All of the material has been covered in lectures.

MARKS

A1 Dish $\theta \sim \lambda/D$ $\quad d\Omega \simeq \pi\left(\frac{\lambda}{2D}\right)^2$ [1]

$$\text{Gain} = \frac{4\pi}{d\Omega} = \frac{4\pi}{\pi}\left(\frac{2D}{\lambda}\right)^2 = 4.4.\left(\frac{D}{\lambda}\right)^2$$ [1]

$$= 16.\left(\frac{20}{0.01}\right)^2 = 16.2000^2 = 6.4\times10^7$$ [1]

A2 Source counts in time τ, $S = N_S.\tau\,\delta\nu$ [½]

Background " " " " $B = N_B\,\tau\,\delta\nu$ [½]

$$S/N = \frac{S}{\sqrt{S+B}} = \frac{N_S\,\tau\,\delta\nu}{\sqrt{(N_S+N_B)\,\tau\,\delta\nu}} = \frac{N_S}{\sqrt{N_S+N_B}}(\tau\,\delta\nu)^{1/2}$$ [1]

Here $N_S = 1$ $\quad N_B = 500\times0.2 = 100$, $\quad \delta\nu = 10^4$ $\quad \tau = 10$ [½]

$$\therefore S/N = \frac{1}{(101)^{1/2}}.10^2\,10^{0.5} = 31.5$$ [½]

A3 $\lambda_{max}T = \text{const} = 2.7\times1.1\times10^{-3}$ (MWB)

For Saturn $T = 95$ $\therefore \lambda_{max} = \frac{2.7\times1.1\times10^{-3}}{95} = 3.13\times10^{-5}$ m

(ie 31.3 μm). [1]

A4 $\delta\nu = 2.8\times10^{10}\times10^{-9} = 28$ Hz [½]

$\frac{1}{2}mv^2 = \frac{3}{2}kT$ $\quad \therefore V = \left(\frac{3kT}{m_p}\right)^{1/2} = \left(\frac{3.1.38.10^{-23}.100}{1.67\times10^{-27}}\right)^{1/2}$

$= 1575$ m s^{-1} [1]

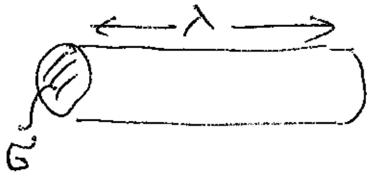
$\frac{\delta\nu}{\nu} = \frac{V}{c}$ $\quad \therefore \delta\nu = \nu.\frac{V}{c} = \frac{c}{\lambda}.\frac{V}{c} = \frac{V}{\lambda} = \frac{1575}{0.21}$

$= 7498$ Hz [1]

So no chance of distinguishing 28 Hz [½]

A5 Need time between collisions, Δt, to be less than Lifetime, τ. [1]

$$\Delta t = \frac{\text{mean free path}, \lambda}{v}$$ [1]



Need vol × n = 1 ie $\sigma \lambda n = 1$

$$\lambda = \frac{1}{n\sigma}$$

$$\therefore \Delta t = \frac{1}{n\sigma}\frac{1}{v}$$

or $n = \dfrac{1}{\Delta t\ \sigma\ v}$ or $< \dfrac{1}{\tau\ \sigma\ v}$ [½]

$$v = \left(\frac{3kT}{16m_p}\right)^{1/2} = \left(\frac{3.1{\cdot}38.10^{-23}.2000}{16.1{\cdot}67.10^{-27}}\right)^{1/2} = 1760$$ [1]

$$\therefore n < \frac{1}{2{\cdot}8.10^{4}.\ \pi.10^{-20}.\ 1760}$$ ie $< 6{\cdot}46 \times 10^{11}$ [½]

