UNIVERSITY OF SOUTHAMPTON PHYS3003W1

SEMESTER 1 EXAMINATIONS 2012/13

LIGHT & MATTER

DURATION 120 MINS

Answer all questions in Section A and two and only two questions in Section B.

Section A carries 1/3 of the total marks for the exam paper and you should aim to spend about 40 mins on it. Section B carries 2/3 of the total marks for the exam paper and you should aim to spend about 80 mins on it.

An outline marking scheme is shown in brackets to the right of each question.

A Sheet of Physical Constants is provided with this examination paper. Only university approved calculators may be used.

Gauss $\nabla \cdot \mathbf{D} = \rho$ $\quad \oiint \mathbf{D} \cdot d\mathbf{S} = \oiint \rho dV$

$\nabla \cdot \mathbf{B} = 0$ $\quad \oiint \mathbf{B} \cdot d\mathbf{S} = 0$

Faraday $\nabla \times \mathbf{E} = -\frac{\partial \boldsymbol{B}}{\partial t}$ $\quad \oint \mathbf{E} \cdot d\mathbf{l} = -\oiint \frac{\partial \boldsymbol{B}}{\partial t} d\mathbf{S}$

Ampere $\nabla \times \mathbf{H} = \mathbf{J} + \frac{\partial \mathbf{D}}{\partial t}$ $\quad \oint \mathbf{H} d\boldsymbol{l} = \oiint \left(\boldsymbol{J} + \frac{\partial \mathbf{D}}{\partial t}\right) \boldsymbol{dS}$

Constitutive equations:

$\mathbf{P} = \varepsilon_0 \chi \ \mathbf{E}$, *and* $\mathbf{D} = \varepsilon_0 \mathbf{E} + \mathbf{P} = \varepsilon_0 \varepsilon_r \mathbf{E}$

$\mathbf{B} = \mu_0 \mu_r \mathbf{H}$, *and* $\mathbf{B} = \mu_0 (\mathbf{H} + \mathbf{M})$, *where* $\mu = \mu_0 \mu_r$ and $\varepsilon = \varepsilon_0 \varepsilon_r$

Number of Pages **7**

SECTION A

A1. Explain polarization and displacement. You may use the context of a capacitor filled with a dielectric. Write down Gauss's law for displacement and polarization. Describe a situation where polarization charges can result in an electrostatic force; you may want to use a sketch. [4]

A2. Write down equations that relate the electric and magnetic fields to the scalar and vector potentials. Discuss where the potentials are useful in the theory of electromagnetism. State what it means to choose gauges and what is gauge invariance. State two gauges that you know. [3]

A3. Discuss carefully whether (i) the phase velocity, (ii) the group velocity can exceed the speed of light. Write down the relation that gives the group velocity from a dispersion relation. [2]

A4. Find an approximate expression for an arbitrary potential function, $V(x)$ that is valid in the vicinity of a minimum at x_0. Use your result to explain why the harmonic oscillator is so frequently encountered in physics. [4]

A5. The wave equation, given by,

$i\frac{\partial^2 y(x,t)}{\partial x^2} = \frac{\partial y(x,t)}{\partial t}$, has a solution of, $y(x,t) = y_0 \exp[\mathrm{i}(\omega t - kx)]$.

State if the given wave equation is linear or non-linear. Find its dispersion relation that links ω and k and also the phase velocity of the wave. Is the phase velocity constant? [3]

A6. Sketch (a) the refractive index and (b) the absorption in a diffuse atomic vapour as a function of wavelength in the region of two resonance lines. [2]

Give the equations that determine the relationship between the real and imaginary parts of the dielectric constant. [2]

