UNIVERSITY OF SOUTHAMPTON PHYS3003W1

SEMESTER 1 EXAMINATIONS 2013/14

LIGHT & MATTER

DURATION 120 MINS (2 Hours)

Answer all questions in Section A and two and only two questions in Section B.
Section A carries 1/3 of the total marks for the exam paper and you should aim to spend about 40 mins on it. Section B carries 2/3 of the total marks for the exam paper and you should aim to spend about 80 mins on it.
An outline marking scheme is shown in brackets to the right of each question.
A Sheet of Physical Constants is provided with this examination paper. Only university approved calculators may be used.

Gauss $\nabla \cdot \mathbf{D} = \rho$ $\oiint \mathbf{D} \cdot d\mathbf{S} = \iiint \rho dV$

$\nabla \cdot \mathbf{B} = 0$ $\oiint \mathbf{B} \cdot d\mathbf{S} = 0$

Faraday $\nabla \times \mathbf{E} = -\frac{\partial \boldsymbol{B}}{\partial t}$ $\oint \mathbf{E} \cdot d\mathbf{l} = -\oiint \frac{\partial \boldsymbol{B}}{\partial t} \cdot d\mathbf{S}$

Ampere $\nabla \times \mathbf{H} = \mathbf{J} + \frac{\partial \mathbf{D}}{\partial t}$ $\oint \mathbf{H} \cdot d\boldsymbol{l} = \oiint \left(\boldsymbol{J} + \frac{\partial \mathbf{D}}{\partial t}\right) \cdot \boldsymbol{dS}$

Constitutive equations:

$\mathbf{P} = \varepsilon_0 \chi \ \mathbf{E}$, *and* $\mathbf{D} = \varepsilon_0 \mathbf{E} + \mathbf{P} = \varepsilon_0 \varepsilon_r \mathbf{E}$

$\mathbf{B} = \mu_0 \mu_r \mathbf{H}$, *and* $\mathbf{B} = \mu_0 (\mathbf{H} + \mathbf{M})$

Number of Pages **9**

SECTION A

A1. The wave equation, given by,

$i\frac{\partial^2 y(x,t)}{\partial x^2} = \frac{\partial y(x,t)}{\partial t}$, has a solution of, $y(x,t) = y_0 \exp[i(\omega t - kx)]$.

State if the given wave equation is linear or non-linear. Find its dispersion relation that links ω and k and also the phase velocity of the wave. Is the phase velocity constant? Calculate the group velocity of the wave. [3]

A2. Write down the rate equations for a two-level system for spontaneous emission, absorption and stimulated emission using the Einstein A and B coefficients. Use diagrams to explain the processes of spontaneous emission, absorption and stimulated emission. [4]

A3. Calculate the minimum thickness of a half wave-plate for use at a wavelength of 633 nm, if the used material to realise the wave-plate has an ordinary and extraordinary refractive index of 1.7 and 1.5, respectively. [2]

If the thickness is too thin for conventional fabrication methods to be used can you propose two alternative methods for creating the wave plate. [3]

A4. Explain Kerr lensing and how a Kerr lens mode-locked laser works. Please make a sketch of a possible mode-locked laser that works with the Kerr lens. [4]

A5. Explain why phase matching is needed for efficient second harmonic generation. Explain what quasi phase matching is and how it is achieved using periodic poling. Show in a graph the differences of second harmonic power versus crystal length for the case of phase unmatched, phase matched and quasi phase matched conditions. [4]

TURN OVER

SECTION B

B1. **(i)** The energy densities W_e and W_m associated with electric and magnetic fields are defined to be,

$$W_e = \frac{1}{2}\boldsymbol{D}\cdot\boldsymbol{E}\,,$$

$$W_m = \frac{1}{2}\boldsymbol{B}\cdot\boldsymbol{H}.$$

Show that $W_e = W_m$ and that the total energy density $W = W_e + W_m$ is equal to $\epsilon_0\epsilon_r E^2$. You can use the constitutive equations for an isotropic, linear material. For a plane electromagnetic wave, you can use the relation $\boldsymbol{H} = \omega/k\cdot\boldsymbol{D}$, and that the speed of light is $v = \frac{1}{\sqrt{\epsilon_0\epsilon_r\mu_0\mu_r}}$. [4]

(ii) Using conservation of energy $\nabla\boldsymbol{S} + \frac{\partial W}{\partial t} = 0$ and the definition of total energy density $W = W_e + W_m = \frac{1}{2}\boldsymbol{D}\cdot\boldsymbol{E} + \frac{1}{2}\boldsymbol{B}\cdot\boldsymbol{H}$ derive a Poynting vector **S** that can satisfy this relation.