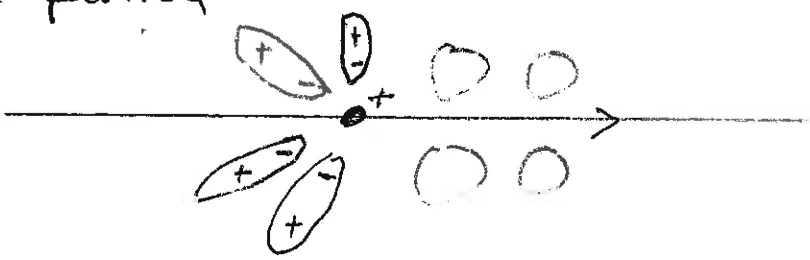
A6 Angular momentum, $L = n\hbar = m_e v b$ (note, e^-) [½]

min when $n = 1$ ie $b_{min} = \dfrac{\hbar}{mv}$ [½]

$$v = \left(\frac{3kT}{m_e}\right)^{1/2} = \left(\frac{3.1{\cdot}38\times10^{-23}.10^{6}}{9{\cdot}1.10^{-31}}\right)^{1/2} = 6{\cdot}75 \times 10^{6}$$ [½]

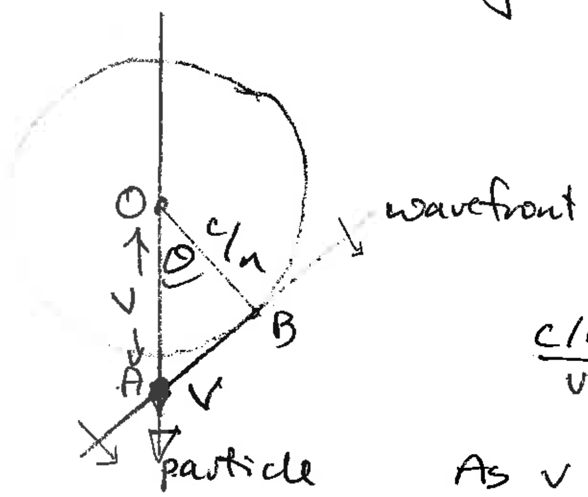
$$\therefore b_{min} = \frac{1{\cdot}05.10^{-34}}{9{\cdot}1.10^{-31}.\ 6{\cdot}75.10^{6}} = 1{\cdot}7\times10^{-11}$$ [½]

A7 TeV photon Compton scatters charged particles to velocity greater than local speed of light in atmosphere. Surrounding atoms + molecules are polarised asymetrically. ~~Particles~~ Atoms in front don't know about the particle



[2]

When atoms relax they emit Cerenkov radn (blue)



Particle emits light at O.

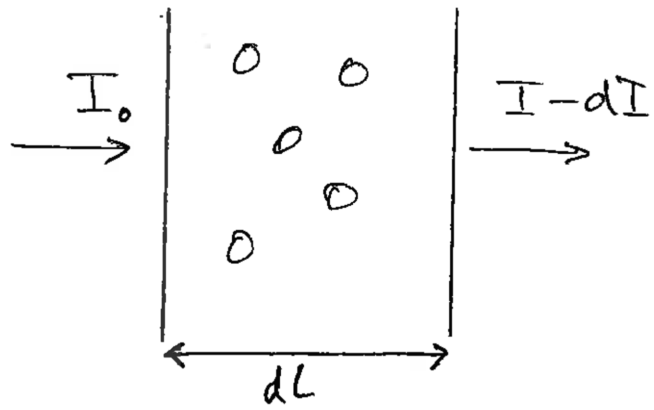
1 sec later light is at B, particle " " A.

$$\frac{c/n}{v} = \cos\theta$$

As $v \sim c$ ∴ $\cos\theta \simeq \frac{1}{n}$ *

∴ Here $n = 1.0008$ ∴ $\theta = 2.3^\circ$ [2]

B1 (a)



$$dI = -I n \sigma dL$$

$$\therefore \int_{I_0}^{I} \frac{dI}{I} = \int -n\sigma dL \qquad \therefore I = I_0 e^{-n\sigma L}$$ [3]

$n\sigma L = \tau$ – optical depth [1]

Here $I_0 = I e^{+n\sigma L} = 1.\ \exp(10^{22}.\ 2.10^5.\ 10^{-26}\ 5^{-8/3})$

$= 1.\ \exp(2.74.\ 10^{1-2}) = \exp(0.274)$

$= 1.31$ counts/s. [3]

b) Increase density, increase L, increase τ, as long as τ small ($\lesssim 1$). At higher τ, photons scatter before emerging and eventually, at very high τ, all photons have multiple interactions and are in equilib with matter and we reach a maximum limiting luminosity [2]

B1(b) cont.