SECTION B

B1. (i) By solving the time-dependent Schrödinger equation for a two-level system we get a system of two equations describing the time evolution of the amplitude coefficients $c_1(t)$ and $c_2(t)$ of the wave-function for level 1 and 2, respectively:

$$\dot{c_1}(t) = \frac{i\Omega_R}{2}\left(e^{i(\omega-\omega_0)t} + e^{-i(\omega+\omega_0)t}\right)c_2(t)$$

$$\dot{c_2}(t) = \frac{i\Omega_R}{2}\left(e^{-i(\omega-\omega_0)t} + e^{+i(\omega+\omega_0)t}\right)c_1(t)$$

Where, $\Omega_R = \left|\frac{\mu_{12}E_0}{h}\right|$ is the Rabi frequency, μ_{ij} are the dipole matrix elements, E_0, ω are the amplitude and angular frequency of the incident electric field and ω_0 is the resonance of the two-level system. Use this system to derive the Rabi oscillations solutions for $c_1(t)$ and $c_2(t)$. Explain and justify the assumptions that you need to make for this derivation. Is population inversion possible for a two-level system? Why is it difficult to experimentally observe Rabi oscillations? Explain by using your knowledge about Rabi oscillations. [8]

(ii) Next write down the rate equations for a two-level system for spontaneous emission, absorption and stimulated emission using the Einstein A and B coefficients. Use diagrams to explain the processes of spontaneous emission, absorption and stimulated emission. [5]

(iii) Explain what happens in the system in (i) if the amplitude of the electromagnetic field, E_0, is zero. Based on your result explain if in the system in (i) spontaneous emission is taken into account. [2]

(iv) In the interacting system of a two-level system with an optical field, explain what is a so-called dressed state. Consider a two-level system inside a resonant cavity. In this context, discuss what is a polariton and explain what kind of coupling we need between the two-state system and the light field to have a polariton state. [5]

B2. **(i)** Write down the electrical field of linearly polarized light at $45°$ and circularly polarised light by analysing its *x* and *y* components. [1]

(ii) Explain the Jones and Stokes vectors by giving examples of how they are used to describe a polarization state. When is it better to use Jones vectors and when Stokes vectors? Can Stokes vectors be experimentally measured? [2]

Show that the Jones vectors of left and right circularly polarized light are orthogonal to each other. (Hint: Use their inner product.) [2]

(iii) Derive the Jones matrix for a vertical polarizer. [3]

(iv) Draw the Poincaré sphere and label the associated coordinate axes and points. [2]

Show with an example how the effect of a wave plate can be found using the Poincare sphere as a graphic calculation tool. [3]

(v) Explain briefly the phenomenon of birefringence and the concept of the optical axis. Then explain how a birefringent material can be used to construct a wave-plate. Explain, briefly the difference between, zero-order, compound zero-order and multiple order wave-plates. [4]

(vi) Calculate the minimum thickness of a quarter wave-plate for use at a wavelength of 550 nm, if the used material to realise the wave-plate has an ordinary and extraordinary refractive index of 1.7 and 1.5, respectively. [3]

B3. **(i)** Write down the relation linking the polarization and the electric field for an anisotropic medium**.** [2]

State the so-called von Neumann principle and explain why it is considered important in the theory concerning the susceptibility tensor. [3]

(ii) Describe the linear and the quadratic electro-optic effect and outline the used non-linearities as mathematically derived by the series expansion of polarisation density. [3]

Explain the effect of Kerr lensing and briefly illustrate its importance for ultrafast lasers. Give a sketch of a cavity that uses Kerr lensing to achieve mode-locking. [3]

(iii) Show how the phenomena of second-harmonic generation and DC optical rectification stem from the existence of a second order susceptibility $\chi^{(2)}$. Explain the origin of the symmetry criterion for the appearance of second-order nonlinear effects based on the series expansion of the polarization density. [5]

(iv) What is the wavelength of the second-harmonic generated from light of a laser with wavelength of 1064 nm? Furthermore, explain phase-matching and using a sketch describe how it can be achieved using birefringence. [2]

Draw a diagram of the power of the second-harmonic for the two cases, when (a) $n_\omega = n_{2\omega}$ and (b) $n_\omega \neq n_{2\omega}$. [2]

END OF PAPER