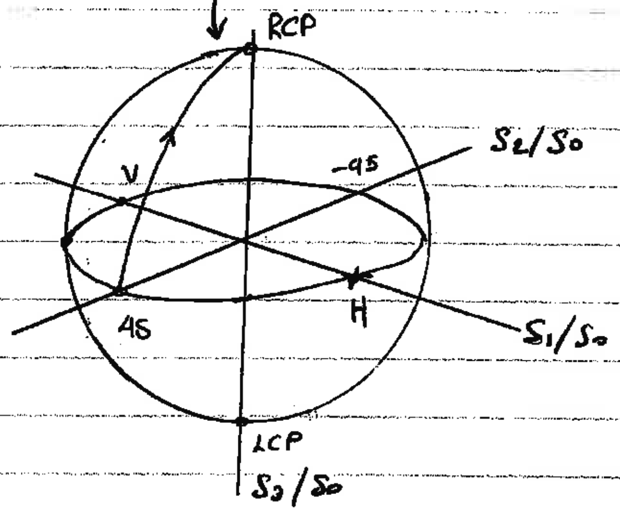
Problem, done in classroom
Partly also in Notes

[1]

A1

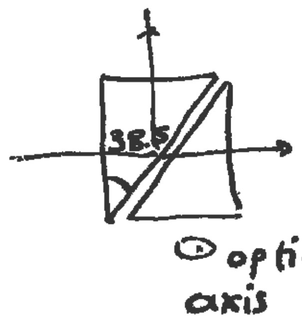


For calculations we need to know the angle of the fast axis at the wave plate.
Then the initial polarisation state,
and finally the angle of retardation of the plate.
Let's say fast axis H, initial polarisation 45°, and angle 90° (QWP)
We draw a great arc with starting point the polarisation of the wave, for an angle as much as the retardation (90°) and center the fast axis angle (H).

[2]

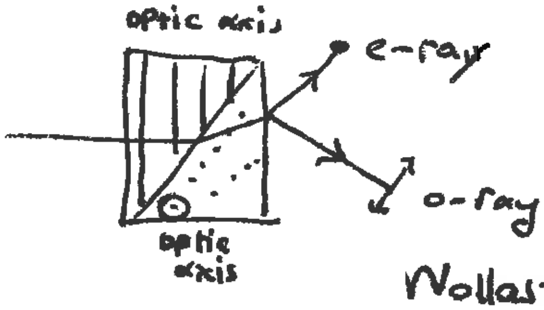
from Lecture Notes

A2.



Glan Foucault

Total internal reflection of one polarisation and not the other



Wollaston prism

just based to different diffraction angles to e-ray and o-ray

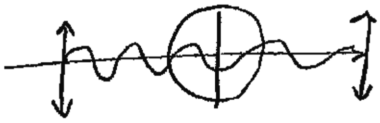
1 mark for each explanation

1 mark for each schematic

Problem, done in classroom

A3

Vertical polarizer



→ vertical light will pass
→ horizontal light will be blocked

$$\begin{bmatrix} x & y \\ z & w \end{bmatrix}\begin{bmatrix} 0 \\ 1 \end{bmatrix} = \begin{bmatrix} 0 \\ 1 \end{bmatrix} \Rightarrow$$

$$\Rightarrow \begin{aligned} y &= 0 \\ w &= 1 \end{aligned}$$

$$\begin{bmatrix} x & 0 \\ z & 1 \end{bmatrix}\begin{bmatrix} 1 \\ 0 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix} \Rightarrow$$

$$\Rightarrow \begin{aligned} x &= 0 \\ z &= 0 \end{aligned}$$

[2 marks]

$$\begin{bmatrix} 0 & 0 \\ 0 & 1 \end{bmatrix}$$

Horizontal polarizer $\begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix}$

Problem, done in Class

A3 Matrix $\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & -1 \\ 0 & 0 & 1 & 0 \end{bmatrix} \cdot \overset{H}{\begin{bmatrix} 1 \\ 1 \\ 0 \\ 0 \end{bmatrix}} = \overset{H}{\begin{bmatrix} 1 \\ 1 \\ 0 \\ 0 \end{bmatrix}}$ etc