The brightness temperature is the temperature of a black body which would produce the same flux, per unit bandwidth, as that which is observed, at the given bandwidth [1]

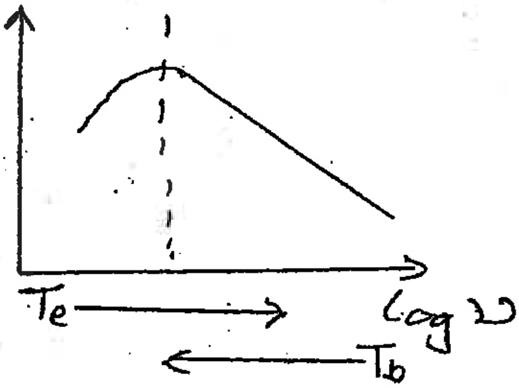
The effective temperature is the temperature of a black body with the same bolometric luminosity, and emitting area, as the observed source [1]

$$S = \frac{2kT_b}{\lambda^2}\Omega$$ [1]

$$L = 1.5\times10^{26} = 4\pi.(10^7)^2 . 5.67.10^{-8}. (T_e)^4$$

$\therefore T_e^4 = 2.10\times10^{18}$ $\therefore T_e = 38{,}000K$ [2]

B1 c)



For typical sync spectrum, T_b rises as ν decreases, but T_e falls. T_b cannot exceed T_e so below freq where $T_e = T_b$, source becomes self-absorbed [2]

$$\therefore kT_e \sim \gamma m_e c^2 \sim \left(\frac{\nu}{\nu_g}\right)^{1/2} m_e c^2 \sim \nu^{1/2}\left(\frac{eB}{2\pi m}\right)^{-1/2} mc^2$$

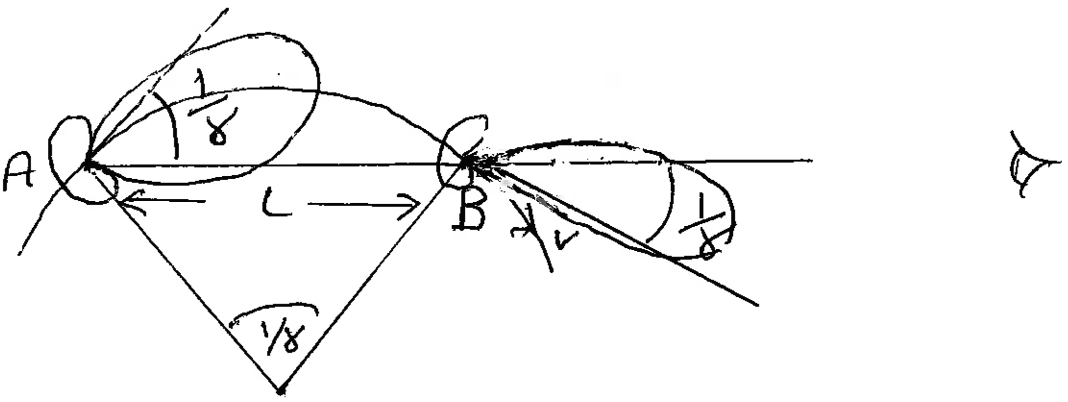
$$\therefore T_e = \nu^{1/2} B^{-1/2} \left(\frac{2\pi m_e}{e}\right)^{1/2} \frac{m_e c^2}{k}$$

Putting $T_e = T_b$ then

$$\nu^{1/2} B^{-1/2} \left(\frac{2\pi}{e}\right)^{1/2} \frac{m^{3/2} c^2}{k} = T_b = \frac{S\, c^2}{2k\nu^2 \Omega}$$

$$\therefore S = \nu^{5/2} B^{-1/2} \Omega \; 2 \left(\frac{2\pi}{e}\right)^{1/2} m_e^{3/2}$$ [4]

B2 a)



Relativistic e^- has "beam" of width $\frac{1}{\gamma}$

We see pulse of radiation of duration Δt.

