SEMESTER 2 EXAMINATIONS 2013-2014

BSc SYNOPTIC PHYSICS

DURATION 120 MINS (2 Hours)

This paper contains 9 questions

Answer **all** questions in **Section A** and **only two** questions in **Section B.**

Section A carries 1/3 of the total marks for the exam paper and you should aim to spend about 40 minutes on it.

Section B carries 2/3 of the total marks for the exam paper and you should aim to spend about 80 minutes on it.

An outline marking scheme is shown in brackets to the right of each question.

A Sheet of Physical Constants is provided with this examination paper.

Only University approved calculators may be used.

A foreign language translation dictionary (paper version) is permitted provided it contains no notes, additions or annotations.

Section A

A1. The average power radiated by an oscillating electric dipole is given in SI units by

$$\mathrm{W} = \frac{4\pi^3}{3}\frac{c}{\varepsilon_0}\frac{p_0^2}{\lambda^4},$$

where p_0 is the amplitude of the oscillating dipole, λ is the wavelength of the radiated field, ε_0 is the permittivity of free space and c is the speed of light. Show that this equation is dimensionally consistent. [4]

A2. The spin wave function of a spin-½ particle may be described in terms of the basis states

$$e_\uparrow = \begin{pmatrix}1\\0\end{pmatrix} \quad e_\downarrow = \begin{pmatrix}0\\1\end{pmatrix}$$

which are eigenfunctions of the spin angular momentum operator S_z. Show that

$$S_x e_\uparrow = \frac{\hbar}{2} e_\downarrow$$

where

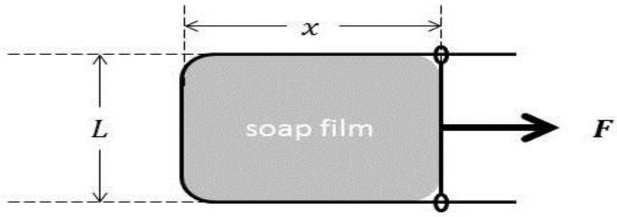
$$S_x = \frac{\hbar}{2}\begin{pmatrix}0 & 1\\1 & 0\end{pmatrix}.$$

Find the eigenstate of S_x in which the particle spin points in the $+x$ direction, expressed as a superposition of $e_\uparrow$ and $e_\downarrow$. Hence show that if the particle spin is initially oriented along $+x$, there is a probability of $\frac{1}{2}$ that a subsequent measurement of S_z will yield the value $\frac{\hbar}{2}$. [4]

A3. According to the highway code, the typical stopping distance for a motor vehicle travelling at 70 mph is 75 m. (This is the distance travelled after the driver has hit the brakes.) Estimate the g-force experienced by the driver while the car is braking; i.e. the average deceleration in units of the acceleration due to gravity. Note that 1 mile = $^{8}/_{5}$ km. [4]

A4. Sketch a ray diagram that explains the operating principle of a hand magnifying lens. Explain the type of image (real, virtual, upright, inverted) observed by the user looking through the lens. State the condition on the distance between the lens and the object that must be satisfied for a magnified image to be formed. [4]

A5.

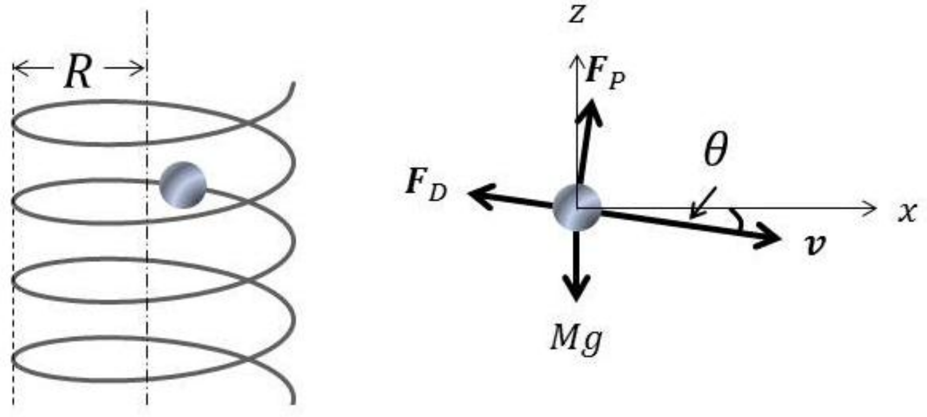


Consider a soap film stretched across a rigid wire framework closed by a sliding wire on the right, as shown in the diagram. In equilibrium a force $F = 2\gamma L$ is applied to the sliding wire to balance the force due to the surface tension γ. Write down the 1st & 2nd laws of thermodynamics as applied to an infinitesimal change dA in the total area $A = 2Lx$ of the top and bottom soap film surfaces. Hence prove that in an adiabatic stretching of the film (no heat transferred), the internal energy U satisfies $(\partial U/\partial A)_S = \gamma$. [4]

TURN OVER

SECTION B

B1.



