SEMESTER 2 EXAMINATION 2014-2015

APPLIED NUCLEAR PHYSICS - model answers

Duration: 120 MINS (2 hours)

This paper contains 10 questions.

Answers to Section A and Section B must be in separate answer books

Answer **all** questions in **Section A** and **only two** questions in **Section B**.

Section A carries 1/3 of the total marks for the exam paper and you should aim to spend about 40 mins on it.

Section B carries 2/3 of the total marks for the exam paper and you should aim to spend about 80 mins on it.

An outline marking scheme is shown in brackets to the right of each question.

A Sheet of Physical Constants is provided with this examination paper.

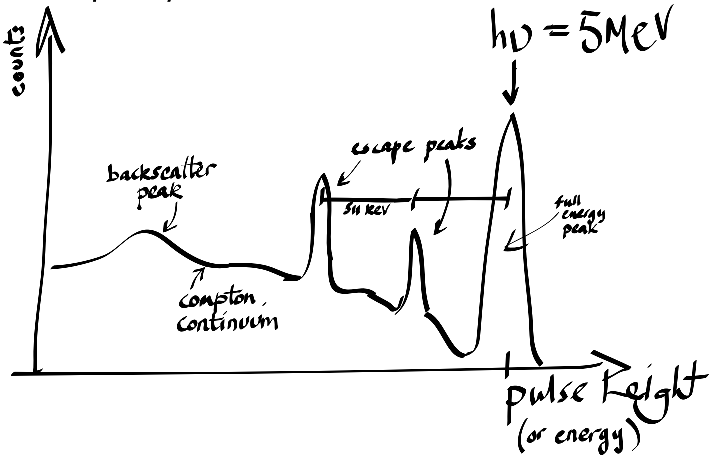
Only university approved calculators may be used.

A foreign language translation dictionary (paper version) is permitted provided it contains no notes, additions or annotations.

Section A

A1. Draw a labelled diagram showing the pulse-height spectrum you would obtain from a germanium gamma-ray spectrometer illuminated by a 5 MeV gamma-ray source. Mark the energy of any lines or other features in the spectrum, and very briefly describe their origin. [4]

Example spectra were sketched and discussed in lectures.



Plot should show:

- appropriate overall shape and ratio of lines **[1]**

- correctly labelled axes **[1/2]**

- labelled photopeak at 5 MeV **[1/2]**

- labelled escape peaks at 3.978 and 4.489 **[1/2]**

- Compton continuum **[1/2]**

- Compton edge(s) **[1/2]**

- labelled Backscatter peak at 200 keV **[1/2]**

A2. What is the fundamental difference between **r-process** and **s-process** nucleosynthesis, and under what conditions might these processes occur? [3]

Standard bookwork.

The fundamental difference is the neutron flux **[1]** *under which these neutron-capture processes occur.*

In s-process, this flux is low, and unstable nuclei can decay, so do not become neutron-rich **[1/2]** *while in r-process the neutron flux is so high that unstable neutron-rich nuclei are created* **[1/2]**.

s-process may occur in red giant cores **[1/2]** *while r-process can probably only occur in supernova explosions* **[1/2]**.

A3. What is an isochron? Write down the two essential features that must be possessed by isochronic samples used in geological dating. [3]

Standard bookwork

Literally, a line of 'equal time' joining samples of equal age but different chemical composition constructed during radiochemical rock dating **[1]**.
Isochron dating overcomes some limitations of rock dating, namely that the initial amount of a daughter isotope is unknown **[1/2]**, *or that some daughter isotope may escape after production* **[1/2]**.
Isochron dating avoids this by comparing the parent and daughter isotope concentrations with that of another chemically identical isotope of the daughter product **[1]**.