Most of emitted power is at $\nu \simeq 1/\Delta t$

Δt = time taken by e^- to travel $A \to B$

— " " "radiation" " " "

$$= \frac{L}{v} - \frac{L}{c} = \frac{L}{v}\left(1 - \frac{v}{c}\right)$$

If period of orbit is $T = 2\pi/\omega_0$ — observed

$$\frac{L}{v} = \frac{1/\gamma}{2\pi} T = \frac{1}{\gamma \omega_0}$$

$$\omega_0 = \frac{eB}{m} = \frac{eB}{\gamma m_0} = \frac{\omega_g}{\gamma} \text{ — gyrofreq}$$

$$\therefore \frac{L}{v} \simeq \frac{1}{\omega_g}$$

Now $1/\gamma^2 = 1 - v^2/c^2 = (1 - v/c)(1 + v/c) \simeq 2(1 - v/c)$

$\therefore 1 - v/c \sim 1/2\gamma^2$

$$\therefore \Delta t \simeq \frac{1}{\omega_g} \frac{1}{2\gamma^2}$$

$\therefore \nu \sim \gamma^2 \nu_g.$ [5]

32 (b) Power radiated by 1 e, $P_1 = \sigma_T c \gamma^2 U_{mag}$.

e^- scatters mag quanta. Vol $= \sigma_T c$ as field rushes past.

Energy density in lab frame is U_{mag} but γ^2 (of e) arises, as in IC derivation, from transformation first to e^- frame for scattering, then back to lab

[2]

(c) Power radiated into $\nu \to \nu + d\nu$, $P(\nu)d\nu$

= no of e^- radiating into that freq range

× energy loss rate of each e

$= N_0 E^{-m} dE \times P_1$

Now $E \sim \gamma \sim \left(\frac{\nu}{\nu_g}\right)^{1/2} \sim \nu^{1/2} B^{-1/2}$

$\therefore dE \sim B^{-1/2} \nu^{-1/2} d\nu$

$\therefore P(\nu)d\nu \propto N_0 \gamma^{-m} B^{-1/2} \nu^{-1/2} d\nu \; \sigma_T c \frac{\nu}{B} . B^2$

$\propto N_0 \nu^{-m/2} B^{+m/2} B^{-1/2} \nu^{-1/2} \nu B \, d\nu$

$\propto N_0 \nu^{-m/2 + 1/2} B^{+m/2 + 1/2} d\nu$

$\propto N_0 \nu^{\frac{1-m}{2}} B^{\frac{1+m}{2}} d\nu$ [6]

If $m = 2$, then $P(\nu) \propto \nu^{-0.5}$, (ie "$\alpha$" $= +0.5$) [1]

B2(d)

$$u_B = \frac{B^2}{2\mu_0}$$ [1].

$$u_p = \int N(E) \times E \, dE$$

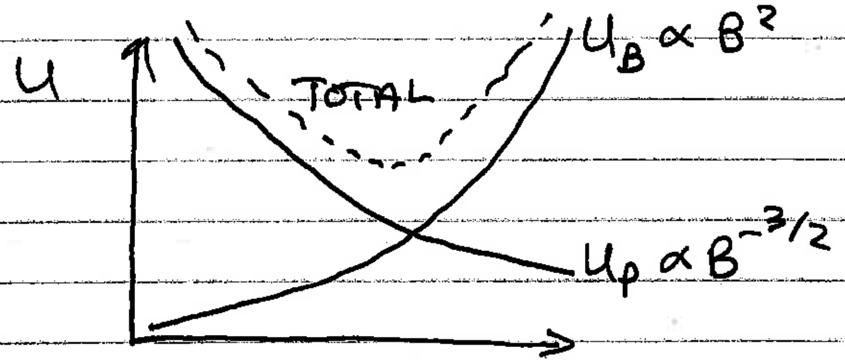
$$= \int N_0 E^{-m+1} \, dE$$ (don't worry about limits)

