SEMESTER 2 EXAMINATION 2014-2015

STELLAR EVOLUTION: **MODEL ANSWERS**

Duration: 120 MINS (2 hours)

This paper contains 8 questions.

Answer **all** questions in **Section A** and **only two** questions in **Section B**.

**Section A** carries 1/3 of the total marks for the exam paper and you should aim to spend about 40 mins on it.

**Section B** carries 2/3 of the total marks for the exam paper and you should aim to spend about 80 mins on it.

An outline marking scheme is shown in brackets to the right of each question.

| A Sheet of Physical Constants is provided with this examination paper. |
|---|

Only university approved calculators may be used.

A foreign language translation dictionary (paper version) is permitted provided it contains no notes, additions or annotations.

# Formula Sheet

Continuity of mass:

$$\frac{\partial m}{\partial r} = 4\pi r^2 \rho(r)$$

Hydrostatic Equilibrium:

$$\frac{\partial P}{\partial m} = -\frac{Gm(r)}{4\pi r^4}$$

Ideal gas equation of state:

$$P_{\rm gas} = nkT = \frac{\mathcal{R}}{\mu}\rho T = \frac{k}{\mu m_u}\rho T$$

Physical constants:

| | | |
|---|---|---|
| c | $= 3 \times 10^{10}$ cm/s | $= 3 \times 10^{8}$ m/s |
| $k_B$ | $= 1.38 \times 10^{-16}$ erg K$^{-1}$ | $= 1.38 \times 10^{-23}$ J K$^{-1}$ |
| R | $= 8.31 \times 10^{7}$ erg K$^{-1}$mol$^{-1}$ | = 8.31 J K$^{-1}$ mol$^{-1}$ |
| $m_u$ | $= 1.66 \times 10^{-24}$ g | $= 1.66 \times 10^{-27}$ kg |
| $m_e$ | $= 9.11 \times 10^{-28}$ g | $= 9.11 \times 10^{-31}$ kg |
| h | $= 6.63 \times 10^{-27}$ erg s | $= 6.63 \times 10^{-34}$ J s |
| G | $= 6.67 \times 10^{-8}$ cm$^3$ g$^{-1}$ s$^{-2}$ | $= 6.67 \times 10^{-11}$ m$^3$ kg$^{-1}$ s$^{-2}$ |
| 1 eV | $= 1.60 \times 10^{-12}$ erg | $= 1.60 \times 10^{-19}$ J |
| a | $= 7.56 \times 10^{-15}$ erg cm$^{-3}$ K$^{-4}$ | $= 7.56 \times 10^{-16}$ J m$^{-3}$ K$^{-4}$ |
| $\sigma_{\rm SB}$ | $= 5.67 \times 10^{-5}$ erg cm$^{-2}$ s$^{-1}$ K$^{-4}$ | $= 5.67 \times 10^{-8}$ J m$^{-2}$ s$^{-1}$ K$^{-4}$ |
| AU | $= 1.496 \times 10^{13}$ cm | $= 1.496 \times 10^{11}$ m |
| $M_\odot$ | $= 1.99 \times 10^{33}$ g | $= 1.99 \times 10^{30}$ kg |
| $R_\odot$ | $= 6.96 \times 10^{10}$ cm | $= 6.96 \times 10^{8}$ m |
| $L_\odot$ | $= 3.9 \times 10^{33}$ erg/s | $= 3.9 \times 10^{26}$ J/s |
| $T_\odot$ | = 5780 K | = 5780 K |

## Section A

**A1.** Complete the following nuclear reactions drawn from the first half of the CNO cycle. Note that there might be more than one particle missing in place of the "?". [ 5 ]

$$
\begin{aligned}
{}^{12}C + ? &\rightarrow {}^{13}N + \gamma \\
{}^{13}N &\rightarrow ? + e^{+} + \nu \\
? + {}^{1}H &\rightarrow {}^{14}N + \gamma \\
{}^{14}N + ? &\rightarrow {}^{15}O + \gamma \\
{}^{15}O &\rightarrow {}^{15}N + ? \\
{}^{15}N + {}^{1}H &\rightarrow {}^{12}C + ? \\
{}^{15}N + {}^{1}H &\rightarrow ? + \gamma \\
{}^{16}O + {}^{1}H &\rightarrow {}^{17}F + ? \\
? &\rightarrow {}^{17}O + e^{+} + \nu \\
{}^{17}O + {}^{1}H &\rightarrow {}^{14}N + ?
\end{aligned}
$$