Horizontal polarisation unchanged
vertical polarisation similarly is unaffected

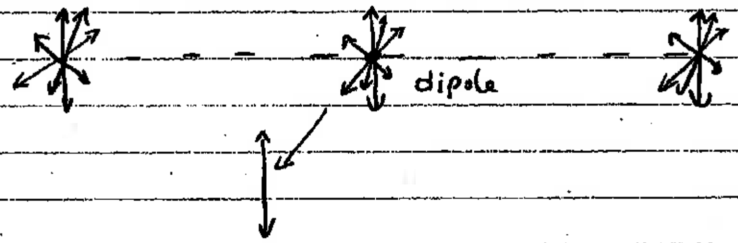
45° polarisation is converted to circular, therefore this is a QWP

I expect to see at least part of the calculations

[3]

A4

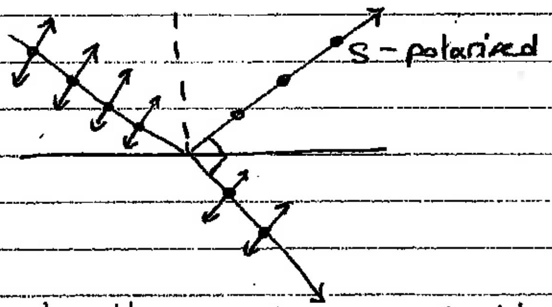
Polarisation by scattering



When unpolarised light (sunlight) is scattered by a dipole then the reemitted light in the same direction is unpolarised, but light at an angle is polarised.

[2 marks]

by reflection: light at an S and p polarisation are affected differently in reflection. Extreme example is the Brewster angle where due to the orientation of dipoles light cannot be scattered at the direction of reflection



At Brewster angle the orientation of the dipoles is such that cannot emit light in the direction of the reflected beam. (see drawing)

[2 marks]

A5

Lecture Notes

χ^2 is present only in materials that do not have centrosymmetry,

$$P_{SH} = \chi^2 E^2 \quad \text{where} \quad E = E_0 \cos\omega t$$

if we change the sign of the excitation then by symmetry the response should be the opposite $-P_{SH}$, but $\chi^{(2)}(-E)^2 = \chi^2 E^2$

and then $P_{SH} = -P_{SH}$ therefore 0 [3]

so χ^2 has to be 0.

NO SHG

No POCKELS

YES KERR $\chi^{(3)}$ process [1]

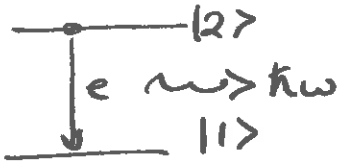
Lecture Notes

B1 (i) In a two level system we have

(a) spontaneous emission of a photon as the e^- drops from $|2\rangle$ to $|1\rangle$

$$\frac{dN_2}{dt} = -A_{21} N_2 \Rightarrow N_2(t) = N_2(0)\exp\left(-A_{21} t\right)$$

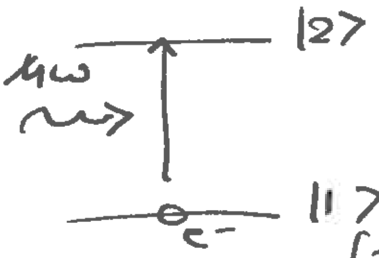
Lifetime is $\tau = \frac{1}{A_{21}}$



[2]

(b) Absorption of an incoming photon which matches the energy separation of the states

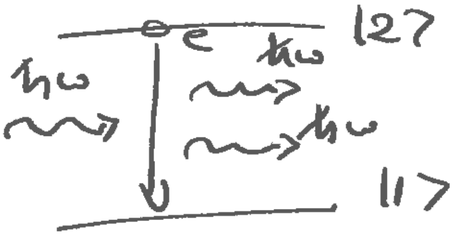
$$\frac{dN_1}{dt} = -B^{\omega}_{12} \cdot N_1 \cdot \underbrace{u(\omega)}_{\text{Spectral energy density}}$$



[2]

(c) Stimulated emission of a photon as the electron drops from $|2\rangle$ to $|1\rangle$ because of disturbance of a photon matching the energy of the transition

$$\frac{dN_2}{dt} = -B^{\omega}_{21} N_2 \, u(\omega)$$



[2]

