

SEMESTER 1 EXAMINATION 2014-2015

CRYSTALLINE SOLIDS

Duration: 120 MINS (2 hours)

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This paper contains 9 questions.

Answer **all** questions in **Section A** and **only two** questions in **Section B**.

**Section A** carries 1/3 of the total marks for the exam paper and you should aim to spend about 40 mins on it.

**Section B** carries 2/3 of the total marks for the exam paper and you should aim to spend about 80 mins on it.

An outline marking scheme is shown in brackets to the right of each question.

A Sheet of Physical Constants is provided with this examination paper.

Only university approved calculators may be used.

A foreign language translation dictionary (paper version) is permitted provided it contains no notes, additions or annotations.

## Section A

- A1.** Explain what the structure factor is in relation to X-ray diffraction. What effect does it have on the measured diffraction pattern? [ 4 ]
- A2.** Describe the assumptions made in the tight-binding theory of atomic bonding. [ 4 ]
- A3.** Explain the Born-Oppenheimer approximation indicating why it is often appropriate to make. [ 4 ]
- A4.** Write down and explain Bloch's equation for the electron wavefunction in the periodic potential of a crystal. Define all symbols used. [ 4 ]
- A5.** Explain what is meant by the terms: dispersion relation, Fermi-energy , Fermi-surface, allowed  $k$ -state. [ 4 ]

## Section B

- B1.** (a) Explain why X-ray diffraction, measured from a crystalline material in powder form, produces conical shaped beams. [ 3 ]
- (b) Name three properties of a crystal that can be determined using X-ray diffraction. [ 3 ]
- (c) Three different crystalline materials (A, B and C) in powder form are known to have either face-centred cubic (fcc), body-centred cubic (bcc) or diamond cubic structure. Diffraction experiments performed on these materials gives a number of diffraction rings. The approximate positions ( $2\theta$ ) of the first four diffraction rings are tabulated below:

Sample $2\theta$ angles (degrees)		
A	B	C
45.72°	40.01°	46.43°
53.30°	46.54°	67.76°
96.12°	67.93°	86.12°
127.56°	81.86°	104.07°

- Identify the crystal structure for each case indicating the Miller indices for each peak. Include the selection rules for each crystal structure in your answer. [ 9 ]
- (d) If the wavelength of the incident X-ray beam is 0.16nm what is the conventional unit cell length for materials A, B and C? [ 3 ]
- (e) If electrons are used instead of X-rays for the above scattering experiments, what accelerating voltage is required to achieve the same diffraction angles? [ 2 ]

**TURN OVER**

- B2.** (a) Describe the assumptions made in the free electron (jellium) model. [ 2 ]
- (b) Explain what is meant by electron 'mobility' ( $\mu$ ) and 'conductivity' ( $\sigma$ ). [ 4 ]
- (c) Derive expressions for  $\mu$  and  $\sigma$  for free electrons. [ 8 ]
- (d) Aluminium metal has a face centred cubic (fcc) structure. The conventional cubic unit cell has side lengths of 0.4 nm. If each aluminium atom contributes just one conduction electron, estimate the mobility of conduction electrons if the resistivity is  $20 \times 10^{-9} \Omega \text{ m}$ . [ 6 ]

**B3.** (a) Describe what is meant by ‘paramagnetism’ and ‘ferromagnetism’. [ 4 ]

(b) The resulting magnetisation due to an applied magnetic field is given by:

$$\mu_0 M = \chi B$$

Define the quantity  $\chi$  in the above equation. [ 2 ]

(c) The total effective magnetic field inside a ferromagnetic material is the sum of the inner molecular field ( $\lambda M$ ) and the external magnetic field ( $B_0$ ), and is given by:

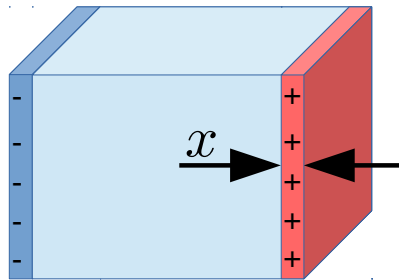
$$B_e = B_0 + \lambda M$$

What are the physical mechanisms described by this equation? [ 4 ]

(d) Use the equations in (b) and (c) to derive the Curie-Weiss law and hence show that the critical temperature,  $T_c$ , is given by the relation  $T_c = \frac{\lambda C}{\mu_0}$ , where  $C$  is the Curie constant. [ 10 ]

**TURN OVER**

- B4.** In this question, we will consider charge oscillations using the jellium model. Consider a metal block in which the electrons are displaced laterally by a small distance  $x$ , as indicated in the diagram below:



- (a) Show that the electric field  $E$  between the two charged regions is given by:

$$E = \frac{nex}{\epsilon_0}$$

where  $n$  is the number density of conduction electrons. State the direction of the electric field. [ 4 ]

- (b) Using the equation above for the electric field  $E$ , show that the plasma frequency is given by:

$$\omega_p^2 = \frac{ne^2}{\epsilon_0 m_e}$$

where  $m_e$  is the mass of a single electron. [ 3 ]

- (c) Copper (Cu) metal has a face-centred cubic (fcc) crystal structure with lattice constant  $\mathbf{a} = 3.61\text{\AA}$ . Given that each Cu atom provides exactly 1 conduction electron, calculate the plasma frequency for Cu. [ 4 ]

- (d) Explain why the value found for  $\omega_p$  is inconsistent with the observation that copper metal has a red appearance in colour. Explain why the model used to determine the plasma frequency breaks down. [ 4 ]

- (e) What is the name of the quasi-particle for charge oscillations? [ 1 ]

- (f) Give two reasons for describing excitations in solids using the concept of quasi-particles. Write down two additional examples of quasi-particles used to describe excitations in crystalline solids. In each case, give the name of the quasi-particle and the type of excitation it is associated with. [ 4 ]

**END OF PAPER**