
SEMESTER 2 EXAMINATION 2013-2014

STELLAR EVOLUTION

Duration: 120 MINS (2 hours)

This paper contains 8 questions.

Answer **all** questions in **Section A** and **only two** questions in **Section B**.

Section A carries 1/3 of the total marks for the exam paper and you should aim to spend about 40 mins on it.

Section B carries 2/3 of the total marks for the exam paper and you should aim to spend about 80 mins on it.

An outline marking scheme is shown in brackets to the right of each question.

A Sheet of Physical Constants is provided with this examination paper.

Only university approved calculators may be used.

A foreign language translation dictionary (paper version) is permitted provided it contains no notes, additions or annotations.

Formula Sheet

Stellar structure equations:

$$
\begin{aligned}
\frac{\mathrm{d}r}{\mathrm{d}m} &= \frac{1}{4\pi r^2 \rho} \\
\frac{\mathrm{d}P}{\mathrm{d}m} &= -\frac{Gm}{4\pi r^4} \\
\frac{\mathrm{d}l}{\mathrm{d}m} &= \epsilon_{\mathrm{nuc}} \\
\frac{\mathrm{d}T}{\mathrm{d}m} &= -\frac{Gm}{4\pi r^4}\frac{T}{P}\nabla \quad \text{with} \quad \nabla = \begin{cases} \nabla_{\mathrm{rad}} = \frac{3}{16\pi acG}\frac{P}{T^4}\frac{\kappa l}{m} & \text{if} \quad \nabla_{\mathrm{rad}} \leq \nabla_{\mathrm{ad}} \\ \nabla_{\mathrm{ad}} & \text{if} \quad \nabla_{\mathrm{rad}} > \nabla_{\mathrm{ad}} \end{cases}
\end{aligned}
$$

Eddington luminosity:

$$L_{\mathrm{Edd}} = \frac{4\pi cGM}{\kappa} = 3.8 \times 10^4 \left(\frac{M}{M_\odot}\right)\left(\frac{0.34\,\mathrm{cm}^2/\mathrm{g}}{\kappa}\right) L_\odot$$

Diffusion equations:

$$F = -D\nabla U = -K\nabla T \quad \text{with} \quad K = D\,C_V = \frac{1}{3} v l_{\mathrm{fp}} C_V$$

Mean free path:

$$l_{\mathrm{fp}} = \frac{1}{\kappa\rho} = \frac{1}{n\sigma}$$

Ideal gas equation of state:

$$P_{\mathrm{gas}} = nkT = \frac{R}{\mu}\rho T = \frac{k}{\mu m_u}\rho T$$

Ideal gas equation of state for multiple particle species:

$$P_{\mathrm{gas}} = \sum_i \frac{X_i}{A_i}\frac{\rho}{m_u} kT$$

Mean molecular weight for electrons in a fully ionised gas:

$$\frac{1}{\mu_e} = \frac{1+X}{2}$$

Total mean molecular weight in a fully ionised gas:

$$\frac{1}{\mu} \approx 2X + \tfrac{4}{3}Y + \tfrac{1}{2}Z$$

Fermi momentum:

$$p_F = h\left(\frac{3}{8\pi}n_e\right)^{1/3}$$

Fermion equation of state (non-relativistic):

$$P_{\rm e,NR} = K_{\rm NR}\left(\frac{\rho}{\mu_e}\right)^{5/3}$$

$$\text{with} \quad K_{\rm NR} = \frac{h^2}{20 m_e m_u^{5/3}}\left(\frac{3}{\pi}\right)^{2/3} = 1.0036\times 10^{13}\ [\text{cgs}] = 1.0036\times 10^{7}\ [\text{SI}]$$

Fermion equation of state (extremely relativistic):

$$P_{\rm e,ER} = K_{\rm ER}\left(\frac{\rho}{\mu_e}\right)^{4/3}$$

$$\text{with} \quad K_{\rm ER} = \frac{hc}{8 m_u^{4/3}}\left(\frac{3}{\pi}\right)^{1/3} = 1.2435\times 10^{15}\ [\text{cgs}] = 1.2435\times 10^{10}\ [\text{SI}]$$