a) **b)**

A metal bead of mass M is free to slide on a rigid insulating wire in the form of a vertical helix of radius R, as shown in a) above. The wire makes a constant angle of θ to the horizontal plane. The forces acting on the bead in the vertical plane containing its velocity $\boldsymbol{v}$ are shown in b): $\boldsymbol{F}_D$ is the drag force due to friction between the bead and the wire, and $\boldsymbol{F}_P$ is one component of the reaction force that the wire exerts on the bead

(i) The bead slides down the helix and reaches a steady state of motion at constant speed v. Write down expressions for F_D and F_P, explaining your reasoning. [4]

(ii) The bead has angular momentum $M(\boldsymbol{r} \times \boldsymbol{v})_z$ about the z-axis**.** Explain why the reaction force that the wire exerts on the bead also contains a horizontal component $\boldsymbol{F}_H$ pointing in towards the axis of the helix. Write down an expression for F_H. [4]

(iii) The magnitude of the drag force F_D is proportional to the magnitude of the *total* reaction force exerted by the wire on the bead. The proportionality constant is α, the coefficient of friction. Show that

$$F_D = \alpha \sqrt{\left(\frac{M(v\ \cos\theta)^2}{R}\right)^2 + (Mg\cos\theta)^2}.$$

[4]

(iv) Show that the constant speed v at which the bead slides in a steady state is given by

$$v^2 = \frac{gR}{\cos\theta} \sqrt{\left[\left(\frac{\tan\theta}{\alpha}\right)^2 - 1\right]}.$$

[4]

(v) A charge Q is applied to the bead, and a uniform magnetic field B pointing vertically upwards is switched on. Explain why the steady-state speed v now depends on B. Discuss whether v increases or decreases in the presence of the field. [4]

TURN OVER

B2

(i) Write down the time-dependent Schrödinger equation for a particle of mass m moving in one dimension in an infinite square-well potential defined by

$$V(x) = 0, \qquad |x| < a$$
$$V(x) = \infty, \qquad |x| \geq a.$$

[4]

(ii) State what it means to say that a wavefunction has well-defined *parity*. Identify the property of the infinite square well potential specified in (i) that causes all the eigenstates of this system to be either even or odd. [4]

(iii) Show that the eigenfunctions

$$\phi_n(x,t) = \frac{1}{\sqrt{a}}\cos\left(\frac{n\pi x}{2a}\right)\exp\left(-\frac{iE_n t}{\hbar}\right) \text{ for } |x| \leq a, n = 1, 3, 5 \ldots$$

$$\phi_n(x,t) = \frac{1}{\sqrt{a}}\sin\left(\frac{n\pi x}{2a}\right)\exp\left(-\frac{iE_n t}{\hbar}\right) \text{ for } |x| \leq a, n = 2, 4, 6 \ldots$$

$$\phi_n(x,t) = 0 \text{ for } |x| > a$$

are solutions of the Schrödinger equation of (i) with appropriate boundary conditions, and find an expression for the energy eigenvalue E_n. [4]

(iv) Suppose that the particle described in (i) occupies the superposition state

$$\psi(x,t) = \frac{1}{\sqrt{2}}\{\phi_2(x,t) + \phi_3(x,t)\}.$$

Sketch a) $\psi(x,0)$ and b) $|\psi(x,0)|^2$ as a function of x for $|x| < a$.

[4]

(v) Write down an expression for $\langle x \rangle$, the expectation value of the position of the particle in the superposition state defined in (iv). Show that $\langle x \rangle$ varies in time as $\cos(\omega t)$, where

$$\omega = \frac{5\pi^2\hbar}{8ma^2}.$$

(Note that you do not need to evaluate any integrals.) [4]

B3. An insulating crystal contains N paramagnetic ions that have a doublet ground state. In an applied magnetic field B, the ground state splits into two levels with energies $E = \pm\mu B$. In thermal equilibrium at temperature T the magnetic degrees of freedom of this system are described by a canonical ensemble, for which the single-molecule partition function Z_1, the free energy F, the internal energy U, and the entropy S are related by

$$Z_1 = \sum_i \exp\left(-\frac{E_i}{k_B T}\right);$$

$$F = U - TS = -Nk_B T \ln Z_1;$$

$$U = N\frac{\sum_i E_i \exp\left(-\frac{E_i}{k_B T}\right)}{Z_1} = Nk_B T^2 \frac{\partial \ln Z_1}{\partial T},$$

where E_i is the energy of level i, and k_B is Boltzmann's constant.