Problem based on lecture

B1 (ii) assuming laser light exactly on resonance and neglecting fast rotating terms, because when integrated $\omega+\omega_0$ demominator is $>> \omega-\omega_0$ we can rewrite the system then as,

$$\dot{c}_1(t) = \frac{i}{2}\Omega_R c_2(t) \qquad (1)$$

$$\dot{c}_2(t) = \frac{i}{2}\Omega_R c_1(t) \qquad (2)$$

if we differentiate (1) and insert in (2), we have

$$\ddot{c}_1 = \left(\frac{i}{2}\Omega_R\right)^2 c_1 \Rightarrow \ddot{c}_1 + \left(\frac{\Omega_R}{2}\right)^2 c_1 = 0 \qquad [2]$$

for initial conditions $c_1(0) = 1$ and $c_2(0) = 0$ and trying a sinusoidal solution

$$c_1(t) = \cos\left(\frac{\Omega_R t}{2}\right)$$

$$c_2(t) = i\sin\left(\frac{\Omega_R t}{2}\right) \qquad [1]$$

If we square these relation they give us the probability of the electron to be in the state $|1\rangle$ or $|2\rangle$. Their behaviour is an oscillatory transition between states $|1\rangle$ and $|2\rangle$ which is Rabi oscillation or Rabi flopping.

[1]

Rabi oscillations give or describe the interplay between stimulated absorption and emission.

Ω_R which defines the Rabi frequency in order to be high enough so to give oscillations faster than lifetime of states need very high laser intensities. Also, any detuning of

frequency creates damping on the oscillations.
For these reasons experimental observations of Rabi oscillations are demanding. [2]

Rabi oscillations show that in a two level system population inversion cannot be achieved or maintained, therefore two-level systems are not lasers.
(1)

iii) In the c_1, c_2 system above spontaneous emission is not taken into account

for $E_0 = 0$ and $C_2(0) = 1$ and

$c_1(0) = 0$ there is no mechanism for the electrons to relax

if $E_0 = 0 \Rightarrow \Omega_e = 0 \Rightarrow$ $\left.\begin{array}{l}\dot{C}_1(t) = 0 \\ \dot{C}_2(t) = 0\end{array}\right\}$ $\begin{array}{l}\Rightarrow C_1(t) = C \\ \Rightarrow C_2(t) = C\end{array}$

[2]

iv)

iv) Dressed states we use when we consider the states of the coupled resonant system of light and atoms, rather than the states of the unperturbed atom.

The states then are $|\varphi ; n\rangle$ where φ is the state of the atom and n is the number of photons. [2]

In general a polariton is the coupling of a photon with an electric dipole excitation In the case of the exciton-polariton a photon and an exciton. It is equivalent to the coupling of two oscillators

Strong coupling is easily explained when an atom is placed inside a cavity resonant with the atom states. If κ is the photon decay rate of the cavity, γ is the non-resonant decay rate and g_0 is the atom photon coupling parameter. These parameters show a characteristic time scale for the dynamics of the atom-photon system Strong coupling limit is when $g_0 \gg \kappa$ or γ [2]

Strong coupling, in this context is needed to make the emission of a photon a reversible process - - the photon is reabsorbed in the atom - and not lost. Therefore the dipole of the atom and the photon become coupled and explained as a new composite object, the polariton.

[1]

Lecture Notes

B 2(i)

We assume an electron, bound in a potential $V = \frac{m\omega_0^2}{2} x^2$ and an applied oscillatory electric field $E = E_0 \sin\omega t$ [1 mark]

The equation of motion is going to be

$m\ddot{x} + m\omega_0^2 x + eE_0 \sin\omega t = 0$ [2marks]

We assume that the solution will follow the excitation, so $x = x_0 \sin\omega t$ and we replace it in the equation to calculate x_0

We have $-m x_0 \omega^2 + m\omega_0^2 x_0 + eE_0 = 0 \Rightarrow$

$\Rightarrow x_0 = \dfrac{-eE_0}{m(\omega_0^2-\omega^2)}$ [2marks]

the atomic polarization is $\bar{P} = -ex = \dfrac{e^2 E_0}{m(\omega_0^2-\omega^2)} \sin\omega t =$

$= \dfrac{e^2}{m(\omega_0^2-\omega^2)} \bar{E}$

So for N concentration of atoms

$\vec{P} = \dfrac{Ne^2}{m(\omega_0^2-\omega^2)} \vec{E}$ [2 marks]

We know that $\vec{P} = \varepsilon_0 \chi \vec{E}$ so $\chi = \dfrac{Ne^2}{m\varepsilon_0(\omega_0^2-\omega^2)}$

and also that $\vec{D} = \varepsilon_0 \vec{E} + \vec{P} = \varepsilon_0 \vec{E} + \varepsilon_0 \chi \vec{E} =$

