SEMESTER 2 EXAMINATION 2014-2015

BSc SYNOPTIC PHYSICS

Duration: 120 MINS (2 hours)

This paper contains 9 questions.

Answer **all** questions in **Section A** and **only two** questions in **Section B**.

Section A carries 1/3 of the total marks for the exam paper and you should aim to spend about 40 mins on it.

Section B carries 2/3 of the total marks for the exam paper and you should aim to spend about 80 mins on it.

An outline marking scheme is shown in brackets to the right of each question.

A Sheet of Physical Constants is provided with this examination paper.

Only university approved calculators may be used.

A foreign language word to word® translation dictionary (paper version) is permitted provided it contains no notes, additions or annotations.

Section A

A1. Explain what is meant by dispersion in wave propagation and define the concepts of phase velocity and group velocity. [4]

A2. A diffraction grating is used to observe the sodium D lines of wavelengths $\lambda_1 = 589.0$ nm and $\lambda_2 = 589.6$ nm. If the grating has 300 rulings per millimeter, what is the angular separation of the two lines at first order? And at the highest order which can be observed? [4]

A3. The components of angular momentum obey the relation $[J_x, J_y] = i\hbar J_z$. Briefly illustrate the meaning of this relation. Why is it not possible, in general, to have states for which more than one component of $\boldsymbol{J}$ has a definite value, while it is possible to have states with simultaneously well-defined values of $\boldsymbol{J}^2$ (where $\boldsymbol{J}^2 = J_x^2 + J_y^2 + J_z^2$) and one component of $\boldsymbol{J}$? [4]

A4. Explain the difference between bosons and fermions. Why must there be two kinds of exchange symmetry? Show that in one of the two cases a doubly occupied state cannot exist (the Pauli exclusion principle). [4]

A5. Estimate the temperature (in degrees Kelvin) at which most of the atoms in a hydrogen gas in equilibrium will be ionised. [4]

Section B

B1. (a) Write down Maxwell's equations in vacuum in a region free of charges and currents. [4]

(b) By evaluating the curl of Faraday's law in differential form, obtain the wave equation for the electric field propagating through free space. What is the wave speed in this equation? [4]

(c) A monochromatic plane electromagnetic wave is incident along the normal to the plane interface between two dielectric media of electric permittivities ε_1 and ε_2.

(i) Write down the boundary conditions for the electric and magnetic fields at the interface between the two media. [4]

(ii) Applying the boundary conditions, show that the electric field amplitudes E_R and E_T for the reflected and transmitted waves as functions of the incident electric field amplitude E_I and of the permittivities ε_1 and ε_2 are given by

$$E_R = E_I \frac{\sqrt{\varepsilon_1} - \sqrt{\varepsilon_2}}{\sqrt{\varepsilon_1} + \sqrt{\varepsilon_2}} \quad , \qquad E_T = E_I \frac{2\sqrt{\varepsilon_1}}{\sqrt{\varepsilon_1} + \sqrt{\varepsilon_2}} \quad .$$

(Assume magnetic permeabilities $\mu_1 \simeq \mu_2 \simeq \mu_0$.) [4]

(iii) Determine the power transported per unit area by the incident, reflected and transmitted waves as a function of E_I, E_R and E_T. Verify that energy is conserved between the three waves. [4]

TURN OVER

B2. (a) Write down the time-independent and the time-dependent Schrödinger equations. [2]

Is it necessary for the wave function of a system to satisfy the time-dependent Schrödinger equation? Under what circumstances does the wave function of a system satisfy the time-independent Schrödinger equation? [2]

What is the significance of the Schrödinger equation being first-order in time, rather than second-order like Newton's equations of motion? [2]