(i) Show that the partition function of this magnetic system is given by

$$Z_1 = 2\cosh\left(\frac{\mu B}{k_B T}\right).$$

[4]

(ii) Show that the entropy is given by

$$\frac{S}{Nk_B} = -\frac{\mu B}{k_B T}\tanh\left(\frac{\mu B}{k_B T}\right) + \ln\left[2\cosh\left(\frac{\mu B}{k_B T}\right)\right].$$

[4]

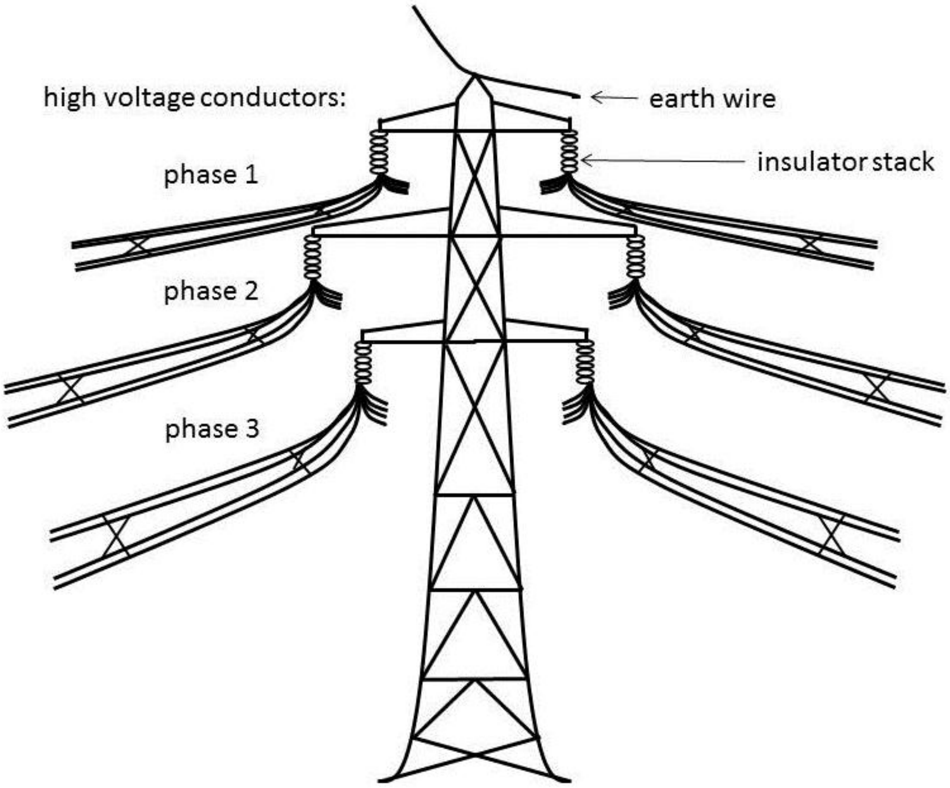
(iii) Explain qualitatively why the entropy of the system decreases monotonically with increasing magnetic field at all temperatures. [4]

(iv) Find values for the entropy of the system a) in the limit as $B/T \to 0$, and b) in the limit as $B/T \to \infty$. [4]

(v) Suppose that the crystal is placed in an intense magnetic field B_i while it is cooled to a low temperature T_i. The magnetic field is then switched off abruptly, so that no heat flows in or out of the crystal while the field is changing (adiabatic demagnetisation). After the switch-off, there remains a small residual magnetic field $B_f \ll B_i$. Explain why the magnetic system cools down during the demagnetisation process, and write down an expression for the final temperature that it attains.

[4]

B4.



The diagram shows the structure of a pylon supporting 400-kV overhead electrical power transmission lines. The left and right-hand sides of the pylon carry separate power transmission circuits, with three phases per circuit. Each phase is carried by 4 wires connected together by cross-shaped supports.

(i) The E-field at the surface of the high-voltage conductors is sufficiently intense to ionise the surrounding air, leading to *corona discharge*, which creates audible and electromagnetic noise and wastes power. Use Gauss's theorem to suggest a reason why the effects of corona discharge are reduced by conducting each high voltage phase along 4 linked wires as shown in the diagram. [4]

(ii) Explain a) why the pylon tends to attract lightning strikes, and b) how the earth wire at the top protects the installation from the resulting electrical discharge. [4]

(iii) The high voltage conductors carry alternating currents which may be written

$$I = I_0 \cos\left(2\pi\nu\left(\frac{z}{c} - t\right) - \phi\right)$$

where I_0 is the same for each phase; the frequency ν is 50 Hz; z is the distance propagated along the conductor; t represents time, and $\phi = 0°, 120°$ and $240°$ for phases 1, 2 and 3 respectively. By means of a phasor diagram, or otherwise, calculate the sum (or resultant) of the currents flowing in all three phases at any point. Hence explain why the transmission line does not include a neutral conductor to carry a return current. [4]

(iv) Calculate a value for I_0, the peak-to-peak current amplitude on one phase in a transmission line that is carrying a total average power of 1 GW divided between 2 circuits at an rms voltage of 400 kV. [4]

(v) Suppose that the transmission line described in (iv) has resistive power losses of 0.1 % over a distance of 30 km. Estimate the percentage power transmission loss over the same distance for the same transmission line now operated at an rms voltage of only 40 kV with the same power. [4]

END OF PAPER