$$\propto \frac{N_0 E^{2-m}}{2-m}$$ i.e. $\propto N_0 (\gamma)^{2-m}$

Just concentrating on B-dependence, from previous page, $N_0 \propto B^{-\left(\frac{1+m}{2}\right)}$ and $\gamma \sim \left(\frac{\nu}{B}\right)^{1/2}$

$$\therefore u_p \propto B^{-\left(\frac{1+m}{2}\right)} B^{-\left(\frac{2-m}{2}\right)}$$

$$\propto B^{\frac{-1-m-2+m}{2}}$$ ie $\propto B^{-3/2}$ [4]

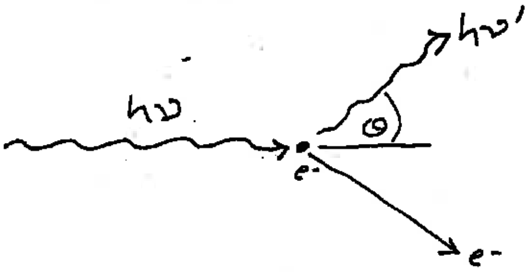


Total energy has a minimum near where $u_p = u_B$ (equipartition).

$\frac{du_{TOT}}{dB} = 0$ gives minimum energy and value of B at that energy [2]

B3 :)

Compton Scattering ($h\nu \gtrsim m_e c^2$)



	Initially	After Scattering
photon energy	$h\nu$	$h\nu'$
photon mom.	$h\nu/c$	$h\nu'/c$
e^- energy	$m_e c^2$	$(m_e^2 c^4 + p^2c^2)^{1/2}$
e^- mom	0	p

Energy Conservation

$$h\nu + m_e c^2 = h\nu' + (m^2c^4 + p^2c^2)^{1/2} \quad \ldots\ldots (1)$$

Momentum – DO IT WITH VECTORS!!

$$\underline{p} = \underline{h\nu/c} - \underline{h\nu'/c}$$ [4]

$$\therefore p^2 = \underline{p}\cdot\underline{p} = (h\nu/c)^2 + (h\nu'/c)^2 - 2.\underline{(h\nu/c)}.\underline{(h\nu'/c)}$$

$$= \qquad - 2\frac{h\nu}{c}\frac{h\nu'}{c}\cos\theta \quad \ldots (2)$$

Square (1)

$$\cancel{m^2c^4} + p^2c^2 = h^2(\nu-\nu')^2 + \cancel{m^2c^4} + 2h(\nu-\nu')mc^2$$

$$p^2c^2 = \cancel{h^2\nu^2} + \cancel{h^2\nu'^2} - 2h\nu h\nu' + 2(h\nu - h\nu')mc^2$$

$$(2) \Rightarrow p^2c^2 = \cancel{h^2\nu^2} + \cancel{h^2\nu'^2} - 2h\nu h\nu'\cos\theta = \text{RHS}$$

Equate (1)+(2), ÷ by 2

$$\therefore h\nu'(-h\nu\cos\theta + h\nu + mc^2) = h\nu mc^2$$

÷ mc^2 $$\therefore \nu' = \frac{\nu}{\frac{h\nu}{mc^2}(1-\cos\theta) + 1}$$

... answer.

[6]

B3 cont)

(i) ν' indep of ν : need $\frac{h\nu}{mc^2}(1-\cos\theta) \gg 1$.

So $h\nu \gg mc^2$ and θ not near zero. [1]

(ii) $\nu' = \nu$: either $h\nu \ll mc^2$ or $\theta = 0$.

