PHYS6004 Space Plasma Physics $\boxed{2014\text{-}15}$

Section A

A1. (i) Heat the gas so that average kinetic energy

$$\langle W \rangle = \frac{3}{2}kT > I_0$$

$$\frac{3}{2} \times 1.38 \times 10^{-23}\, T > 21.6 \times 1.6 \times 10^{-19}\, J$$

$$\underline{T > 1.7 \times 10^{5}}\, K \qquad [2]$$

(ii) Radiate gas with photons of energy $> I_0$

$$E = \frac{hc}{\lambda} > 21.6 \times 1.6 \times 10^{-19}$$

$$\lambda < \frac{6.6 \times 10^{-34} \times 3 \times 10^{8}}{21.6 \times 1.6 \times 10^{-19}}$$

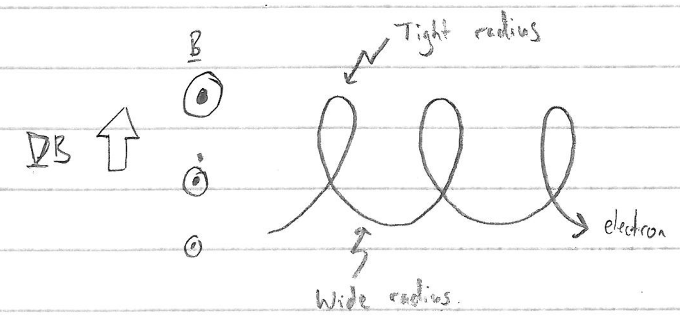
Similar problems done in lectures

$$\lambda < 5.7 \times 10^{-8}\, m$$

$$\lambda < 57\, nm \qquad [2]$$

A2. Magnetic gradient drift:

a)



Gradient in B field strength ⊥ to B

Larger B gives smaller gyroradius.

[1] (diagram)

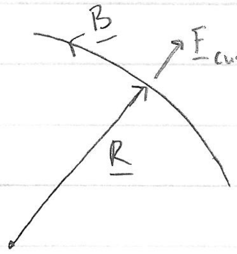
Leads to drift ⊥ to B and ∇B

[1]

Ions and electrons gyrate in opposite sense, so gradient drift in opposite directions.

b) Magnetic curvature drift:

When field lines are curved, so field direction changes along $\underline{B}$, there is curvature drift.



$$\underline{F}_{curv} = \frac{m v_{\parallel}^2 \underline{R}_c}{R_c^2}$$

[1] (diagram)

Particles moving along B with parallel velocity $v_{\parallel}$ experience centrifugal force in $\underline{R}$ direction.

Produces drift perpendicular to $\underline{B}$ and its radius of curvature.

[1]

c) Drift velocity depends on q so ions and electrons drift in opposite directions. [1]

Gradient and curvature drifts in magnetosphere lead to westward drift for +ve ions, eastward drift for electrons, around the Earth.

This produces ring current around the Earth. [1]

Ring current generates magnetic field which opposes Earth's field, so magnetic field measured at Earth's equator decreases during enhanced ring current. [1]

Bookwork

A3. Plasma beta is ratio of thermal and magnetic pressures, given by

$$\beta = \frac{P_{th}}{P_B} = \frac{nkT}{B^2/2\mu_0}$$

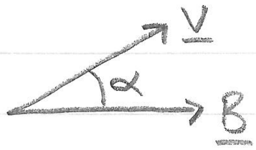
Low β, $\beta \ll 1$ magnetic field dominates

High β, $\beta \gg 1$ plasma behaves as fluid

$\boxed{\text{Bookwork}}$

[2]

A4. Pitch angle α is angle between particle velocity and $\underline{B}$



[1]

Magnetic moment of particle

$$\mu_m = \frac{\left(\frac{1}{2} m v_\perp^2\right)}{B}$$ is constant

(could easily derive from I.A)

$$= \frac{mv^2 \sin^2\alpha}{B}$$

[1]

Since v is constant

$$\frac{\sin^2\alpha}{B} = \text{constant}$$

[1]

Particle reverses direction when

$v_\parallel = 0$ and $\alpha = 90°$

A4 cont.

$$\therefore \quad \frac{\sin^2 \alpha_{eq}}{B_{eq}} = \frac{\sin^2 90^\circ}{B_M}$$

$$\therefore \quad B_M = \frac{B_{eq}}{\sin^2 \alpha_{eq}} \qquad [2]$$

Bookwork

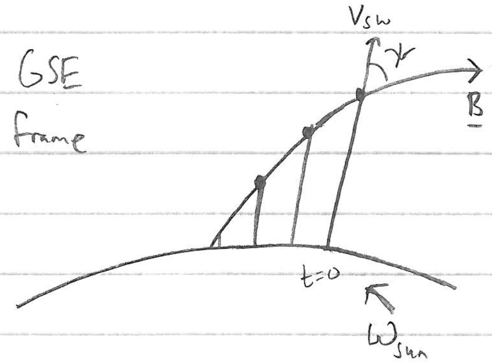
A5. Closed field lines connect the north + south magnetic hemispheres of Earth's surface and do not pass through the magnetopause. [1]