You may want to use Faraday and Ampere laws and also the vector relation $\nabla\cdot(\boldsymbol{A}\times\boldsymbol{B}) = \boldsymbol{B}\cdot(\nabla\times\boldsymbol{A}) - \boldsymbol{A}\cdot(\nabla\times\boldsymbol{B})$. [7]

(iii) Show that the average Poynting vector can be written as $\bar{S} = \frac{E_0^2}{2z_0}$, where $z_0 = \sqrt{\frac{\mu_0}{\epsilon_0}}$ is the impendence of free space. You may want to use that the Poynting vector is $\boldsymbol{S} = \boldsymbol{E}\times\boldsymbol{H}$, also that $\boldsymbol{H} = \omega/k\cdot\boldsymbol{D}$, the speed of light $v = \frac{1}{\sqrt{\epsilon_0\epsilon_r\mu_0\mu_r}}$ and the constitutive equations. [3]

(iv) The BBC longwave Radio 4 is transmitting at a frequency of 198 KHz with a power of 400 kW. Find the energy density and the amplitude of the electric field 1 wavelength away from the transmitter. Use the relation for the average Poynting vector from question (iii) and the impendence of free space $z_0 = 377\ Ohms$. [6]

TURN OVER

B2. (i) Explain the physical meaning of polarisation density $\boldsymbol{P}$ and electric field $\boldsymbol{E}$. In a non-uniform electric field such as one that would be generated by a point charge, would a dielectric sphere feel a force? Would the force be attractive or repulsive? Explain your answer with a sketch. [4]

(ii) By considering a complex plane wave solution derive the dispersion relation arising from the telegraphy equation,

$$\nabla^2 \mathbf{E} = \mu_0 \mu_r \varepsilon_0 \varepsilon_r \frac{\partial^2 \mathbf{E}}{\partial t^2} + \mu_0 \mu_r \sigma \frac{\partial \mathbf{E}}{\partial t}$$

and define the complex dielectric constant,

$$\varepsilon = \varepsilon_1 + i\varepsilon_2 = \varepsilon_r + i\frac{\sigma}{\varepsilon_0 \omega}.$$

[3]

Using the equation for a plane wave with a complex wavenumber, $k=k_1+ik_2$, show how absorption occurs. Use your result to sketch how the electric field is attenuated inside the material. [2]

For a good conductor, show that the attenuation length δ, after which the electric field strength has fallen by a factor of e, may be approximated by $(2/\sigma\omega\mu_0\mu_r)^{1/2}$.

You may want to use that the imaginary part of the wavenumber is,

$$k_2 = \sqrt{\frac{\mu_0 \mu_r \varepsilon_0 \omega^2}{2}}\, \varepsilon_2 \Big/ \sqrt{\varepsilon_1 + \sqrt{\varepsilon_1^2 + \varepsilon_2^2}},$$

,where, ε_1, ε_2 are the real and imaginary parts of the complex dielectric constant given above. [5]

(iii) Using $(2/\sigma\omega\mu_0\mu_r)^{1/2}$ for the attenuation depth, calculate the intensity attenuation presented by the 2 mm thick alloy skin of an airliner to long wave radio transmissions at 200 kHz. The electrical conductivity of aluminium alloy is approximately 3 x 10^7 $(\Omega.\text{m})^{-1}$ and μ_r, may be taken to be unity. [3]

Pure aluminium has a slightly higher conductivity of 4 x 10^7 $(\Omega.\text{m})^{-1}$. Estimate the minimum thickness of aluminium that should be deposited onto panes of glass in the manufacture of bathroom mirrors (you can assume a wavelength of 500 nm). [3]

TURN OVER

B3. **(i)** Write a definition of polarization. Write the electric field E of linearly polarized light at 45^{o}, and circularly polarised light. The light should be polarised in the xy-plane and propagating in the positive z-axis.

Write a definition of birefringence, explain its origin and explain the concept of the optical axis. [4]

Show with a sketch, and explain, how light propagates in calcite for an ordinary and for an extraordinary polarization. Then explain how a birefringent material can be used to construct a wave-plate. [3]

Show how linear polarization can be expressed as addition of circularly polarized waves. [2]

(ii) Give the general definitions of Jones and Stokes vectors by giving examples of how they are used to describe a polarization state. [2]

Show how we can use Jones vectors and the inner product to prove that left and right circularly polarized states are orthogonal. When is it better to use Jones vectors and when Stokes vectors? Can Stokes vectors be experimentally measured? [2]

Construct a "right circular polarizer", a device that will only let through right circularly polarised light. Explain how it is working using a sketch. [3]

(iii) Explain how polarization can be affected by reflection, give the definition of Brewster's angle and show with a sketch the polarization and angle that this happens. [2]

Explain how polarization can be affected by scattering, use a drawing [2]

END OF PAPER