[1]

In energy, rather than frequency:

$$E' = \frac{30}{1 + \frac{30}{0.511}(1-\cos 15)} = 10.0 \text{ MeV}$$

$\therefore$ KE of $e^- = 30 - 10 = 20$ MeV. [3]

b) Need $E_1 E_2 > (M_e c^2)^2$ [1]

$M_e c^2 = 511$ keV. Optical photon (or calculate) ~ 2 eV. (accept 1).

MWB: $\lambda = 1.1$ mm $\therefore E = \frac{hc}{\lambda} = \frac{6.62\times10^{-34}}{1.1\times10^{-3}} \times \frac{3.10^8}{1.6\times10^{-19}}$ ↑ to eV from Joule

$= 1.1\times10^{-3}$ eV

[2]

So, from optical, $E_2 > \frac{(5.11\times10^5)^2}{2} = 1.3\times10^{11}$ eV.

from MWB $E_2 > \frac{(5.11\times10^5)^2}{1.1\times10^{-3}} = 2.37\times10^{14}$ eV

[2]

B4 (a)

X-ray mirrors.

At very high frequencies (i.e. X-rays), refractive index $\underline{\underline{n_2 \simeq 1 - \delta}}$ [call free space $n_1 = 1$]

(Where $\delta \propto N_e$ and typically $\sim 10^{-4}$)

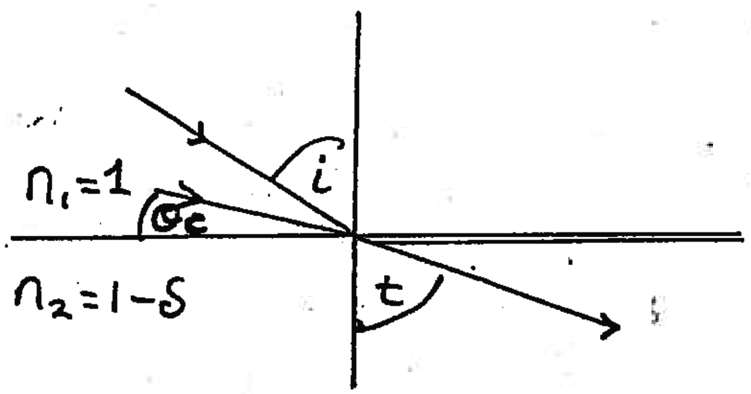
Thus Reflection coeff (direct incidence) $= \dfrac{n_1 - n_2}{n_1 + n_2} \sim 0$

and $T \sim 1.$ $\left(= \dfrac{2n_1}{n_1 + n_2} \right)$

So normal incidence cannot be used.

Instead we use total "external" reflection. [2]

Remember $n_i \sin i = n_t \sin t$



So at critical θ_c, we get total reflection. ($\sin t = 1$)

$$\therefore 1 . \sin(90 - \theta_c) = (1 - \delta) . 1 \qquad [2]$$

$$\therefore \cos\theta_c = 1 - \delta$$

Now $\cos\left(\frac{\theta}{2} + \frac{\theta}{2}\right) = \cos^2\left(\frac{\theta}{2}\right) - \sin^2\left(\frac{\theta}{2}\right) = 1 - 2\sin^2\left(\frac{\theta}{2}\right) = 1 - \frac{\theta^2}{2}$

$$\therefore \theta_c = \sqrt{2\delta} \qquad [2]$$

Now $n = (1 - \delta) = \left(1 - \frac{\omega_p^2}{\omega^2}\right)^{1/2} \simeq 1 - \frac{1}{2}\frac{\omega_p^2}{\omega^2}$

$$\therefore \delta \simeq \frac{1}{2}\frac{\omega_p^2}{\omega^2}$$

B4 a cont)

But $\theta = \sqrt{2\delta} = \sqrt{2 \cdot \frac{1}{2}\frac{\nu_p^2}{\nu^2}} = \frac{\nu_p}{\nu}$ [1]

Metal with $N = 5 \times 10^{28}$

$$\therefore \nu_p = \left(\frac{N e^2}{4\pi^2 \epsilon_0 m_e}\right)^{1/2} = \left(\frac{5 \cdot 10^{28} \ (1.6 \cdot 10^{-19})^2}{4\pi^2 \cdot 8.85 \times 10^{-12} \cdot 9.1 \times 10^{-31}}\right)^{1/2}$$

