SEMESTER 2 EXAMINATION 2014-2015

STELLAR EVOLUTION

Duration: 120 MINS (2 hours)

This paper contains 8 questions.

Answer **all** questions in **Section A** and **only two** questions in **Section B**.

Section A carries 1/3 of the total marks for the exam paper and you should aim to spend about 40 mins on it.

Section B carries 2/3 of the total marks for the exam paper and you should aim to spend about 80 mins on it.

An outline marking scheme is shown in brackets to the right of each question.

A Sheet of Physical Constants is provided with this examination paper.

Only university approved calculators may be used.

A foreign language word to word® translation dictionary (paper version) is permitted provided it contains no notes, additions or annotations.

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Formula Sheet

Continuity of mass:

$$\frac{\partial m}{\partial r} = 4\pi r^2 \rho(r)$$

Hydrostatic Equilibrium:

$$\frac{\partial P}{\partial m} = -\frac{Gm(r)}{4\pi r^4}$$

Ideal gas equation of state:

$$P_{\rm gas} = nkT = \frac{\mathcal{R}}{\mu}\rho T = \frac{k}{\mu m_u}\rho T$$

Physical constants:

c	$= 3 \times 10^{10}$ cm/s	$= 3 \times 10^{8}$ m/s
k_B	$= 1.38 \times 10^{-16}$ erg K^{-1}	$= 1.38 \times 10^{-23}$ J K^{-1}
R	$= 8.31 \times 10^{7}$ erg K^{-1}mol^{-1}	= 8.31 J K^{-1} mol^{-1}
m_u	$= 1.66 \times 10^{-24}$ g	$= 1.66 \times 10^{-27}$ kg
m_e	$= 9.11 \times 10^{-28}$ g	$= 9.11 \times 10^{-31}$ kg
h	$= 6.63 \times 10^{-27}$ erg s	$= 6.63 \times 10^{-34}$ J s
G	$= 6.67 \times 10^{-8}$ cm^3 g^{-1} s^{-2}	$= 6.67 \times 10^{-11}$ m^3 kg^{-1} s^{-2}
1 eV	$= 1.60 \times 10^{-12}$ erg	$= 1.60 \times 10^{-19}$ J
a	$= 7.56 \times 10^{-15}$ erg cm^{-3} K^{-4}	$= 7.56 \times 10^{-16}$ J m^{-3} K^{-4}
$\sigma_{\rm SB}$	$= 5.67 \times 10^{-5}$ erg cm^{-2} s^{-1} K^{-4}	$= 5.67 \times 10^{-8}$ J m^{-2} s^{-1} K^{-4}
AU	$= 1.496 \times 10^{13}$ cm	$= 1.496 \times 10^{11}$ m
$M_\odot$	$= 1.99 \times 10^{33}$ g	$= 1.99 \times 10^{30}$ kg
$R_\odot$	$= 6.96 \times 10^{10}$ cm	$= 6.96 \times 10^{8}$ m
$L_\odot$	$= 3.9 \times 10^{33}$ erg/s	$= 3.9 \times 10^{26}$ J/s
$T_\odot$	= 5780 K	= 5780 K

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Section A

A1. Complete the following nuclear reactions drawn from the first half of the CNO cycle. Note that there might be more than one particle missing in place of the "?". [5]

$$
\begin{aligned}
^{12}C + ? &\rightarrow {}^{13}N + \gamma \\
^{13}N &\rightarrow ? + e^{+} + \nu \\
? + {}^{1}H &\rightarrow {}^{14}N + \gamma \\
^{14}N + ? &\rightarrow {}^{15}O + \gamma \\
^{15}O &\rightarrow {}^{15}N + ? \\
^{15}N + {}^{1}H &\rightarrow {}^{12}C + ? \\
^{15}N + {}^{1}H &\rightarrow ? + \gamma \\
^{16}O + {}^{1}H &\rightarrow {}^{17}F + ? \\
? &\rightarrow {}^{17}O + e^{+} + \nu \\
^{17}O + {}^{1}H &\rightarrow {}^{14}N + ?
\end{aligned}
$$

A2. Explain the main principles behind the 'r-' and the 's-process' in nucleosynthesis. [3]

A3. Explain two major effects played by convection in the evolution of stars. [4]

A4. Name and describe the four main physical processes that contribute to the opacity of hot stellar interiors involving electrons. [4]

