SEMESTER 1 EXAMINATION 2012/13

PHOTONS IN ASTROPHYSICS

Duration: 120 MINS

*Answer **all** questions in **Section A** and two **and only two** questions in **Section B**.*

Section A carries 1/3 of the total marks for the exam paper and you should aim to spend about 40 mins on it. Section B carries 2/3 of the total marks for the exam paper and you should aim to spend about 80 mins on it.

A Sheet of Physical Constants will be provided with this examination paper.

An outline marking scheme is shown in brackets to the right of each question.

Only university approved calculators may be used.

Section A

A1. A single dish operating at a wavelength of 1.3cm has a diameter of 25m. Calculate the forward gain of this dish. [3]

A2. An astronomical object is observed through a filter of bandwidth $\delta\nu$. The image of this object fills an area A on a CCD detector. The total count rate from the object is N_S photons per second per Hz of bandwidth. The background count rate on the detector in an area of the same size is N_B photons per second per Hz of bandwidth. Derive an expression for the signal to noise ratio of the detection if the observation time is τ. [2]

If A=0.3 cm^2, $N_S = 1$ per second per Hz, $N_B = 1000$ per square cm per second per Hz, $\delta\nu = 10^4$ Hz and $\tau = 10$ seconds, calculate the signal to noise ratio of the detection. [1]

A3. The binding energy of Cooper pairs in a superconductor is $\sim 10^{-3}$eV. Estimate the uncertainty, in eV, with which the energy of a 10eV photon incident on this superconductor might, in principle, be determined. [2]

A4. Why are planets used as flux calibration sources in the far-infrared and sub-millimetre observing bands? [2]

The surface temperature of Jupiter is approximately 130K. Given that the wavelength of peak emission of the Cosmic Microwave Background is at 1.1mm, estimate the wavelength of the peak emission from Jupiter. [1]

A5. A forbidden emission line from an excited state of an oxygen atom (mass of $16x$ mass of a proton) whose normal lifetime is 2.8×10^4s is detected from gas at a temperature of 5000K. If the cross-sectional area of the oxygen atom is $\pi \times 10^{-20}$ m^{-2}, place a lower limit on the gas density. [5]

A6. Briefly describe the interaction when TeV ($> 10^{12}$eV) photons interact with the atmosphere. With the aid of a diagram, determine a relationship between the refractive index of the atmosphere, n, and the accuracy with which the direction of arrival of a TeV photon can be determined. Hence estimate that accuracy, in degrees, if $n = 1.0003$. [4]

Section B

B1. (a) Explain why radiation is emitted by an accelerated electric charge. [1]

Show that the transverse electric field E_T, detected at a distance R from a charge q is given by

$$E_T = \frac{q\ a\ sin\theta}{4\ \pi\ \epsilon_0\ R\ c^2}$$

where a is the acceleration of the charge, θ is the angle between the direction of acceleration and the line of sight and c is the speed of light. [5]

Explain how the state of polarisation of the emitted radiation is related to the motion of the accelerated charge. In the case of electrons circling in a magnetic field, what type of polarisation would we see if we looked (a) down the field lines (b) at right angles to the field lines. [3]

(b) Define a differential scattering cross-section and a total scattering cross-section. [2]

By considering the effect of an externally applied electric field on a population of free electrons and protons, determine the ratio of the scattering cross-sections of protons and electrons, as applicable to the scattering of low energy photons. [3]

(c) The X-ray flux emitted by a neutron star is attenuated by passage through cold gas surrounding the source. The equivalent line of sight neutral hydrogen column density ($n_{\rm H}\ l$) through the obscuring gas is 3.0×10^{28} m^{-2}. The observed flux at 10 keV is 0.5 counts m^{-2} s^{-1} keV^{-1}. Calculate the unobscured 10 keV X-ray flux. [4]

Describe qualitatively how the observed X-ray flux is affected if the material is fully ionised. [2]

(You may assume that the photoelectric absorption cross-section for a cosmic elemental abundance is $\sigma_{pe} = 10^{-26}E^{-8/3}$ m^2, where E is the energy in keV).