Boson equation of state:

$$P_\gamma = \frac{1}{3}aT^4$$

Mass-luminosity relationship for main sequence stars:

$$\frac{L}{L_\odot} = \left(\frac{M}{M_\odot}\right)^3$$

TURN OVER

Lane-Emden equation for a polytrope $P = K\rho^{\gamma} = K\rho^{(n+1)/n}$:

$$\frac{1}{z^2}\frac{\mathrm{d}}{\mathrm{d}z}\left(z^2\frac{\mathrm{d}w}{\mathrm{d}z}\right) + w^n = 0$$

with $\rho = \rho_c w^n$

and $r = \alpha z$, where $\alpha = \sqrt{\frac{n+1}{4\pi G}K\rho_c^{(1-n)/n}}$.

Physical properties of the Lane-Emden equations:

$$R = \alpha z_n$$

$$M = 4\pi\alpha^3\rho_c\Theta_n$$

$$K = N_n G M^{(n-1)/n} R^{(3-n)/n} \quad \text{with} \quad N_n = \frac{(4\pi)^{1/n}}{n+1}\Theta_n^{(1-n)/n} z_n^{(n-3)/n}$$

$$P_c = W_n \frac{GM^2}{R^4}$$

Numerical values for polytropic models with index n:

n	z_n	Θ_n	ρ_c/ρ	N_n	W_n
0	2.44949	4.89898	1.00000	...	0.119366
1	3.14159	3.14159	3.28987	0.63662	0.392699
1.5	3.65375	2.71406	5.99071	0.42422	0.770140
2	4.35287	2.41105	11.40254	0.36475	1.638183
3	6.89685	2.01824	54.1825	0.36394	11.05068
4	14.97155	1.79723	622.408	0.47720	247.559
4.5	31.8365	1.73780	6189.47	0.65798	4921.84
5	∞	1.73205	∞	∞	∞

Physical constants:

c	$= 3 \times 10^{10}$ cm/s	$= 3 \times 10^{8}$ m/s
k_B	$= 1.38 \times 10^{-16}$ erg K^{-1}	$= 1.38 \times 10^{-23}$ J K^{-1}
R	$= 8.31 \times 10^{7}$ erg K^{-1}mol^{-1}	= 8.31 J K^{-1} mol^{-1}
m_u	$= 1.66 \times 10^{-24}$ g	$= 1.66 \times 10^{-27}$ kg
m_e	$= 9.11 \times 10^{-28}$ g	$= 9.11 \times 10^{-31}$ kg
h	$= 6.63 \times 10^{-27}$ erg s	$= 6.63 \times 10^{-34}$ J s
G	$= 6.67 \times 10^{-8}$ cm^3 g^{-1} s^{-2}	$= 6.67 \times 10^{-11}$ m^3 kg^{-1} s^{-2}
1 eV	$= 1.60 \times 10^{-12}$ erg	$= 1.60 \times 10^{-19}$ J
a	$= 7.56 \times 10^{-15}$ erg cm^{-3} K^{-4}	$= 7.56 \times 10^{-16}$ J m^{-3} K^{-4}
$\sigma_{\rm SB}$	$= 5.67 \times 10^{-5}$ erg cm^{-2} s^{-1} K^{-4}	$= 5.67 \times 10^{-8}$ J m^{-2} s^{-1} K^{-4}
AU	$= 1.496 \times 10^{13}$ cm	$= 1.496 \times 10^{11}$ m
$M_\odot$	$= 1.99 \times 10^{33}$ g	$= 1.99 \times 10^{30}$ kg
$R_\odot$	$= 6.96 \times 10^{10}$ cm	$= 6.96 \times 10^{8}$ m
$L_\odot$	$= 3.9 \times 10^{33}$ erg/s	$= 3.9 \times 10^{26}$ J/s
$T_\odot$	= 5780 K	= 5780 K

TURN OVER

Section A

A1. What is the source of pressure support in white dwarfs? What kind of equation of state describes a typical white dwarf? What is the associated polytropic index for the ones that have masses much lower than the Chandrasekhar mass limit? [3]