A4. What are the relative advantages and disadvantages of radiochemical and Cherenkov detectors for studying solar neutrinos? [3]

Standard bookwork

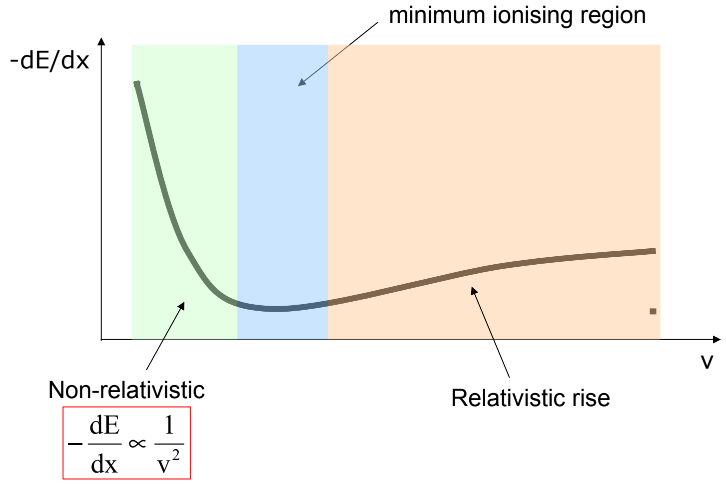
Cherenkov detectors only work at higher energies (>5MeV) so cannot see the most abundant solar neutrinos **[1/2]**, *but work in real-time (SN bursts!) and can reconstruct the direction of the neutrino* **[1/2]**, *and can also (using heavy water) observe all 3 flavours* **[1/2]**.
Radiochemical systems can operate at lower energies (even p-p chain neutrinos) **[1/2]** *but are not directional. The chemical extraction of the daughter product is complicated, and the efficiency of the overall process is harder to calibrate* **[1/2]**. *Usually use exotic chemicals (like Ga) and so are very expensive* **[1/2]**.

A5. With the help of a labelled graph, describe how the energy lost per unit distance by a heavy charged particle moving through matter varies with the particle velocity. Use your answer to explain the term *minimum ionising particle*. [4]

TURN OVER

Standard bookwork

Labelled sketch should show $1/v^2$ behaviour, minimum, relativistic rise, and flattening, with appropriately labelled axes **[2]***:*



Relativistic rise is due to greater impact potential of particle due to compression of E-field in direction of travel **[1/2]***, ultimately limited by polarisation of material shielding electric field* **[1/2]***.*
A minimum ionising particle is one that has a velocity such that -dE/dx is at the minimum value, ie. kinetic energy is approx rest mass energy, before relativistic rise starts **[1/2]***. Regardless of the particle, this is about 2 MeV/gm* **[1/2]***.*

A6. What are the two main reactions used in fusion research reactors? State the advantages and disadvantages of each one. [3]

Standard bookwork

D-D reaction **[1/2]***: only low energy neutrons produced ; almost unlimited fuel availability* **[1/2]** *higher ignition temp* **[1/2]**
D-T reaction **[1/2]** *larger cross-section, lower ignition temperature and more energy per fusion* **[1/2]** *but energetic neutrons produced* **[1/2]**

Section B

B1. **(a)** Outline the qualitative naming conventions used to describe the energy of neutrons. [4]

Standard Bookwork

Neutrons may be described according to their temperature, their velocity (their speed really), or their energy.

At low energies, the neutrons energy is dictated by the thermal environment they are in equilibrium with, giving rise to terms like 'ultra-cold', 'cold' or 'thermal'. **[1]**

Thermal neutrons are the most common, being in thermal equilibrium at room temperature, with an energy of 0.025 eV, that being the modal value of a Maxwellian distribution at 273K. **[1]**

At somewhat higher energies (up to MeV energies), speed-like notation is used, in this case the most important being fast neutrons at ~1MeV, which are typically produced in nuclear reactions such as fission. **[1]**

Above that, energy becomes the most common descriptor, with terms like 'very high energy neutron' in use. **[1]**

(b) Describe the interactions of neutrons with matter, being sure to discuss the energy dependence of each interaction, and the neutron energy range over which it is most applicable. [6]

Standard bookwork, and lecture discussion.

General comments: lacking charge, or the ability to interact via weak forces, neutrons interact only via strong forces with the nuclei of the absorbing material **[1/2]**. *This makes them a very penetrating and damaging/disruptive radiation* **[1/2]**.