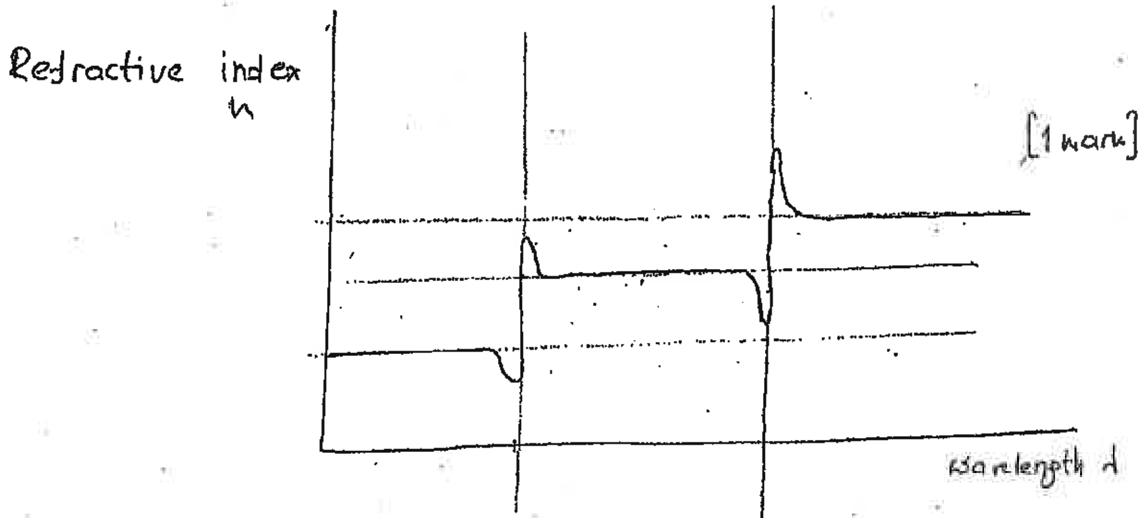
$= \varepsilon_0 (1+\chi) \vec{E} = \varepsilon_0 \varepsilon_r \vec{E}$ [1 mark]

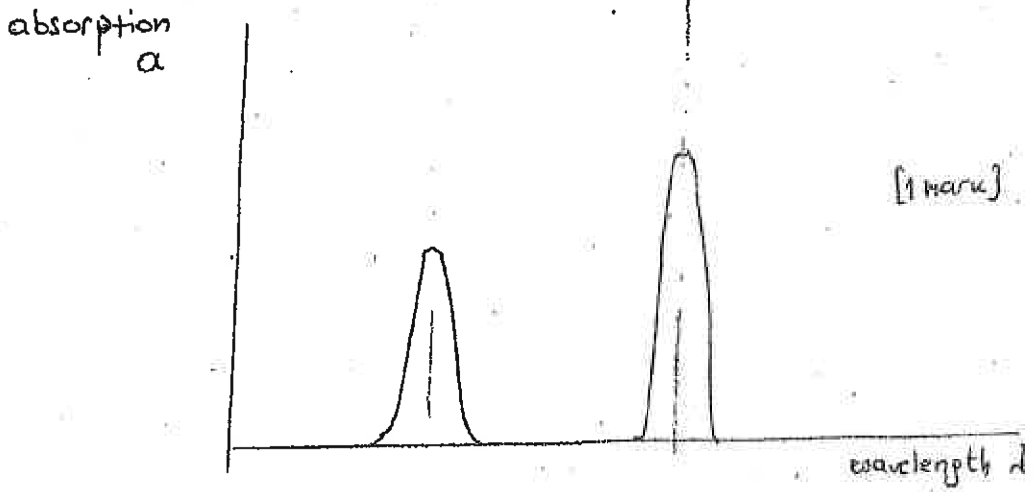
$\Rightarrow \varepsilon_r = \chi + 1$

So comparing $\varepsilon_r = 1 + \dfrac{Ne^2}{\varepsilon_0 m(\omega_0^2-\omega^2)}$ (2 marks)

(ii)