A2. Assuming that a typical star is in hydrostatic equilibrium, derive an approximate expression for its central pressure and calculate an order-of-magnitude value for this quantity. Justify your assumptions if necessary. [5]

A3. In two to four sentences explain what is the helium flash? Your answer must specify the type of star that is involved, the stage of evolution and why it does not happen to every star. [4]

A4. Suppose fusion stopped at the centre of the Sun, how long could it maintain its current luminosity? What timescale does this correspond to? [4]

A5. What is the Schönberg-Chandrasekhar limit? Explain how it affects the evolution of stars? What particular feature of the colour-magnitude diagram does it explain? [4]

Section B

B1. (a) In one sentence explain the main observational difference between type I and type II supernovae. [1]

(b) What are the progenitors of type Ia, type Ib, type Ic and type II supernovae? Which one(s) would you find associated with old stellar populations? [5]

(c) In two or three sentences explain why empirically and theoretically we believe that type Ia supernovae can be used as standard candles for distance measurements. [3]

(d) Sketch the typical light curve evolution of a type Ia supernova on a magnitude/log(Flux) vs time plot. Indicate the rough time scale on the time axis. [2]

(e) In a type II supernova, the core of a massive star collapses to form a typical neutron star of mass $1.4\,M_{\odot}$ and radius 12 km. Find an upper limit to the bolometric luminosity of such a supernova. [5]

(f) In reality, we only see about 1% of the total energy available in a supernova. Where does the rest of the energy go? How is this "missing" energy produced, and why is it also important in terms of ensuring that the supernova does not fail? [4]

TURN OVER

B2. Consider a star made of hydrogen, helium and traces of heavy elements. The hydrogen distribution is given by:

$$X(m) = \begin{cases} 0.1 & \text{for } m < m_c\,, \\ 0.7 & \text{for } m \geq m_c\,. \end{cases}$$

(a) In this star a discontinuous jump in the composition profile occurs at $m = m_c$. What could have caused such a chemical profile? Why is the outer part hydrogen-rich? Write down the expected stellar composition in the two different regions. [4]

(b) Explain why the pressure and temperature must be continuous functions. [3]

(c) Calculate the jump in density $\Delta\rho/\rho$. Assume that the star behaves like an ideal gas composed of a mixture of hydrogen and helium (see formula sheet). [3]

(d) Assuming the amount of heavy elements remains constant throughout the star, calculate the jump in opacity, $\Delta\kappa/\kappa$, due to:

i) Kramers: $\kappa_{\rm bf} = Z(1 + X)\rho T^{-3.5}$. [2]

ii) Electron scattering: $\kappa_e = 0.2(1 + X)$. [2]

(e) In two or three sentences, explain why nuclear burning occurs only one stage at a time in any given region of the star (i.e. hydrogen burning does not mix with helium burning)? [3]

(f) In two or three sentences, explain why a star develops an onion shell-like structure as it evolves. [3]

B3. (a) Show that for a star in hydrostatic equilibrium, the pressure scales with density as $P \propto \rho^{4/3}$ if we consider that the gas and radiation pressure ratio is constant throughout the star. [6]

(b) What is the range of $\gamma_{\rm ad} = \left(\frac{\partial \log P}{\partial \log \rho}\right)_{\rm ad}$ in stars supported by gas and radiation? If $\gamma_{\rm ad} < 4/3$ a star becomes dynamically unstable. Explain why. [4]

(c) Which type of stars have $\gamma_{\rm ad} \approx 4/3$? [2]

(d) What is the effect of partial ionisation (for example $\mathrm{H} \rightleftharpoons \mathrm{H}^+ + \mathrm{e}^-$) on $\gamma_{\rm ad}$? What is the effect of ionisation on the stability of a star? [3]

(e) Pair creation and photo-disintegration of iron have a similar effect on $\gamma_{\rm ad}$. In what type of stars, and in what phase of their evolution, do these processes play a role? [2]

(f) Explain in your own words what homologous contraction means and why it is useful? [3]

END OF PAPER