SEMESTER 1 EXAMINATION 2014-2015

PHOTONS IN ASTROPHYSICS

Duration: 120 MINS (2 hours)

This paper contains 11 questions.

Answer **all** questions in **Section A** and **only two** questions in **Section B**.

Section A carries 1/3 of the total marks for the exam paper and you should aim to spend about 40 mins on it.

Section B carries 2/3 of the total marks for the exam paper and you should aim to spend about 80 mins on it.

An outline marking scheme is shown in brackets to the right of each question.

A Sheet of Physical Constants is provided with this examination paper.

Only university approved calculators may be used.

A foreign language translation dictionary (paper version) is permitted provided it contains no notes, additions or annotations.

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Section A

A1. A single dish operating at a wavelength of 1.0 cm has a diameter of 20 m. Calculate the forward gain of this dish. [3]

A2. An astronomical object is observed through a filter of bandwidth $\delta\nu$. The image of this object fills an area A on a CCD detector. The total count rate from the object is N_S photons per second per Hz of bandwidth. The background count rate on the detector in an area of the same size is N_B photons per second per Hz of bandwidth. Derive an expression for the signal to noise ratio of the detection if the observation time is τ. [2]

For a particular observation A=0.2 cm^2, $N_S = 1.0$ per second per Hz, $\delta\nu = 10^4$ Hz and $\tau = 10$ seconds. Here the background count rate is 500 counts per second per Hz per square cm of detector. Calculate the signal to noise ratio of the detection. [1]

A3. The surface temperature of Saturn is approximately 95K. Given that the wavelength of peak emission of the 2.7 K Cosmic Microwave Background is at 1.1mm, estimate the wavelength of the peak emission from Saturn. [1]

A4. The separation, $\delta\nu$ Hz, between two Zeeman-split components of the 21cm hydrogen line is $\delta\nu = 2.8 \times 10^{10}$B Hz where B is the magnetic field strength. Determine by how much the linewidth would be broadened in interstellar gas at a temperature of 100K and hence determine whether it will be possible to detect the Zeeman-splitting if B=10^{-9}T. [3]

A5. A forbidden emission line from an excited state of an oxygen atom (mass of $16\times$ mass of a proton) whose normal lifetime is 2.8×10^4s is detected from gas at a temperature of 2000K. Given that the mean free path, λ, for an oxygen atom is approximately $1/n\sigma$ where n is the gas density and σ, the cross-sectional area of the oxygen atom, is $\pi \times 10^{-20}$ m^{-2}, place an upper limit on n. [4]

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A6. What is the minimum possible impact parameter for the collision of an electron and an ion in a plasma of temperature 10^6K? [2]

A7. Briefly describe the interaction when TeV ($> 10^{12}$eV) photons interact with the atmosphere. [2]

With the aid of a diagram, determine a relationship between the refractive index of the atmosphere, n, and the accuracy with which the direction of arrival of a TeV photon can be determined. Hence estimate that accuracy, in degrees, if $n = 1.0008$. [2]

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Section B

B1. (a) For radiation incident upon a shell of gas of thickness l and number density n, and where the cross-sectional area of each gas atom is σ, derive a relationship between the intensity of the radiation entering (I_0) and leaving (I) the gas shell. You may assume that any photon hitting a gas atom is either absorbed or else scattered completely out of the line of sight. [3]

Describe how the above relationship defines the optical depth through the gas. [1]

Such a gas shell, with $n = 10^{22}$ m^{-3} and $l = 2 \times 10^5$m, surrounds an X-ray binary source. The observed X-ray flux from the source at 5 keV is 1 counts m^{-2} s^{-1} keV^{-1}. If $\sigma = 10^{-26} E^{-8/3}$ m^2, where E is the energy in keV, calculate the flux that would have been observed if there had been no gas shell in the way. [3]

(b) Including the concept of optical depth in your discussion, explain why radiation emitted by any mechanisms cannot exceed that from a black body of the same area at the same temperature. [2]

Define what is meant by brightness temperature, T_b, and by effective temperature, T_e, making clear the difference between the two. [2]

A flux density, S, is received at a wavelength, λ, from a radio source which subtends a solid angle, Ω, at the observer. Write down the relationship between T_b, S, λ and Ω. [1]

A star has a radius of 10^7m, luminosity of 10^{26}W in the ultraviolet band and 5×10^{25}W in the optical band with no emission in any other band. What is its effective temperature? [2]

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(c) Explain the phenomenon of synchrotron self-absorption. [2]