Specific interesting/useful reactions are:

Nuclear reactions - many options for neutron to disrupt nucleus such that one or more particle is emitted (n,p) (n,α) (n,t) etc **[1]**. *All have 1/v cross sections.***[1/2]**

TURN OVER

Fission - a special case of nuclear reaction resulting in splitting of the nucleus into fission fragments and large energy output **[1/2]**. *Again, a 1/v cross section* **[1/2]**

Radiative capture - capture results in formation of an excited (radioactive) nucleus that decays by emission of one or more photons **[1/2]** *that are useful in element analysis* **[1/2]**

Elastic and inelastic scatter involve a collision with a nucleus in which some energy is transferred from the neutron. At higher energies (~ 1 MeV) the excited nucleus may emit a photon that is lost from the system, and hence the collision is inelastic **[1]**.

At even higher energies, the nucleus is likely to be totally disrupted, with the creation of a hadron shower **[1/2]**.

(c) Show that, for the simple case of a direct head-on elastic collision of a neutron with a nucleus of atomic mass A, the ratio of the neutron's energy after the collision, E, to that before the collision, E_0, is given by:

$$\frac{E}{E_0} = \left(\frac{A-1}{A+1}\right)^2$$ [5]

More complex derivation involving angles was covered in lectures. This question is simpler, with no angles. Should be doable from first principles.

Neutron mass m, nucleus M. In lab frame, nucleus is stationary and neutron has velocity v in +X direction.

In Centre of Mass frame, momentum conservation gives:

$$m(v - V_{cm}) = MV_{cm}$$

$$i.e. \quad V_{cm} = \frac{mv}{M+m}$$

After collision, neutron rebounds with velocity $v - V_{cm}$ in CM frame in -X direction, ie with velocity $v - 2V_{cm}$ (in -X direction) in lab frame.

$v - 2V_{cm} = v - 2\frac{mv}{M+m} = v\left(\frac{M+m-2m}{M+m}\right) = v\left(\frac{M-m}{M+m}\right)$

Therefore $\frac{E}{E_0} = \left(\frac{M-m}{M+m}\right)^2 = \left(\frac{A-1}{A+1}\right)^2$ **[5]**

(d) The number, n, of collisions needed to reduce the energy of a neutron from E_0 to E is given approximately by

$$n = \frac{1}{\zeta} ln \frac{E_0}{E}$$

where

$$\zeta = 1 + \frac{(A+1)^2}{2A} ln \frac{A-1}{A+1}.$$

If E=1 MeV, calculate ζ and n for collisions, with both deuterium and carbon, to bring neutrons down to the energy which is most effective for inducing fission in ^{235}U. [3]

Putting in the values:
For deuterium, ie A=2, ζ=0.72.
For carbon, ie A=12, ζ=0.158. **[1]**

To induce fission in ^{235}U we require 'thermal' neutrons, ie 0.025 eV (mentioned many times in lectures), ie $ln\frac{E_0}{E} = 17.5$. **[1]**

Thus, for deuterium, $n = 24.3$ and for carbon $n = 110$. **[1]**

(e) Discuss the various contributions to the total neutron scattering cross section, including how the cross section varies with atomic number and with energy. [2]

Discussed in class. There are multiple components including for absorption, for scattering (both coherent and incoherent) and magnetic, arising from the interaction between the magnetic dipole moment of the neutron

TURN OVER

and of unpaired electrons in the interacting atoms. Large absorption cross-sections are often associated with low energy resonances in the compound nucleus formed by neutron capture.

There is no systematic change with either Z or A and the cross section is even different for different isotopes of the same element. **[2]**

B2. **(a)** Describe how radioactive ^{14}C is produced, distributed within the ecosystem and incorporated into organic samples. [4]

Standard bookwork

- *energetic neutrons are produced by cosmic ray interactions in the upper atmosphere* **[1]**
- *C-14 production is by neutron capture on atmospheric N-14* **[1]**
- *C-14 is stored in 'reservoirs' such as the atmosphere, oceans etc* **[1/2]** *which maintain an equilibrium by exchanging carbon* **[1/2]**
- *living organic material maintains equilibrium of C-14 levels by absorption in form of CO_2* **[1]**