Lectures – Bookwork





Refractive index and absorption are related via the Kramers–Kronig relations which relate the real and imaginary part of the dielectric constant [1 mark]

$$\varepsilon_1(\omega) = 1 + \frac{2}{\pi}\int_0^\infty \frac{\omega' \varepsilon_2(\omega')}{\omega'^2-\omega^2}\, d\omega'$$

$$\varepsilon_2(\omega) = \frac{2\omega}{\pi}\int_0^\infty \frac{\varepsilon_2(\omega')}{\omega'^2-\omega^2}\, d\omega'$$

[1 mark]

(for any relation that has the general dependence)

e.g. $\varepsilon_1(\omega) = c\int_0^\infty \frac{\varepsilon_2}{\omega'^2-\omega^2} d\omega'$

iii) $\sqrt{\dfrac{2}{\sigma \omega \mu_0 \mu_r}}$

0.8mm thick

$\omega = 2\pi \cdot 2 \cdot 10^9\ Hz = 4\pi\, 10^9\ Hz$

$\mu_0 = 4\pi \cdot 10^{-7}\ H/m$

$\sigma = 2 \cdot 10^8 \dfrac{1}{\Omega \cdot m}$

$\sqrt{\dfrac{2}{2 \cdot 10^8 \cdot 4\pi \cdot 10^9 \cdot 4\pi \cdot 10^{-7}}}$ μ_r $= [1\tfrac{1}{2}]$

$= \dfrac{1}{4\pi} \cdot 10^{-5}\ m = 0.08 \cdot 10^{-5}\ m = 0.8 \cdot 10^{-6}\ m$

after $0.8\ mm = 0.8 \cdot 10^{-3}\ m$

$e^{-0.8 \cdot 10^{-3} / 0.8 \cdot 10^{-6}}$

and for intensity $e^{-2 \cdot 0.8 \cdot 10^{-3} / 0.8 \cdot 10^{-6}}$

$[1\tfrac{1}{2}]$

$$\sigma = 3.8 \cdot 10^{7} \frac{1}{\Omega \cdot m}$$

$$\omega = \frac{2\pi c}{\lambda} \quad \text{with } \lambda = 500\,nm \quad \omega = 3.8 \cdot 10^{15}\,Hz$$

$$\delta = \sqrt{\frac{2}{4\pi \cdot 10^{-7} \cdot 3.8 \cdot 10^{7} \cdot 3.8 \cdot 10^{15}}} \cong 3\,nm \qquad [1]$$

So if we require $e^{-2x/\delta} \simeq 10^{-3} \Rightarrow$

$$\Rightarrow \quad -\frac{2x}{\delta} = \ln(0.001) \Rightarrow$$

$$\Rightarrow \quad x = -\frac{\delta}{2} \ln(0.001) = 10\,nm \qquad [2]$$

derivations from the lecture notes

B3 (i) $W_e = \frac{1}{2} \vec{D} \cdot \vec{E}$

$W_m = \frac{1}{2} \vec{B} \cdot \vec{H}$

Using $\vec{D} = \varepsilon_0 \varepsilon_r \vec{E}$, $\vec{B} = \mu_0 \mu_r \vec{H}$ we have $W_e = \frac{1}{2} \varepsilon_0 \varepsilon_r E^2$, $W_m = \frac{1}{2} \mu_0 \mu_r H^2$ [2]

$$\vec{H} = \underbrace{\frac{\omega}{k}}_{v_p} \vec{D} = \frac{1}{\sqrt{\varepsilon_0 \varepsilon_r \mu_0 \mu_r}} \cdot \vec{D} = \frac{\varepsilon_0 \varepsilon_r}{\sqrt{\varepsilon_0 \varepsilon_r \mu_0 \mu_r}} \vec{E}$$ [1]

if we use that in W_m,

$$W_m = \frac{1}{2} \mu_0 \mu_r \cdot \frac{\varepsilon_0^2 \varepsilon_r^2}{\varepsilon_0 \varepsilon_r \mu_0 \mu_r} E = \frac{1}{2} \varepsilon_0 \varepsilon_r E^2$$

So $W_e = W_m$ [1]

(ii) $$\nabla \vec{S} + \frac{\partial w}{\partial t} = 0$$

and $$w = \frac{1}{2} \vec{D} \vec{E} + \frac{1}{2} \vec{B} \cdot \vec{H}$$

$$\frac{\partial W_e}{\partial t} = \frac{1}{2} \vec{D} \frac{\partial E}{\partial t} + \frac{1}{2} E \frac{\partial \vec{D}}{\partial t}$$