B2. (a) Explain the difference between cyclotron (or gyro-) radiation and synchrotron radiation and derive an expression for the gyro frequency, ν_g, of an electron of mass m and charge e spiralling in a magnetic field of strength B. [3]

(b) Write down, and briefly explain, the synchrotron power radiated by one electron of energy factor γ in a magnetic field of strength B. [2]

(c) For synchrotron radiation, the frequency, ν, at which the bulk of the radiation is emitted, is related to the γ factor of the electron (energy = γmc^2) by $\nu = \gamma^2 \nu_g$.

If synchrotron radiation is produced by electrons with a power-law energy distribution

$$N(E)dE = N_0 E^{-m} dE$$

radiating in a magnetic field of strength B, show that the power radiated into frequency interval ν to $\nu + d\nu$ is given by

$$P(\nu)d\nu \propto N_0\, \nu^{\frac{1-m}{2}}\, B^{\frac{m+1}{2}}\, d\nu\ .$$

[7]

Hence determine the observed spectral index of the synchrotron radiation if the slope of the electron energy distribution is m = 2. [1]

(d) Write down the relationship between the energy density in the magnetic field, U_B, and the magnetic field strength, B. [1]

Show that, for a given observed flux density at a given frequency, the energy density in emitting particles, U_P, in the radio source is proportional to $B^{-3/2}$. [4]

Hence explain the concept of 'minimum energy' in a radio source and show how it can lead to an estimate of B. [2]

B3. (a) A photon of frequency ν is incident upon a stationary electron of mass m. Write down the equations for the conservation of momentum and energy before and after the collision. [4]

Hence show that the frequency of the scattered photon, ν', as a function of scattering angle θ is given by

$$\nu' = \frac{\nu}{1 + \frac{h\nu}{mc^2}(1 - cos\theta)} \ .$$

[6]

Give one circumstance each under which the frequency of the scattered photon is (i) independent of the frequency of the incident photon and (ii) the same as the frequency of the incident photon? [2]

If the incoming photon has an energy of 20 MeV and is scattered through 10°, what is the kinetic energy of the recoil electron? [3]

(b) Electron-positron pairs can be produced by photon-photon collisions. State, or derive, the relationship between the energies of the two photons and the electron rest mass energy. [1]

State, or calculate, the electron rest mass energy in electron volts and the energies, also in electron volts, of (i) an optical photon and (ii) a photon of the Cosmic Microwave Background. [2]

Hence calculate the minimum photon energy, in electron volts, required to produce an electron-positron pair by collision with (i) an optical starlight photon and (ii) a photon from the Cosmic Microwave Background. [2]

B4. (a) Describe qualitatively the principles underlying the design of mirrors for imaging at low X-ray energies (∼few keV). [2]

The refractive index, n, of a metal at frequencies, ν, above the plasma frequency of the metal, ν_p, is given by $n = \left(1 - \frac{\nu_p^2}{\nu^2}\right)^{1/2}$ where $\nu_p = \left(\frac{Ne^2}{4\pi^2\epsilon_0 m_e}\right)^{1/2}$, N is the free electron number density, e is the electronic charge, m_e is the electron mass and ϵ_0 is the permittivity of a vacuum. Show that, for grazing incidence X-ray reflection, the maximum grazing angle for total reflection of X-rays, is θ where

$$\theta = \frac{\nu_p}{\nu} \ .$$

[6]

Calculate ν_p for a metal for which $N = 1.2 \times 10^{29}$ m^{-3}. [1]

Hence determine the value of θ, in degrees, for photons of energy 1 keV incident on mirrors coated with this metal. [1]

The total collecting area of an imaging X-ray telescope at 1 keV is 0.1 m^2. Calculate the maximum useful energy at which the telescope can operate if a collecting area of greater than 0.02m^2 is required. [2]

(b) Describe with the aid of diagrams how photons are detected, how the position of a photon's origin is located and how its energy is estimated at energies of i) 100 keV and ii) 10 GeV. [6]

What is the main source of background events at energies above 100 keV and how might such events might be distinguished from real source photons? [2]

END OF PAPER