$$
\begin{aligned}
{}^{12}C + {}^{1}H &\rightarrow {}^{13}N + \gamma \\
{}^{13}N &\rightarrow {}^{13}C + e^{+} + \nu \\
{}^{13}C + {}^{1}H &\rightarrow {}^{14}N + \gamma \\
{}^{14}N + {}^{1}H &\rightarrow {}^{15}O + \gamma \\
{}^{15}O &\rightarrow {}^{15}N + e^{+} + \nu \\
{}^{15}N + {}^{1}H &\rightarrow {}^{12}C + {}^{4}He \\
{}^{15}N + {}^{1}H &\rightarrow {}^{16}O + \gamma \\
{}^{16}O + {}^{1}H &\rightarrow {}^{17}F + \gamma \\
{}^{17}F &\rightarrow {}^{17}O + e^{+} + \nu \\
{}^{17}O + {}^{1}H &\rightarrow {}^{14}N + {}^{4}He
\end{aligned}
$$

**TURN OVER**

*(10 "pieces" were missing, 0.5 mark per good answer)*

**A2.** Explain the main principles behind the 'r-' and the 's-process' in nucleosynthesis. [ 3 ]

*The heavy element nucleosynthesis mostly relies on the capture of neutrons by heavy nuclei.[1] In the s-process, the time interval between the capture of neutrons is long enough for the nuclei to beta-decay and hence go up the proton number ladder.[1] In the r-process, the capture of neutrons happens quickly and does not leave enough time for the nuclei to experience radioactive decay towards more stable elements and hence become neutron-rich.[1]*

**A3.** Explain two major effects played by convection in the evolution of stars. [ 4 ]

*Convection can mix the content of the stellar core in massive stars having convective core, thus bringing fresh elements and sustaining the various nuclear cycles for a longer period.[2]*

*Convection can bring nuclear burning products from the boundary of the stellar core to the surface in low-mass stars having convective envelopes.[2]*

*(Other good solutions may be considered, though they should not repeat each other.)*

**A4.** Name and describe the four main physical processes that contribute to the opacity of hot stellar interiors involving electrons. [ 4 ]

*0.5 marks should be given for the name and 0.5 marks for the description of each process.*

*(a). Electron scattering - The scattering of a photon by a free electron.*

*(b). Free-free absorption - The absorption of a photon by a free electron which is in the vicinity of a nucleus or ion.*

*(c). Bound-free absorption - Photoionisation; the removal of an electron from an atom (or ion) caused by the absorption of a photon.*

*(d). Bound-bound absorption - The excitation of an atom (or ion) due to the transition of a bound electron to a higher energy level by the absorption of a photon.*

**A5.** Explain qualitatively the origin of the thin shell instability. Explain why it is important for understanding late stages of stellar evolution. [ 4 ]

*The thin shell instability sets in because in a shell which is thin compared to the radius of the star, the pressure in the shell will be dominated by the weight of the gas above the shell.[2] As a result, the shell will not expand in a normal manner in response to an increase in temperature.[1] In late stages of stellar evolution, there will often be thin shells of hydrogen and helium burning, and the ignition of fusion in these shells triggers thermal pulses that lead to the extreme mass loss in these phases.[1]*

**TURN OVER**

## Section B

**B1.** (a) Stellar formation originates from the collapse of a cold molecular cloud. One of the critical conditions for this process to occur is that the mass enclosed within a certain volume of the cloud be larger than the Jeans mass. In two or three sentences, explain what is the Jeans mass and why it represents the criterion for collapse. [ 3 ]

*At a given temperature and density, there is a mass such that the self-gravity of the system becomes larger than the kinetic energy which we call the Jeans mass.[1.5] Beyond this this limit, gravity overcomes the pressure support.[1.5]*

(b) Show that for a spherically symmetric gas cloud with total mass, $M$, and radius, $R$, and assuming a uniform density, $\rho$, throughout the cloud that the gravitational potential energy, $E_{gr}$, of the cloud is given by: [ 3 ]

$$E_{gr} = -\frac{3}{5}\frac{GM^2}{R}$$

**[Assigned as a problem sheet problem]**

*Total gravitational potential energy of a gas cloud is given by integrating thin shells of gas over the whole cloud:*

$$E_{gr} = \int_0^M -\frac{Gm(r)}{r}dm \qquad \text{[0.5 mark]}$$

*From the continuity of mass equation we know* $\partial m = 4\pi r^2\rho\partial r$ *and for a uniform density we have:*

$$\begin{aligned} m(r) &= \rho V \\ &= \rho\frac{4}{3}\pi r^3 \qquad \text{[0.5 mark]} \end{aligned}$$