(b) A simplistic approach to radiocarbon dating might assume that the abundance of ^{14}C in natural carbon is the same at all times and places. Discuss the problems with this assumption, and the methods used to deal with any changes to the ^{14}C abundance. [6]

Standard bookwork

Radiocarbon production changes with time due to

- *cosmic ray rate correlated to solar activity* **[1]**
- *exchange rate with reservoirs (climate change)* **[1]**
- *human effects (Suess effect, nuclear tests)* **[1]**

Radiocarbon levels depend on location as exchange with deep oceans is slow, so oceans are depleted in C-14 **[1]**

We can overcome these problems by calibration against samples of known age, typically by:

- *Tree ring dating (dendrochronology) allows calibration back 7000 years* **[1]**

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– *For aquatic samples, values must be adjusted by normally 500 years (this is dodgy)* **[1]**

(c) The half-life of ^{14}C is 5730 years, and young natural carbon contains ^{14}C at a concentration of 1 part in 10^{12}. How many decays will occur per gram of young natural carbon per minute. [2]

Trivial (unseen) calculation of radioactive decay

12g of C contains 6.02x10^{23} atoms

Hence 1g of C contains $\frac{6.02\times10^{23}}{12\times1\times10^{12}} = 5\times10^{10}$ *C-14 atoms* **[1]**

decay rate is just $\lambda N = N \times \frac{ln2}{t_{1/2}} = \frac{5\times10^{10}\times ln2}{5730\times365\times24\times60}$

decay rate is therefore 11.55 counts/g/minute **[1]**

A sample of 50mg of carbon of unknown age produces 0.09 counts/minute. Use this information to estimate its age. [4]

Again, a trivial calculation, and easy to sanity check

From previous section, we know that 1mg of young carbon should disintegrate at 0.01155 c/minute **[1/2]**

If we convert our sample to same units, we get 0.0018 c/mg/min **[1/2]**.

(so can immediately estimate that our sample is a bit more than two half-lifes old - no marks for that, but answers should be at least sensible as a result!)

$N = N_0 e^{-\lambda t}$ *and* $\lambda = \frac{ln2}{t_{1/2}} = 1.21\times10^{-4}$ **[1]**

Rearranging, $-\lambda t = ln(0.0018/0.01155) = -1.859$ **[1]**

and hence t = 15362 years (or could accept 15000-15500 as a sensible estimate!) **[1]**

(d) Discuss various ways in which the decay rate of small samples of carbon can be measured. [4]

Standard bookwork

Original Radiometric method converts C into CO_2 and incorporates into a gas proportional counter **[1/2]** *but this is very insensitive due to low count rate from long half life* **[1/2]**. *Can need samples up to 1000g and is completely destructive* **[1/2]**

Modern AMS (accelerator mass spectroscopy) **[1/2]** *allows counting of much lower levels of radiocarbon because it counts the nuclei* **[1/2]**, *rather than waiting for them to decay, so allows use of much smaller samples down to mg levels* **[1/2]**. *Like mass spectroscopy, but uses additional techniques to remove similar-mass isotopes and molecules because:*

- *14N cannot form stable negative ions* **[1/2]**
- *12CH_2, 13CH are removed by stripper (which causes molecular break-up by removal of many electrons)* **[1/2]**

B3. **(a)** In a fission reactor, the neutron reproduction factor can be described by the four-factor formula, normally written as $k_\infty = \eta\epsilon p f$.

Starting with a population of N thermal neutrons in the fuel of a fission reactor core, describe the neutron reproduction cycle, defining all the terms of the four-factor formula. Quantify all neutron loss and gain processes that can occur in an infinite-sized reactor. [8]

Standard bookwork

(a). In the fuel, the neutrons can be absorbed or induce fission, and furthermore each fission generates > 1 thermal neutrons **[1]**. *The combination of these two effects is to increase the neutron flux by a factor $\eta \sim 1.3 - 2.1$ depending on the fuel enrichment etc* **[1]**.

$$\eta = \nu \frac{\sigma_f}{\sigma_f + \sigma_a}$$

where ν is the number of neutrons per fission, and σ_f and σ_a are the cross-sections for fission and absorption respectively. **[1 1/2]**