(b) A particle is confined in a potential well such that its allowed energies are $E_n = n^2\varepsilon$, where $n = 1, 2, \ldots$ is an integer and ε is a positive constant. Let $u_1, u_2, \ldots, u_n, \ldots$ be the corresponding orthonormal energy eigenfunctions. At time $t = 0$ the particle is in the state described by the wave function

$$\psi(0) = 0.2u_1 + 0.3u_2 + 0.4u_3 + 0.843u_4 \quad .$$

(i) What is the probability, if the energy is measured at $t = 0$, of finding a value smaller than 6ε? [3]

(ii) What is the expectation value $\langle E \rangle$ of the energy of the particle at $t = 0$? [2]

(iii) What is its root mean square deviation ΔE? [2]

(c) Write down the wave function of the particle at time $t > 0$. [4]

(d) When the energy is measured it turns out to be 16ε. After the measurement, what is the wave function of the system? What result is obtained if the energy is measured again? [3]

B3. (a) The power radiated from a black body at temperature T per unit area can be written as

$$\Phi = \frac{c}{4}\int_0^\infty \hbar\omega\; n(\omega)\; g(\omega)\; d\omega \;,$$

where c is the speed of light, $n(\omega) = [\exp(\beta\,\hbar\,\omega) - 1]^{-1}$ is the thermal occupancy of a photon mode with energy $\hbar\omega$, $\beta = (k_B T)^{-1}$, and $g(\omega)$ is the density of modes as a function of frequency ω.

(i) By writing down the number of modes with wave number in the interval $(k, k + dk)$ per unit volume, and using that photons have two polarizations and the dispersion relation is $\omega = ck$, show that $g(\omega) = \pi^{-2}c^{-3}\omega^2$. [4]

(ii) Evaluate the above integral over frequency ω. [4]

(iii) Show that $\Phi = \sigma T^4$, where $\sigma = \pi^2 k_B^4/(60\,\hbar^3 c^2)$. [2]

[Use the result $\int_0^\infty dx\; x^3(e^x - 1)^{-1} = \pi^4/15$.]

(b) A spherical satellite of radius 1 m is in circular orbit about the Sun (radius $R_S = 6.955\times10^8$ m) at a distance of 1 Astronomical Unit (= 1.496×10^{11} m).

(i) If energy is radiated isotropically from the Sun, and the satellite can be treated as a black-body disk of radius $r = 1$ m, what is the power absorbed by the satellite? [4]

(ii) Estimate the equilibrium temperature of the satellite assuming it to behave as a black body. [4]

[The surface temperature of the Sun is T_S = 5,700 K.]

(c) When the power supply for the instruments on the satellite is turned on, heat is generated at a rate of 100 W. What is the new equilibrium temperature of the satellite? [2]

TURN OVER

B4. (a) The equation for the energy of a satellite of mass m in an elliptical orbit around the Earth under the effect of its gravitational attraction is

$$E = \frac{m\dot{r}^2}{2} + \frac{J^2}{2mr^2} - \frac{GM_E m}{r} \ .$$

Briefly illustrate the meaning of this equation, explaining the symbols and the physical significance of each term. [3]

(b) Explain the meaning of the effective potential, defined as

$$U(r) = \frac{J^2}{2mr^2} - \frac{GM_E m}{r} \ .$$

Sketch $U(r)$ for a general elliptical orbit as a function of r. [4]

On the same diagram indicate the total energy and the effective potential energy of the satellite at perigee (closest point to the Earth), apogee (furthest point), and a point elsewhere on the orbit. [3]

(c) A geostationary orbit is a circular orbit around the Earth which has a period of 24 hours. Calculate the orbital velocity for this orbit. [3]

Show that its radius is approximately 40,000 km. [3]

(d) A satellite is to be inserted in to a geostationary orbit from an elliptical orbit with perigee at a geocentric radius of 8,000 km and apogee at 40,000 km. When it is at apogee, a brief firing of its rocket motor places it into the circular orbit. What is the change in velocity the motor needs to provide? [4]

[Take the mass of the Earth to be 6×10^{24} kg.]

END OF PAPER