*Combining these we get:*

$$\begin{aligned} E_{gr} &= -\frac{16}{3}G\pi^2\rho^2\int_0^R r^4 dr \\ &= -\frac{16}{15}G\pi^2\rho^2R^5 \qquad \text{[1 mark]} \end{aligned}$$

*And substituting for the density:*

$$E_{gr} = -\frac{16}{15}G\pi^2 R^5 \left(\frac{3M}{4\pi R^3}\right)^2$$
$$= -\frac{3}{5}\frac{GM^2}{R} \qquad \text{[1 mark]}$$

(c) Assume that the gas cloud behaves as an ideal gas where kinetic energy per particle is $\frac{3}{2}kT$ for a temperature, $T$. Starting from the viral theorem which relates the internal gas energy, $E_{int}$, and the gravitational potential energy, $E_{gr}$:

$$2E_{int} + E_{gr} = 0$$

derive a quantitative expression for the Jeans mass in terms of the temperature, $T$, density, $\rho$ and mean molecular weight of the gas, $\mu$. [ 5 ]

**[Derived in class]**

*The Jeans mass corresponds to the minimum mass required for the gravity to overcome the internal energy of the gas (which is the sum of the particle energies):*

$$
\begin{aligned}
-E_{gr} &> 2E_{int} && \text{[1 mark]}\\
\frac{3}{5}\frac{GM^2}{R} &> 3NkT \\
\frac{3}{5}\frac{GM^2}{R} &> 3\frac{M}{\mu m_u}kT && \text{[1 mark]}\\
M &> \frac{5kT}{\mu m_u G}R && \text{[1 mark]}\\
M > M_J &= \frac{5kT}{\mu m_u G}R
\end{aligned}
$$

*If we now replace R with the density $\rho$:*

$$M_J = \frac{5kT}{\mu m_u G}\left(\frac{3M_J}{4\pi\rho}\right)^{\frac{1}{3}} \qquad \text{[0.5 mark]}$$

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$$M_J^3 = \left(\frac{5kT}{\mu m_u G}\right)^3 \left(\frac{3M_J}{4\pi\rho}\right) \qquad \text{[0.5 mark]}$$
$$M_J^2 = \left(\frac{5kT}{\mu m_u G}\right)^3 \left(\frac{3}{4\pi\rho}\right) \qquad \text{[0.5 mark]}$$
$$M_J = \left(\frac{5kT}{\mu m_u G}\right)^{\frac{3}{2}} \left(\frac{3}{4\pi\rho}\right)^{\frac{1}{2}} \qquad \text{[0.5 mark]}$$

(d) The Jeans criterion for collapse of a gas cloud can also be defined in terms of the sound speed of the cloud, $c_s$, such that:

$$c_s^2 < \frac{GM}{5R}$$

From this define the sound travel time across the cloud, $t_s = \frac{R}{c_s}$, in terms of the free-fall timescale, $t_{ff} = \sqrt{\frac{3}{8\pi G\rho}}$. What is the physical interpretation of this relationship? [ 3 ]

*The sound crossing time, $t_s = \frac{R}{c_s}$:*

*Start by substituting the mass for the density:*

$$\begin{aligned} c_s^2 &< \frac{G}{5R}\frac{4}{3}\pi R^3\rho \\ &< \frac{4\pi G\rho R^2}{15} \\ \frac{R}{c_s} &> \sqrt{\frac{15}{4\pi G\rho}} \\ t_s &> \sqrt{\frac{15}{4\pi G\rho}} \\ t_s &> \sqrt{\frac{15}{4}}\sqrt{\frac{8}{3}}t_{ff} \\ t_s &> \sqrt{10}t_{ff} \end{aligned}$$

*[1.5 marks]*

*This implies then that the condition for collapse is that the free-fall time must be be less than the sound speed. Hence if a dynamical disturbance* *occurs, the instability will keep growing instead of being damped.[1.5 marks]*

(e) What is the Hayashi forbidden zone and what is its significance for star formation? [ 3 ]

