SEMESTER 1 EXAMINATIONS 2014-2015

LIGHT AND MATTER

DURATION 120 MINS (2 Hours)

This paper contains 8 questions

Answer **ALL** questions in **Section A** and **only TWO** questions in **Section B.**

Section A carries 1/3 of the total marks for the exam paper and you should aim to spend about 40 minutes on it.

Section B carries 2/3 of the total marks for the exam paper and you should aim to spend about 80 minutes on it.

An outline marking scheme is shown in brackets to the right of each question.

A Sheet of Physical Constants is provided with this examination paper.

Only University approved calculators may be used.

A foreign language translation dictionary (paper version) is permitted provided it contains no notes, additions or annotations.

Gauss $\nabla \cdot \mathbf{D} = \rho$ $\quad \oiint \mathbf{D} \cdot \mathrm{d}\mathbf{S} = \iiint \rho \mathrm{d}V$

$\nabla \cdot \mathbf{B} = 0$ $\quad \oiint \mathbf{B} \cdot \mathrm{d}\mathbf{S} = 0$

Faraday $\nabla \times \mathbf{E} = -\frac{\partial \boldsymbol{B}}{\partial t}$ $\quad \oint \mathbf{E} \cdot \mathrm{d}\mathbf{l} = -\oiint \frac{\partial \mathbf{B}}{\partial t} \cdot \mathrm{d}\mathbf{S}$

Ampere $\nabla \times \mathbf{H} = \mathbf{J} + \frac{\partial \mathbf{D}}{\partial t}$ $\quad \oint \mathbf{H} \cdot \mathrm{d}\boldsymbol{l} = \oiint \left(\mathbf{J} + \frac{\partial \mathbf{D}}{\partial t}\right) \cdot \mathrm{d}\boldsymbol{S}$

Constitutive equations:

$\mathbf{P} = \varepsilon_0 \chi \; \mathbf{E}$, *and* $\mathbf{D} = \varepsilon_0 \mathbf{E} + \mathbf{P} = \varepsilon_0 \varepsilon_r \mathbf{E}$, $\mathbf{B} = \mu_0 \mu_r \mathbf{H}$, *and* $\mathbf{B} = \mu_0 \left(\mathbf{H} + \mathbf{M}\right)$

TURN OVER

SECTION A

A1. Draw the Poincaré sphere and label the associated coordinate axes and points. [1]

Show with an example how the effect of a wave plate can be found using the Poincaré sphere as a graphic calculation tool. [2]

A2. Describe two different types of optical polariser, draw a schematic for each and explain the principle of operation of each. [4]

A3. Calculate the Jones matrix for a horizontal polariser.

Identify the optical element with a Mueller matrix of $\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & -1 \\ 0 & 0 & 1 & 0 \end{bmatrix}$

by calculating how it transforms horizontal, vertical and 45 degrees polarisation. [5]

A4. Explain how polarisation is affected by scattering, use also a schematic. Explain how polarisation is affected by reflection and with the aid of a diagram explain Brewster's angle. [4]

A5. Explain and derive the symmetry criterion for second order nonlinear processes. Can a centro-symmetric material exhibit second harmonic generation? Can it exhibit the Pockels effect and finally can it exhibit the Kerr effect? Briefly explain your answers. [4]

SECTION B

B1. (i) Write down the rate equations for a two-level system for spontaneous emission, absorption and stimulated emission using the Einstein A and B coefficients. Use diagrams to explain the processes of spontaneous emission, absorption and stimulated emission. [6]

(ii) By solving the time-dependent Schrödinger equation for a two-level system we get a system of two equations describing the time evolution of the amplitude coefficients $c_1(t)$ and $c_2(t)$ of the wave-function for level 1 and 2, respectively:

$$\dot{c_1}(t) = \frac{i\Omega_R}{2}\left(e^{i(\omega-\omega_0)t} + e^{-i(\omega+\omega_0)t}\right)c_2(t)$$

$$\dot{c_2}(t) = \frac{i\Omega_R}{2}\left(e^{-i(\omega-\omega_0)t} + e^{+i(\omega+\omega_0)t}\right)c_1(t)$$

Where $\Omega_R = \left|\frac{\mu_{12}E_0}{h}\right|$ is the Rabi frequency, μ_{ij} are the dipole matrix elements, $E_0 \text{ and } \omega$ are the amplitude and angular frequency of the incident electric field and ω_0 is the resonance frequency of the two-level system. Use this system of equations to derive the Rabi oscillations solutions for $c_1(t)$ and $c_2(t)$. Explain and justify the assumptions that you need to make for this derivation.