A5. Explain qualitatively the origin of the thin shell instability. Explain why it is important for understanding late stages of stellar evolution. [4]

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Section B

B1. (a) Stellar formation originates from the collapse of a cold molecular cloud. One of the critical conditions for this process to occur is that the mass enclosed within a certain volume of the cloud be larger than the Jeans mass. In two or three sentences, explain what is the Jeans mass and why it represents the criterion for collapse. [3]

(b) Show that for a spherically symmetric gas cloud with total mass, M, and radius, R, and assuming a uniform density, ρ, throughout the cloud that the gravitational potential energy, E_{gr}, of the cloud is given by: [3]

$$E_{gr} = -\frac{3}{5}\frac{GM^2}{R}$$

(c) Assume that the gas cloud behaves as an ideal gas where kinetic energy per particle is $\frac{3}{2}kT$ for a temperature, T. Starting from the virial theorem which relates the internal gas energy, E_{int}, and the gravitational potential energy, E_{gr}:

$$2E_{int} + E_{gr} = 0$$

derive a quantitative expression for the Jeans mass in terms of the temperature, T, density, ρ and mean molecular weight of the gas, μ. [5]

(d) The Jeans criterion for collapse of a gas cloud can also be defined in terms of the sound speed of the cloud, c_s, such that:

$$c_s^2 < \frac{GM}{5R}$$

From this define the sound travel time across the cloud, $t_s = \frac{R}{c_s}$, in terms of the free-fall timescale, $t_{ff} = \sqrt{\frac{3}{8\pi G\rho}}$. What is the physical interpretation of this relationship? [3]

(e) What is the Hayashi forbidden zone and what is its significance for star formation? [3]

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(f) Protostars eventually reach the point at which H-core burning starts. They become proper stars and settle on the main sequence. If two distinct populations were to form, one with solar metallicity ($Z = Z_\odot$) and the other with a low-metal abundance ($Z = 0.01 Z_\odot$), how would you be able to differentiate them in the colour-magnitude diagram? [3]

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B2. (a) With the aid of a diagram derive the equation of hydrostatic equilibrium for a spherically symmetric star. [3]

(b) By combining the equations of hydrostatic equilibrium and mass continuity demonstrate that the lower limit for the central pressure P_c of a star (with mass M and radius R) in hydrostatic equilibrium is given by: [5]

$$P_c > \frac{GM^2}{8\pi R^4}$$

(c) Estimate the mean free path of a photon, l_{ph}, within the Sun assuming a uniform density throughout the star; for the opacity coefficient you may assume $\kappa = 0.04$ m^2 kg^{-1}. Consequently explain, by reference to the Sun, why radiative transport in stellar interiors can be treated as a diffusive process. [4]

(d) The diffusive flux J of particles (per unit area and time) between places of different particle density n is given by:

$$J = -D\nabla n$$

where the coefficient of diffusion $D = \frac{1}{3}v l_p$ is determined by the mean velocity, v, and mean free path, l_p, of the particles.

For the case of a stellar interior where there is a net flux of energy, F, across the surface and where photons are the transporting particles with a radiation energy density U we can write:

$$F = -D\nabla U$$

Assuming spherical symmetry, show that for the case where photons have a radiation energy density $U = aT^4$ that the equation of radiative transport is given by:

$$\frac{\partial T}{\partial r} = -\frac{3}{16\pi ac}\frac{\kappa\rho}{r^2T^3}L(r)$$

Where a is the radiation-density constant, κ is the absorption coefficient, ρ is the density and $L(r)$ is the luminosity. [8]

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B3. (a) Define analytic expressions for the three characteristic timescales of stellar evolution and give an example of an evolutionary phase that operates on each timescale. [6]

(b) Describe the main characteristics of the *Upper* ($M > 1.5M_{\odot}$) and *Lower* ($M < 1.5M_{\odot}$) main sequence stars in terms of: fusion reactions; core temperature; stellar structure; lifetime. [5]

(c) Dredge-up occurs in a star when a surface convection zone extends down to regions where material has undergone nuclear fusion and as a result fusion products are mixed into the outer layers of the stellar atmosphere. An intermediate mass ($2M_{\odot} \leq M \leq 8M_{\odot}$) star is believed to experience three dredge-up episodes during its evolution. For each dredge-up, briefly describe (i) the evolutionary state of the star, (ii) its structure and (iii) the products that are brought to the surface. [9]

END OF PAPER

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