*The Hayashi forbidden zone is an area of the colour-magnitude diagram in which there exists no solution for a star in hydrostatic equilibrium.[1] The limit of the zone is more or less a vertical line (i.e. constant temperature) to the left of which stars are fully convective.[1] It is relevant to star formation because protostars go down the Hayashi line as they evolve towards the main sequence because their low temperatures imply large opacities which forces convection to transport energy inside the protostar.[1]*

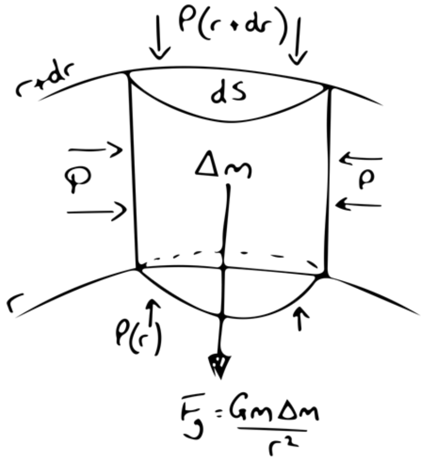
(f) Protostars eventually reach the point at which H-core burning starts. They become proper stars and settle on the main sequence. If two distinct populations were to form, one with solar metallicity ($Z = Z_{\odot}$) and the other with a low-metal abundance ($Z = 0.01Z_{\odot}$), how would you be able to differentiate them in the colour-magnitude diagram? [ 3 ]

*Stars with a lower metal abundance are hotter and more luminous than those with higher metal content for a similar mass and radius. This can be seen from the homology equations.[1] As a result, the isochrones for metal-poor stars will be shifted to the upper left in the colour-magnitude diagram (i.e. hot, blue).[1] One could therefore separate them from isochrone fitting, assuming that their distance can be measured accurately.[1]*

**TURN OVER**

**B2.** (a) With the aid of a diagram derive the equation of hydrostatic equilibrium for a spherically symmetric star. [ 3 ]

**[Derived in class]**




*[1 mark] for the diagram showing all of the forces in action on the volume element of the stellar atmosphere*

*Balance of forces requires that the net force, $F$, is:*

$$F = \ddot{r}\Delta m = -\frac{Gm\Delta m}{r^2} + P(r)dS - P(r+dr)dS$$

*with*

$$\begin{aligned} P(r+dr) &= P(r) + \frac{\partial P}{\partial r}dr \\ \ddot{r}\Delta m &= -\frac{Gm\Delta m}{r^2} - \frac{\Delta m}{\rho}\frac{\partial P}{\partial r} \\ \ddot{r} &= -\frac{Gm}{r^2} - 4\pi r^2\frac{\partial P}{\partial m} \end{aligned}$$

*if star is in equilibrium then $\ddot{r} = 0$ and:*

$$\frac{\partial P}{\partial m} = -\frac{Gm}{4\pi r^4}$$

*[2 marks] for the full derivation; relationship in terms of $\frac{\partial P}{\partial r}$ will be accepted. Partial marks available for partial derivation.*

(b) By combining the equations of hydrostatic equilibrium and mass continuity demonstrate that the lower limit for the central pressure $P_c$ of a star (with mass $M$ and radius $R$) in hydrostatic equilibrium is given by: [ 5 ]

$$P_c \;>\; \frac{GM^2}{8\pi R^4}$$

**[Derived in class]**

*From the equation of hydrostatic equilibrium it follows that we can integrate over the whole star to calculate the pressure gradient from the centre to the surface, $P_{surf} - P_c$. For a star of mass, $M$, and radius, $R$ and using $P_{surf} = 0$ [1 mark] and $R \geq r$ [1 mark] we derive the following:*

$$\begin{aligned}
\frac{dP}{dm} &= -\frac{Gm}{4\pi r^4} \\
\int_{P_c}^{P_{surf}} dP &= -\int_0^M \frac{Gm}{4\pi r^4} dm \qquad \text{[1 mark]} \\
P_{surf} - P_c &> -\frac{G}{4\pi R^4}\int_0^M m\, dm \qquad \text{[1 mark]} \\
P_c &> \frac{GM^2}{8\pi R^4} \qquad \text{[1 mark]}
\end{aligned}$$

*[N.B. 1 mark of the derivation is explicitly for switching to $R$ and $>$ at the right point.]*

