$|K_S\rangle = \frac{1}{\sqrt{2}}\left(|K^0\rangle - |\overline{K}^0\rangle\right)$$

is CP-even. [2]

(f) Explain why K_S decays only into two pions whereas K_L can only decay into three pions. [2]

(g) The mean lifetime of K_S, denoted by τ_S, is much shorter than the mean lifetime of K_L. A K_0 is produced at time $t = 0$, and after a time t, which is much larger than τ_S, it decays semi-leptonically. Explain why in such a case the decay $K_0 \to \pi^+\mu^-\overline{\nu}_\mu$ is just as likely as $K_0 \to \pi^-\mu^+\nu_\mu$. [5]

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