(b). The fast neutrons coming from fission can induce fission in ^{238}U **[1]**. *The neutron flux is enhanced by a small factor of $\epsilon \sim 3\%$.* **[1/2]**

(c). Neutrons will be lost during thermalisation, especially by captures in ^{238}U in the 10-100 eV range. **[1]** *A fraction p (the resonance escape fraction) escape capture.* **[1]**

(d). Thermalised neutrons will be lost in the moderator and fuel assembly. A fraction f (the thermal utilisation factor) return safely to the fuel. **[1]**

Write down and explain an expression that shows how the four-factor formula can be extended to describe k, the neutron reproduction factor for a finite-sized reactor. [2]

Standard bookwork

Leakage of both fast and thermal neutrons can be important, and is taken into account with two extra terms l_f and l_t where these are the fractions *escaping* **[1]**. *Hence:*

$$k = \eta \epsilon p f (1 - l_f)(1 - l_t)$$

[1]

(b) What factors define the lifetime of the prompt neutrons emitted during fission? [1]

Standard bookwork

Time to slow in the moderator (very short) and time to diffuse before capture (longer). **[1]**

By making reasonable estimates of typical reactor parameters, estimate by what factor the neutron flux might rise in one second in a prompt supercritical reactor. [3]

Standard bookwork

The reactor response time is

$$\tau_r = \frac{\tau_p}{k - 1}$$

where τ_p *is the 'generation' or lifetime of ~1ms* **[1]**.

For k=1.01 and $\tau_p \sim 1ms$ *(prompt supercritical),* $\tau_r \sim 0.1s$. **[1]** *Thus the flux can rise by* $e^{10} \sim 20000$ *in a second* **[1]**

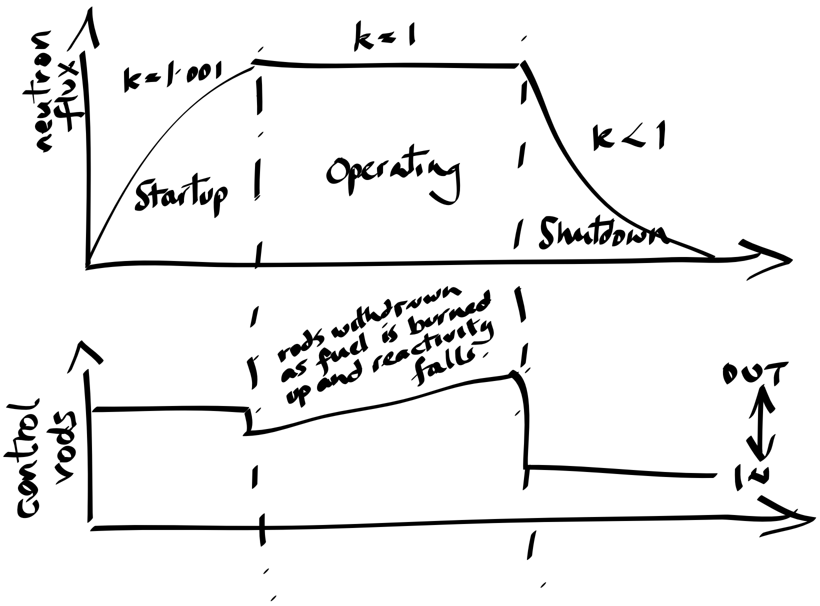
Describe why practical reactor response times can be made as long as a few hundred seconds. [2]

Standard bookwork

A small fraction of fission neutrons (0.65%) are emitted with a significant *delay (~12.5s). As long as the reactor is designed such that it is not critical with prompt neutrons alone, the additional delayed neutrons effectively increase the response time by factors of 100 or so, thereby allowing control.* **[2]**.

(c) Sketch a graph showing how the value of k changes through the start-up, operation and shutdown of a reactor. [2]

Standard bookwork



What practical means are used to control the value of k during reactor operation? [2]

Standard bookwork

See diagram above. Control rods are used to absorb neutrons and change k (within the safe range of 0 - 1.001) **[1]**.