TURN OVER

Is population inversion possible for a two-level system and what does this imply about the ability of such a system to act as a laser? Why is it difficult to experimentally observe Rabi oscillations? Explain by using your knowledge about Rabi oscillations. [7]

(iii) Show what happens in the equation system in (ii) if the amplitude of the electromagnetic field, E_0, is zero. Based on your result explain if in the equation system in (ii) spontaneous emission is taken into account. [2]

(iv) In the interacting system of a two-level system with an optical field, explain what is a "dressed state". Consider a two-level system inside a resonant cavity. In this context, discuss what a polariton is and explain what kind of coupling between the two-state system and the light field is needed to have a polariton state. [5]

B2. **(i)** A plane electromagnetic wave passes through a crystal comprising of a small concentration of bound electrons, *N* per unit volume, each of which behaves like an undamped harmonic oscillator with natural angular frequency ω_0. Using the differential equation of motion for one electron and an appropriate trial solution show that the dielectric constant is approximately given by,

$$\epsilon_r = 1 + \frac{Ne^2}{m\epsilon_0(\omega_0^2 - \omega^2)},$$

where e and m are the charge and mass of the electron ω, is the angular frequency of the wave and ω_0 is the natural angular frequency of electron oscillations. [10]

(ii) Sketch (a) the refractive index and (b) the absorption in a diffuse atomic vapour as a function of wavelength in the region of two resonance lines. [2]

Give the equations that determine the relationship between the real and imaginary parts of the dielectric constant. [2]

(iii) Using $\sqrt{2/\sigma\omega\mu_0\mu_r}$ for the attenuation depth of an electromagnetic wave inside a conducting material, calculate the intensity attenuation presented by the 0.8 mm thick metal of a car to a mobile telephone signal at 2 GHz. The electrical conductivity of the metal is around 2×10^8 $(\Omega\text{m})^{-1}$ and μ_r may be taken to be unity. [3]

TURN OVER

Pure aluminium has a conductivity of 3.8×10^7 $(\Omega.m)^{-1}$. Estimate the minimum thickness of aluminium that should be deposited onto panes of glass in the manufacture of bathroom mirrors if we want to attenuate the intensity to one thousandth of the incident value (you can assume a wavelength of 500 nm). [3]

B3. (i) The energy densities W_e and W_m associated with electric and magnetic fields are defined to be,

$$W_e = \frac{1}{2}\mathbf{D}\cdot\mathbf{E}\,,$$

$$W_m = \frac{1}{2}\mathbf{B}\cdot\mathbf{H}.$$

Show that for a plane electromagnetic wave $W_e = W_m$ and that the total energy density $W = W_e + W_m$ is equal to $\epsilon_0\epsilon_r E^2$. You can use the constitutive equations for an isotropic, linear material. For a plane electromagnetic wave, you can use the relation $\mathbf{H} = \frac{\omega}{k}D$, and that the speed of light is $v = \frac{1}{\sqrt{\epsilon_0\epsilon_r\mu_0\mu_r}}$. [4]

(ii) Using conservation of energy $\nabla\cdot\mathbf{S} + \frac{\partial W}{\partial t} = 0$ and the definition of total energy density,

$$W = W_e + W_m = \frac{1}{2}\mathbf{D}\cdot\mathbf{E} + \frac{1}{2}\mathbf{B}\cdot\mathbf{H},$$

derive a Poynting vector $\mathbf{S}$ that can satisfy this relation.

You may want to use Faraday and Ampere laws and also the vector relation $\nabla\cdot(\mathbf{A}\times\mathbf{B}) = \mathbf{B}\cdot(\nabla\times\mathbf{A}) - \mathbf{A}\cdot(\nabla\times\mathbf{B})$. [7]

TURN OVER

(iii) Show that the average Poynting vector can be written as $\bar{S} = \frac{E_0^2}{2z_0}$, where $z_0 = \sqrt{\frac{\mu_0}{\epsilon_0}}$ is the impedence of free space. You can use that the Poynting vector is $\mathbf{S} = \mathbf{E}\times\mathbf{H}$, also that $\mathbf{H} = \frac{\omega}{\mathrm{k}} \cdot \mathbf{D}$**,** the speed of light $v = \frac{1}{\sqrt{\epsilon_0 \epsilon_r \mu_0 \mu_r}}$ and the constitutive equations. [4]

(iv) Explain the principle of the solar sail. If the sun radiates an optical power of 4×10^{26} W, estimate the greatest acceleration that may be achieved by a spacecraft in earth orbit of mass 10 kg with a 100 m^2 reflective sail. The mean distance between the earth and the sun is approximately 1.5×10^{11} m. [5]

END OF PAPER