(c) Estimate the mean free path of a photon, $l_{ph}$, within the Sun assuming a uniform density throughout the star; for the opacity coefficient you may assume $\kappa = 0.04$ m$^2$ kg$^{-1}$. Consequently explain, by reference to the Sun, why radiative transport in stellar interiors can be treated as a diffusive process. [ 4 ]

*Mean free path of a photon is, $l_{ph} = \frac{1}{\kappa\rho}$, where $\rho$ is the density [1 mark]. Using the average density of the Sun assuming spherical symmetry and the mass and radius given in the constants sheet ($\rho = 1409\ kg\ m^{-3}$) then $l_{ph} = 0.018\ m$ [1 mark].*

*Hence, in the Sun, the photon mean free path is very small relative to the stellar radius and so photons are continually scattered, absorbed and*

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*re-emitted in random directions. As a result, radiation is trapped within the stellar interior and photons diffuse outwards slowly via a 'random walk' process. [2 marks]*

(d) The diffusive flux $J$ of particles (per unit area and time) between places of different particle density $n$ is given by:

$$J = -D\nabla n$$

where the coefficient of diffusion $D = \frac{1}{3}vl_p$ is determined by the mean velocity, $v$, and mean free path, $l_p$, of the particles.

For the case of a stellar interior where there is a net flux of energy, $F$, across the surface and where photons are the transporting particles with a radiation energy density $U$ we can write:

$$F = -D\nabla U$$

Assuming spherical symmetry, show that for the case where photons have a radiation energy density $U = aT^4$ that the equation of radiative transport is given by:

$$\frac{\partial T}{\partial r} = -\frac{3}{16\pi ac}\frac{\kappa\rho}{r^2T^3}L(r)$$

Where $a$ is the radiation-density constant, $\kappa$ is the absorption coefficient, $\rho$ is the density and $L(r)$ is the luminosity. [ 8 ]

**[Derived in class]**

*This derivation requires a number of different stages with marks allocated to each stage as follows:*

$$\begin{aligned} F &= -D\nabla U \\ &= -\frac{1}{3}cl_{ph}\nabla U \end{aligned}$$

*[1 marks]*

$$\nabla U = \left(\frac{\partial U}{\partial T}\right)_V \nabla T$$

*[2 marks]*

$$F = \frac{L(r)}{4\pi r^2}$$

*[1 mark]*

$$\begin{aligned}
F &= -\frac{1}{3} c l_{ph} \nabla U \\
&= -\frac{1}{3} c l_{ph} \left(\frac{\partial U}{\partial T}\right)_V \nabla T \\
&= -\frac{c}{3\kappa\rho} 4aT^3 \frac{\partial T}{\partial r} \\
\frac{L(r)}{4\pi r^2} &= -\frac{c}{3\kappa\rho} 4aT^3 \frac{\partial T}{\partial r} \\
\frac{\partial T}{\partial r} &= -\frac{3}{16\pi ac} \frac{\kappa\rho}{r^2 T^3} L(r)
\end{aligned}$$

*[4 marks]*

**TURN OVER**

**B3.** (a) Define analytic expressions for the three characteristic timescales of stellar evolution and give an example of an evolutionary phase that operates on each timescale. [ 6 ]

**[Analytic expressions derived in class]**

*1 mark for defining each timescale and 1 mark for an example of an evolutionary phase that operates on that timescale.*

- *Dynamical or free-fall timescale: Ratio of stellar radius to free-fall velocity,*

$$\tau_{dyn} = \frac{R}{\sqrt{2GM/R}} = \sqrt{\frac{R^3}{2GM}}$$

*This is the timescale of mechanical changes of the star and is important during stellar collapse/supernova.*

- *Thermal or Kelvin-Helmholtz timescale: the ratio of internal stellar energy to the luminosity,*

$$\tau_{KH} = \frac{GM^2}{2RL}$$

*This is the time it would take a star to emit all of its thermal energy though contraction and is important during the pre-main sequence evolutionary phase.*

- *Nuclear timescale: this is the ratio of convertable rest-mass energy to the luminosity,*

$$\tau_{nuc} = \frac{\phi f_{nuc} M c^2}{L}$$

*where $\phi$ is the mass fraction of the star in hydrogen and $f_{nuc}$ is the fraction of hydrogen that will be fused in the core of the star.*
*This is the timescale over which the star will expend all of it's nuclear fuel, it is the longest of the timescales and hence governs stellar evolution. This timescale determines how long the star remains on the main sequence.*