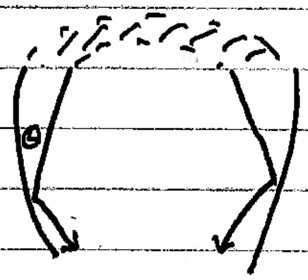
$= 2.00 \times 10^{15}$ Hz. [1]

As $E = h\nu = 1 \cdot 10^3 \times 1.6 \times 10^{-19}$. $\therefore \nu = 2.4 \times 10^{17}$

$$\therefore \theta \text{ (radians)} = \frac{2.00 \times 10^{15}}{2.4 \times 10^{17}} = 4.15 \times 10^{-3}$$

$= 0.24$ degrees [1]

For grazing incidence collecting area $\propto \theta$.

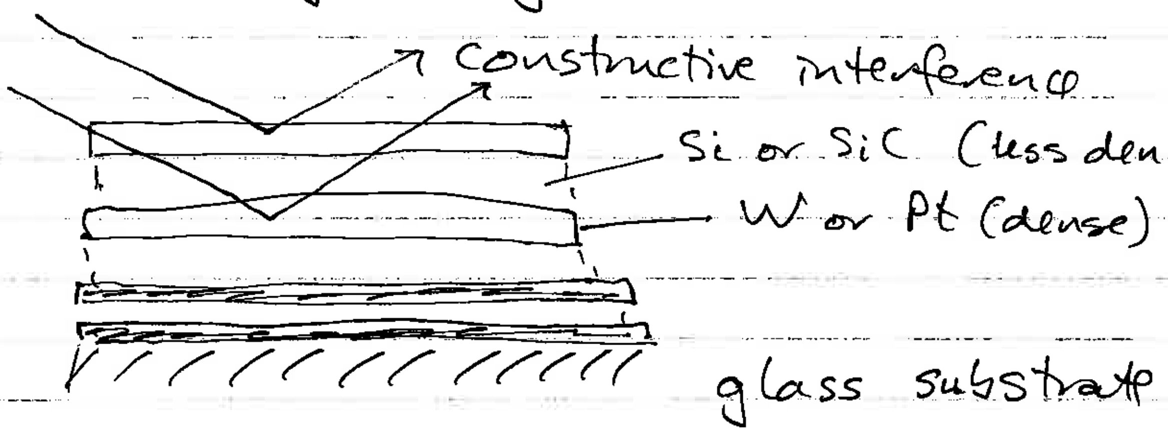


And $\theta \propto \frac{1}{\text{energy}}$ $\therefore$ area $\propto \frac{1}{\text{energy}}$

Thus decreasing area by $\times 2.5$ ($0.1 \to 0.04 m^2$) increases energy by $\times 2.5$ to 2.5 keV [2]

B4 (b) 3-79 keV. NuStar

~200 Multilayers of graded thickness.

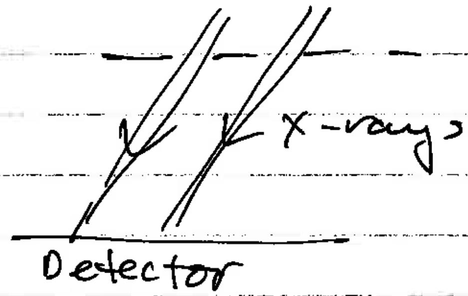


Graded thickness increases bandwidth. ½
Thinest layer at base for highest energies ½
Use many (~133 for Nustar) concentric mirrors.

Detector is ~10m away in separate module.

79 keV is platinum edge so higher energies are absorbed [½]

B4 (c) Describe coded masks.



Cross correlate mask + image pattern to get direction. [2]

For detector - describe crystal scintillator.
eg Thallium doped NaI.
γ-ray produces light flash ∝ photon γ-ray energy [2]

B4(c) cont.

Background – cosmic rays? [1]
– longer tracks, longer light flashes. [1]