, we use, [1]

$$\frac{\partial W_m}{\partial t} = \frac{1}{2} \vec{B} \frac{\partial \vec{H}}{\partial t} + \frac{1}{2} \vec{H} \frac{\partial \vec{B}}{\partial t}$$

$\vec{D} = \varepsilon_0 \varepsilon_r \vec{E}$ and $B = \mu_0 \mu_r H$ for a linear, isotropic

So $$\vec{D} \frac{\partial \vec{E}}{\partial t} = \varepsilon_0 \varepsilon_r E \frac{\partial \vec{E}}{\partial t} = \vec{E} \frac{\partial \vec{D}}{\partial t}$$ [2]

Similarly $$\vec{B} \cdot \frac{\partial \vec{H}}{\partial t} = \vec{H} \cdot \frac{\partial \vec{B}}{\partial t}$$

[1]

So $$\frac{\partial w}{\partial t} = \vec{E} \cdot \frac{\partial \vec{D}}{\partial t} + \vec{H} \cdot \frac{\partial \vec{B}}{\partial t}$$

Faraday $$\frac{\partial \vec{B}}{\partial t} = -\nabla \times \vec{E}$$

Ampère $$\frac{\partial \vec{D}}{\partial t} = \vec{\nabla} \times \vec{H} - \vec{J}$$

[2]

$$\frac{\partial w}{\partial t} = \underbrace{\vec{E} \cdot (\vec{\nabla} \times \vec{H}} - \vec{J}) - \underbrace{\vec{H} \cdot (\vec{\nabla} \times \vec{E})}$$

if we the vector relation given on the underlined terms.

$$\frac{\partial w}{\partial t} = -\vec{\nabla} (\vec{E} \times \vec{H}) - \vec{E} \cdot \vec{J}$$

using now the conservation of energy and $\frac{\partial w}{\partial t}$ we get,

$$\vec{\nabla} \vec{S} - \vec{\nabla}(\vec{E} \times \vec{H}) - \vec{E} \cdot \vec{J} = 0$$

$\vec{E} \cdot \vec{J}$ is resistive energy dissipation (heat)

if $\vec{J} = 0$ (no currents), then

$$\vec{\nabla} \cdot \vec{S} - \vec{\nabla}(\vec{E} \times \vec{H}) = 0 \Rightarrow \vec{S} = \vec{E} \times \vec{H}$$

[1]

part of a solved problem

(iii) $S = \vec{E} \times \vec{H} \Rightarrow S_o = E_o \cdot H_o =$

$$= E_o \cdot c \cdot \varepsilon_o \varepsilon_r E_o = c \cdot \varepsilon_o \varepsilon_r E_o^2 \quad [1]$$

and $\langle S \rangle = \dfrac{c\, \varepsilon_o \varepsilon_r E_o^2}{2}$ because E is a sinusoidal wave, [1]

if $E = E_o \cos(\omega t - kz) \Rightarrow \langle E^2 \rangle = \dfrac{E_o^2}{2}$

$$\langle S \rangle = \frac{\varepsilon_o \cancel{\varepsilon_r}^{1}}{2\sqrt{\varepsilon_o \cancel{\varepsilon_r}^{1} \mu_o \cancel{\mu_r}^{1}}} E_o^2 = \frac{1}{2}\sqrt{\frac{\varepsilon_o}{\mu_o}}\, E_o^2 =$$

$$= \frac{E_o^2}{2 Z_o} \quad \text{because } Z_o^{-1} = \sqrt{\frac{\varepsilon_o}{\mu_o}} \quad [2]$$

(iv) Light carries momentum and therefore exerts a force to whichever surface is incident upon
when absorbed or reflected.
A lightweight mirror of large surface area is attached to a spacecraft and therefore provides acceleration [1]

$$S = \frac{\text{Power}}{\text{Surface}} = \frac{4 \cdot 10^{26}\,W}{4\pi\,(1.5 \cdot 10^{11}\,m)^2} = 1415\,\frac{W}{m^2} \quad [1]$$

Pressure is equal to $2 \cdot W$ (reflection)

$$\frac{2 \cdot S}{c} = \frac{2830\ W/m^2}{3 \cdot 10^8\ m} = 9.43 \cdot 10^{-6}\,\frac{N}{m^2} \quad [1]$$

force on $100\,m^2 = 9.43 \cdot 10^{-4}\,N$

acceleration on $10\,kg = 9.43 \cdot 10^{-5}\,\frac{m}{s^2}$

[2]