(b) Describe the main characteristics of the *Upper* ($M > 1.5M_\odot$) and *Lower* ($M < 1.5M_\odot$) main sequence stars in terms of: fusion reactions; core temperature; stellar structure; lifetime. [ 5 ]

*Nuclear reaction rates are very sensitive to core temperature: P-P chain rate* $\sim T^4$*; CNO cycle rate* $\sim T^{18}$*. This leads to different internal structures. The mass at which the P-P chain and CNO cycle both contribute equally to the total energy generation in the core is* $\sim 1.5M_\odot$*, this marks the division between Upper and Lower Main-Sequences by mass. [1 mark]*

*Upper Main Sequence, mass,* $M > 1.5M_\odot$*:*

- *Core temperature,* $T_{core} > 18 \times 10^6 K$*.*
- *Hydrogen fusion occurs via CNO cycle.*
- *Star has a convective core and a radiative envelope.*
- *Lifetime may only be a few million years*

*[2 marks for 3 properties]*

*Lower Main Sequence, mass,* $M < 1.5M_\odot$*:*

- *Core temperature,* $T_{core} < 18 \times 10^6 K$*.*
- *Hydrogen fusion occurs via P-P chain.*
- *Star has a radiative core and a convective envelope.*
- *Lifetime of order 10 billion years.*

*[2 marks for 3 properties]*

(c) Dredge-up occurs in a star when a surface convection zone extends down to regions where material has undergone nuclear fusion and as a result fusion products are mixed into the outer layers of the stellar atmosphere. An intermediate mass ($2M_\odot \leq M \leq 8M_\odot$) star is believed to experience three dredge-up episodes during its evolution. For each dredge-up, briefly describe (i) the evolutionary state of the star, (ii) its structure and (iii) the products that are brought to the surface. [ 9 ]

**TURN OVER**

 

*We have an Upper main sequence star and so the core is burning H by the CNO cycle with a high core temperature gradient resulting in a convective, well-mixed core. Prior to the 1st dredge up the star will be powered by hydrogen-shell burning around an inert He core.*

- *1st dredge-up: As the star ascends the Red Giant Branch of the HR diagram it develops a huge convection envelope which grows until it reaches down into the H-burning shell. The convective envelope brings up material processed by the CNO cycle to the surface. The C-N cycle reaches equilibrium before the O-N cycle hence N-enriched, C-depleted material is first exposed on the surface. The inert He core reaches* $10^8$ *K and begins to fuse He and the core expands; H-shell burning continues outside the core. Expansion of the core leads to contraction of the envelope as the star evolves down the RGB and towards higher effective temperatures.*
- *2nd dredge-up: The convective core will contract again continuing He-fusion until the He is exhausted in the core which is now composed of C, O and heavier elements. A thick He-burning shell forms which causes the region outside of it to expand and cool, extinguishing the H-burning shell. The contraction of the core leads to a strong expansion of the envelope and the star moves to ascend the Asymptotic Giant Branch. The expansion of the envelope causes it to become convective and reaches down to the extinguished H-burning shell and H-burning products are brought to the surface; these are the CNO products, all H has been burnt to He and C, O to N so the surface He and N abundances rise while C and O decrease. The contracting C-O core becomes degenerate.*
- *3rd dredge-up: The structure of the star is now: a degenerate C-O core; helium-burning shell; helium layer; hydrogen-burning shell; an outer H-rich convective envelope. The two burning-shells can not exist in a steady state and hence the two-shells provide energy cyclicly in what is known as the "thermal pulse" mechanism. For most of the cycle*

*H is burnt in the external shell while the inner shell is extinguished. As result the He-layer separating the shells grows in mass and contracts; the base of the layer heats up until it triggers He-burning in the inner shell causing a He-flash (or "thermal pulse"). Sudden increase in energy causes the outer layers to expand and cool, extinguishing the H-burning shell. The high-temperature and neutrons from the triple-*$\alpha$ *process produces s-process elements. The convective envelope now extends down into the layer where He-burning has occurred dredging up C and s-process elements to the surface. The He-burning shell front advances out through the He-layer and the increased temperature in proximity of the H-burning shell reignites it. Temp and density in the star re-adjust to equilibrium and the He-burning shell is extinguished due to the lower temperature. Thus a new cycle begins.*

*[3 marks are allocated to each dredge-up; with the marks equally awarded for describing the stellar structure, dredged-up products and evolutionary state.]*

**END OF PAPER**