UNIVERSITY OF SOUTHAMPTON PHYS3008W1

SEMESTER 1 EXAMINATION 2012-2013

ATOMIC PHYSICS

Duration: 120 MINS

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*Answer **all** questions in **Section A** and two **and only two** questions in **Section B**.*

Section A carries 1/3 of the total marks for the exam paper and you should aim to spend about 40 mins on it. Section B carries 2/3 of the total marks for the exam paper and you should aim to spend about 80 mins on it.

A Sheet of Physical Constants will be provided with this examination paper.

An outline marking scheme is shown in brackets to the right of each question.

Only university approved calculators may be used.

Section A

A1. The z component of the angular momentum operator for a single particle is given by

$$\hat{L}_z = -i\hbar \frac{\partial}{\partial \varphi},$$

in spherical polar coordinates. How is $\hat{L}_z$ related to symmetry under rotations around the z-axis? By deriving its eigenfunctions, explain how this leads to the existence of a conserved quantum number m for the hydrogen atom. [5]

A2. In quantum mechanics, what is meant by a commutator? What is required in quantum mechanics for a quantity to be conserved? Show that this implies that for the linear momentum of an electron to be conserved in the presence of a potential, the potential must be constant. [6]

A3. One of the electromagnetic emission lines for a hydrogen atom has wavelength 389nm. Assuming that this is a line from one of the Lyman ($n_f = 1$), Balmer ($n_f = 2$) or Paschen ($n_f = 3$) series, what is the initial principal quantum number associated with the transition? The Rydberg energy can be assumed to be 13.6eV. [5]

A4. A Hamiltonian is represented by the matrix:

$$\begin{pmatrix} 5 & 0 & -1 \\ 0 & 5 & 0 \\ -1 & 0 & 5 \end{pmatrix} \mathrm{eV}$$

Show that $\frac{1}{\sqrt{2}}(1\ 0\ 1)$ is an eigenstate of the operator and derive its eigenvalue. Give one other eigenstate of the operator with its eigenvalue. [4]

Section B

B1. (i) What is the central field approximation for multi-electron atoms and how is the central field calculated in practice? [5]

(ii) Describe what is meant by the “quantum defect” as applied to alkali metal atoms and the physical mechanism which leads to the quantum defects. [4]

(iii) Explain why the first four rows of the periodic table have 2, 8, 8, and 18 atoms respectively. [4]

(iv) Name the two perturbations usually applied to the central field Hamiltonians to obtain the first layer of fine structure for multi-electron atoms. Which is more important for small atomic number atoms? [3]

(v) Consider a small atomic number atom in a large enough magnetic field that the strong field Zeeman effect occurs. On which quantum numbers of the atom do the Zeeman energies depend and why? [4]

B2. (i) What are the physical observables associated with the principal quantum numbers n, l and m which are used to describe the electronic states of hydrogen? [3]

(ii) State the inequalities which limit the possible combinations of n, l and m. [2]

(iii) Sketch or otherwise describe the angular dependence of the probability distribution of the hydrogen states $[n = 2, l = 0, m = 0]$, $[n = 2, l = 1, m = 0]$, and $[n = 2, l = 1, m = -1]$. [3]

(iv) How does the spin-orbit correction applied to hydrogen depend on orbital and spin angular momentum? What is the physical mechanism which gives rise to the spin-orbit correction? What effect does the spin-orbit correction have on the quantum numbers used to label the states of hydrogen? [8]

(v) The sum of the relativistic fine structure corrections for hydrogen is given by

$$\Delta E = -\frac{\alpha^2}{n^2}\left[\frac{3}{4} - \frac{n}{j + \frac{1}{2}}\right] E_n^0 .$$

How many distinct spectral lines due to this fine structure are seen for optical transitions between the $n = 4$ and $n = 3$ levels? Give your reasoning. [4]

B3. (i) The Hamiltonian for a single electron in the presence of a nucleus with atomic number Z is

$$\hat{H}_0 = \left[-\frac{\hbar^2}{2m}\nabla^2 - \frac{Ze^2}{4\pi\varepsilon_0 r} \right] .$$

Show that the eigenstates and eigenvalues of this problem are those for the hydrogen atom scaled in the following ways:

$$\psi^Z_{nlm}(\mathbf{r}) = Z^{3/2}\psi^H_{nlm}(Z\mathbf{r}) , \qquad E^Z_{nlm} = Z^2 E^H_{nlm} .$$