The gyrofrequency of the electron, $\nu_g, = \frac{eB}{m}$ where e is the electron charge, m is its mass and B is the magnetic field strength. For synchrotron radiation, the frequency of the emitted radiation, ν, is related to ν_g and the relativistic γ factor of the emitting electron by $\nu \simeq \gamma^2 \nu_g$. Hence show that for a self-absorbed synchrotron source of magnetic field strength B

$$S \propto \nu^{5/2} B^{-1/2} \Omega \, .$$ [4]

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B2. (a) The gyro frequency, ν_g, of an electron of mass m and charge e spiralling in a magnetic field of strength B is given by $\nu_g \propto \frac{eB}{m}$. By considering the duration of pulses of radiation received from relativistic electrons spiralling in a magnetic field show, that for synchrotron radiation, the frequency, ν, at which the bulk of the radiation is emitted, is related to the γ factor of the electron (energy = γmc^2) by $\nu \simeq \gamma^2 \nu_g$. [5]

(b) Write down, and briefly explain, the synchrotron power radiated by one electron of energy factor γ in a magnetic field of energy density U_{mag}. [2]

(c) If synchrotron radiation is produced by electrons with a power-law energy distribution

$$N(E)dE = N_0 E^{-m} dE$$

radiating in a magnetic field of strength B, show that the power radiated into frequency interval ν to $\nu + d\nu$ is given by

$$P(\nu)d\nu \propto N_0\, \nu^{\frac{1-m}{2}}\, B^{\frac{m+1}{2}}\, d\nu\ .$$ [6]

Hence determine the observed spectral index of the synchrotron radiation if the slope of the electron energy distribution is m = 2. [1]

(d) Show that, for a given observed flux density at a given frequency, the energy density in emitting particles, U_P, in the radio source is proportional to $B^{-3/2}$. Hint: Use the relationship found in part (c) in your derivation, [4]

Hence, by comparison with the energy density in the magnetic field, explain the concept of 'minimum energy' in a radio source and show how it can lead to an estimate of B. [2]

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B3. (a) A photon of frequency ν is incident upon a stationary electron of mass m. Write down the equations for the conservation of momentum and energy before and after the collision. [4]

Hence show that the frequency of the scattered photon, ν', as a function of scattering angle θ is given by

$$\nu' = \frac{\nu}{1 + \frac{h\nu}{mc^2}(1 - cos\theta)} \, .$$ [6]

Give one circumstance each under which the frequency of the scattered photon is (i) independent of the frequency of the incident photon and (ii) the same as the frequency of the incident photon? [2]

If the incoming photon has an energy of 30 MeV and is scattered through 15°, what is the kinetic energy of the recoil electron? [3]

(b) Electron-positron pairs can be produced by photon-photon collisions. State, or derive, the relationship between the energies of the two photons and the electron rest mass energy. [1]

State, or calculate, the electron rest mass energy in electron volts and the energies, also in electron volts, of (i) an optical photon and (ii) a photon of the Cosmic Microwave Background. [2]

Hence calculate the minimum photon energy, in electron volts, required to produce an electron-positron pair by collision with (i) an optical starlight photon and (ii) a photon from the Cosmic Microwave Background. [2]

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B4. (a) Describe qualitatively the principles underlying the design of mirrors for imaging at low X-ray energies (~few keV). [2]

The refractive index, n, of a metal at frequencies, ν, above the plasma frequency of the metal, ν_p, is given by $n = \left(1 - \frac{\nu_p^2}{\nu^2}\right)^{1/2}$ where $\nu_p = \left(\frac{Ne^2}{4\pi^2\epsilon_0 m_e}\right)^{1/2}$, N is the free electron number density, e is the electronic charge, m_e is the electron mass and ϵ_0 is the permittivity of a vacuum. Show that, for grazing incidence X-ray reflection, the maximum grazing angle for total reflection of X-rays is θ, where

$$\theta = \frac{\nu_p}{\nu} \, .$$ [5]

Calculate ν_p for a metal for which $N = 5 \times 10^{28}$ m^{-3}. [1]

Hence determine the value of θ, in degrees, for photons of energy 2 keV incident on mirrors coated with this metal. [1]

The total collecting area of an imaging X-ray telescope at 1 keV is 0.1 m^2. Calculate the maximum useful energy at which the telescope can operate if a collecting area of greater than 0.04m^2 is required. [2]

(b) Describe the principles underlying the design of mirrors for imaging at high X-ray energies (~3 to 79 keV). Include a discussion of the wide energy range and of the upper energy limit. [3]

(c) Describe with the aid of diagrams how photons are detected, how the position of a photon's origin is located and how its energy is estimated at energies between 100 keV and a few MeV. [4]

What is the main source of background events at energies above 100 keV and how might such events might be distinguished from real source photons? [2]

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