The rods must be slowly withdrawn during operation ($k = 1$) to compensate for the use of fuel that changes the reactivity. Special rod settings will be used during startup ($k > 1$) and shutdown ($k << 1$)

B4. **(a)** What percentage of the background radiation dose to which we are subject comes from natural sources? [1]

About 90%! **[1]**

Upon which factors does the natural background depend? [3]

Radon (1300 μSv) - so don't live in Cornwall or Aberdeen
Cosmic rays (260 μSv)
Food and drink (300 μSv)
Terrestrial γ-rays (350 μSv)

2 marks for the list, one mark additionally for noting that radon is the largest contributor.

(b) For which purposes is radiation used in medicine? [1]

Mainly in imaging, eg via X-rays, CT scanners, radio-isotope scanning, and also for targetted treatment of cancers. **[1]**

Draw, and describe the operation of, the components of a gamma-camera. [5]

A gamma-camera (a.k.a. Anger camera) is a slab of scintillator about 15-20 cm radius and a few cm thick. A gamma-ray interaction in the scintillator creates a flash of uv/optical light which is detected by the photomultipliers attached to the lower surface of the scintillator. The centroid of the light pulse, which gives the position of the gamma-ray, can be determined from the relative sizes of the signals in the various photomultipliers. The total light intensity is proportional to the energy of the gamma-ray. **[3]**

A collimator is placed above the scintillator to improve spatial resolution. Small holes provide better resolution but obstruct more gamma-rays and so reduce sensitivity. Resolution of about 1.8cm is much worse than for X-ray imaging. **[1]**

Sometimes there may be 2 or even 3 scintillator arrays arranged around

TURN OVER

the patient to allow construction of 3D images. **[1]**

Explain how the gamma-camera is used to diagnose the presence of damaged tissue. [1]

Patients are injected with a radioactive pharmaceutical which produces the gamma-rays. Eg if the heart is damaged, blood will not flow normally into the damaged heart vessels and so damaged vessels will not emit gamma-rays. **[1]**

(c) Compare the use of photons (X-rays or Gamma-rays) with that of particles for radiotherapy in nuclear medicine. Consider different possible particle types separately. [4]

One point for discussion of photons, 3 for any 3 good particle points.

Photons are not good because they interact at all depths in the body and so any tissue between the skin surface and the target will receive a heavy dose. Beams tend to be broad.

Heavy charged particles, such as alpha particles, negative pi-mesons and protons, are better because they have a well defined range and so their dose is localised.

Alpha particles have a short range so not good for surface illumination but good if generated internally, eg by radioactive nuclides or by neutron beam.

Negative pions have long range and give low dose along the path. They interact with nuclei to produce high linear energy transfer charge particles and thus a sharply defined region of activity.

Protons also promising. Can make tight beam, range is quite long and energy deposit localised. **[4]**

(d) Discuss the physical principles behind Magnetic Resonance Imaging (MRI) and compare the use of MRI with X-ray Computed Tomography. [5]

A nucleus, of magnetic moment μ, has a potential energy $-\mu B$ in an external magnetic field, B. Protons are normally used. There are two energy states corresponding to the 2 possible spin states of the proton. Transition between the 2 states, for $B = 1$T, corresponds to a photon of (Larmor) frequency 42.6MHz.

A second B field, perpendicular to the first, and oscillating at the Larmor frequency, is then applied. It excites the protons. When this field is turned off, they relax and emit at the Larmor frequency. The signal is picked up by an external coil.

Slices of, eg, a patient, are selected by adding another external magnetic field, aligned along the z axis with the original field, but with a strength gradient. Thus particular slices can be selected by selecting specific Larmor frequencies.

Within individual slices, the Larmor frequency can again be altered by the addition of yet another magnetic field with a gradient (in the right orientation), allowing the emission from a strip within the slice to be recorded. By rotating the various fields it is possible to rotate the strips, allowing tomographic reconstruction of the full 3D image.

[4]

MRI is non-invasive, with no radiation dose. It is very good for differentiating between different soft tissues, which is difficult for CT. Must ensure, however, that there are nothing magnetically sensitive will go into the scanner, eg pace-maker or shrapnel. **[1]**

END OF PAPER