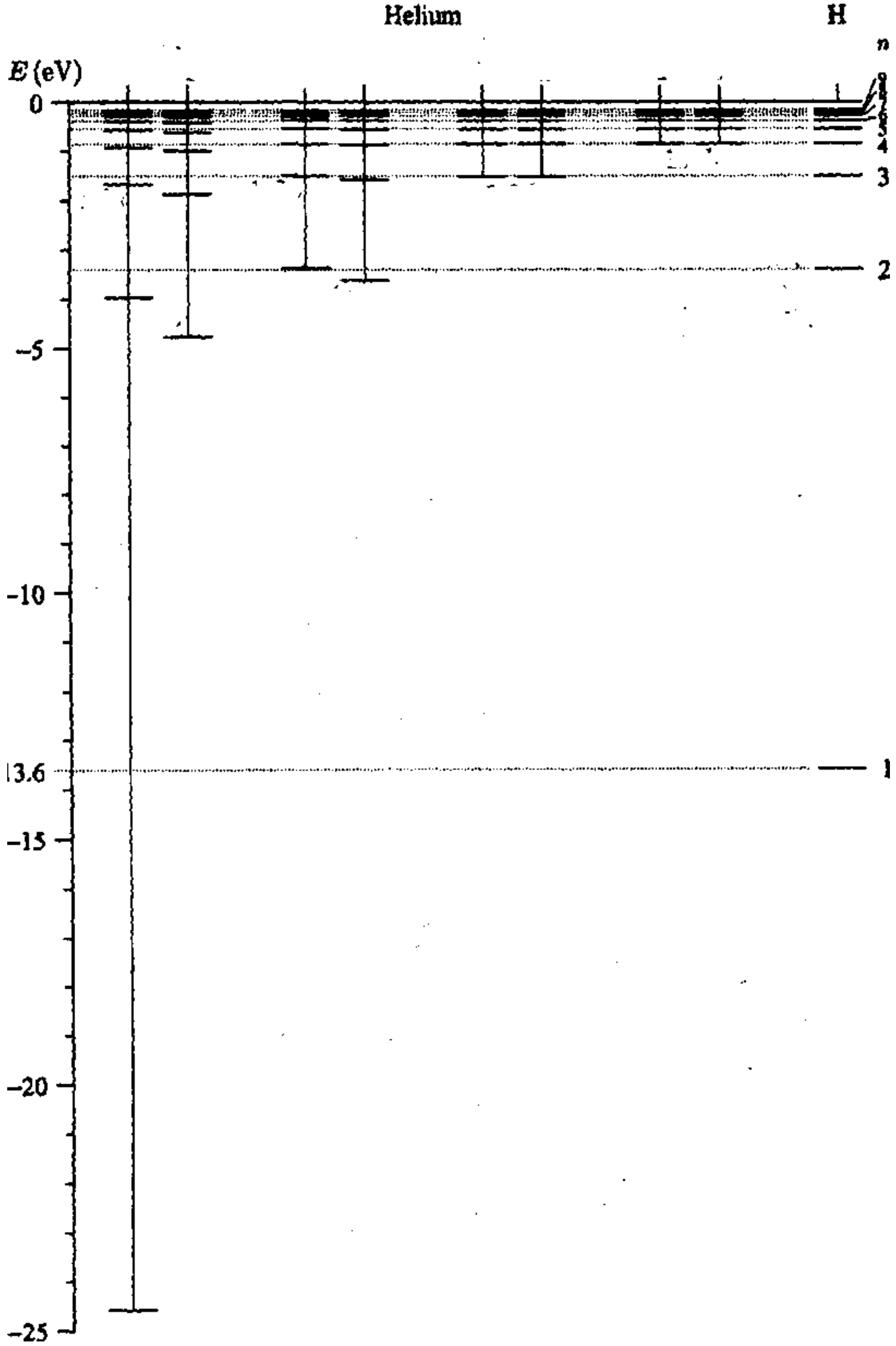
[4]

(ii) If we neglect the electron-electron interaction, the Hamiltonian for the helium atom is the sum of two of the Hamiltonians discussed above with $Z = 2$, one for each electron. Write down the two possible two-particle spatial wavefunctions with definite exchange symmetry, assuming that one electron is in the $1s$ state and one in the $2s$ state. [2]

(iii) Write down the operator for the electron-electron interaction and use it to give an integral formula for the direct and exchange energies using first order perturbation theory. Describe the physical significance of the two energies. [4]

(iv) Which of the two energies in (iii) leads to the energy of the states of helium depending on spin? Why? [5]

(v) The diagram below sets out the energy levels of a helium atom in which at least one electron is in the lowest possible single electron state. Label the three lowest helium energy levels on this diagram with the corresponding single-electron orbitals. In addition label the columns in which these levels are found with their correct term symbol. [5]



B4. (i) State all of the selection rules for an electric dipole optical transition for a multi-electron atom. [6]

(ii) Which of the following transitions are allowed and which forbidden? Where forbidden, state which selection rule or rules they break.

$$(1s)^2\, 2p\, 3p\ {}^3S_1 \ \rightarrow\ (1s)^2\, 2p\, 4d\ {}^3D_1$$
$$(1s)^2\, (2p)^2\ {}^3D_2 \ \rightarrow\ (1s)^2\, 2p\, 3p\ {}^1D_2$$
$$(1s)^2\, 2p\, 3p\ {}^3S_1 \ \rightarrow\ (1s)^2\, 2p\, 4d\ {}^3P_1$$

[6]

(iii) What does Fermi's golden rule predict for optical transitions and what is the formula for that prediction? Clearly identify all the terms in this equation. [4]

(iv) It is possible for optical transitions to occur via the absorption of two photons simultaneously. What are the selection rules for J, M_J and S for such a transition for a simple hydrogen atom. [4]

END OF PAPER