Open field lines thread the magnetopause and so connect the magnetosphere to interplanetary space [1]

Bookwork

Section B

B1. a) Parker spiral



Plasma leaving the sun at $t=0$ follows a radial streamline, reaching

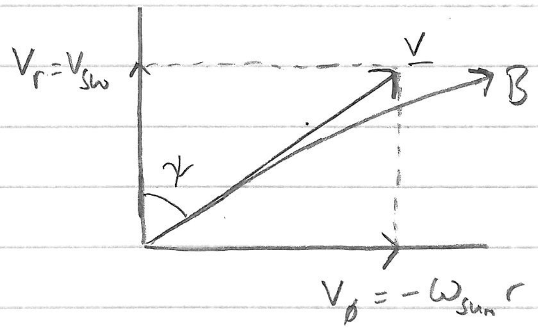
$r = v_{sw} t$ at time t.

Plasma leaving sun at later times also travels radially, but sun has rotated.

Plasma and B field "frozen-in"

[2]

Frame moving with Sun



$$\tan\psi = \frac{\omega_{sun} r}{v_{sw}}$$ [2]

When V_{sw} increases, the spiral angle decreases and the spiral spreads out (becomes more radial). [1]

Bookwork

b) Solar rotation period = 25 days

$$\omega = \frac{2\pi}{T}, \quad r = 5.2\,\text{AU}, \quad V_{sw} = 500\ \text{km/s}$$

$$\tan\psi = \frac{\omega r}{V_{sw}} = \frac{2\pi}{25 \times 24 \times 60 \times 60} \cdot \frac{5.2 \times 1.496 \times 10^{11}}{500 \times 10^3}$$

$$\psi = \arctan(4.526)$$

$$\underline{\psi = 78^\circ} \qquad [1]$$

This is greater than at Earth, where $\psi \sim 45^\circ$
(or at 1AU in above $\psi = 41^\circ$) [1]

Similar problem done in lecture

B1 c) Transit time $dt = 100 \times 60 = 6000s$

Reconnection length $dx = dt\, V_{sw}$

$= 6000 \times 650 \times 10^3\ m$

$= 3.9 \times 10^9\ m$ [1]

Reconnection area $dxdy = 3.9 \times 10^9 \times 7.5 \times 6378 \times 10^3\ m^2$

$= 1.866 \times 10^{17}\ m^2$ [1]

Flux through area $= B_{sw}\, dxdy$

$= 2 \times 10^{-9} \times 1.866 \times 10^{17}\ Wb$

$= 3.73 \times 10^8\ Wb$ [1]

This flux is equal to the flux in the polar cap where the magnetic field is $B_i = 5 \times 10^{-5}\ T$

$$F_{pc} = A_{pc} B_i$$

$$\therefore A_{pc} = \frac{3.73 \times 10^8}{5 \times 10^{-5}}$$

Area of polar cap $= 7.46 \times 10^{12}\ m^2$ [1]

Similar problem done in class

B1 d) $V_{sw} = 650$ km/s in $-X$ direction

$$\underline{V}_{sw} = -6.5 \times 10^5 \, \underline{i} \quad m/s$$

$$\underline{B} = -2 \times 10^{-9} \, \underline{k} \quad T$$

$$\underline{E} = -\underline{V}_{sw} \times \underline{B} \quad \text{in Earth's frame}$$

$$= - \begin{vmatrix} \underline{i} & \underline{j} & \underline{k} \\ V_x & 0 & 0 \\ 0 & 0 & B \end{vmatrix}$$

$$= V_x B \, \underline{j}$$

$$= 6.5 \times 10^5 \times 2 \times 10^{-9} \; V/m \; \underline{j}$$

$$= 1.3 \times 10^{-3} \; V/m \; \underline{j}$$

i.e. in $+y$ direction [2]

Voltage across Stern gap

$$V_{sg} = E_y \Delta Y$$

$$= 1.3 \times 10^{-3} \times 7.5 \times 6378 \times 10^3$$
$$= 62 \text{ kV}$$ [1]

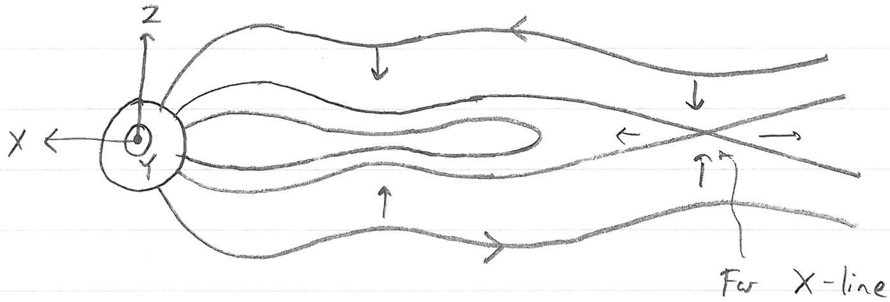
Voltage across whole magnetosphere $= 1.3 \times 10^{-3} \times 60 \times 6378 \times 10^3 \, V$
$= 497$ kV

$\boxed{\text{Similar problem done in class}}$

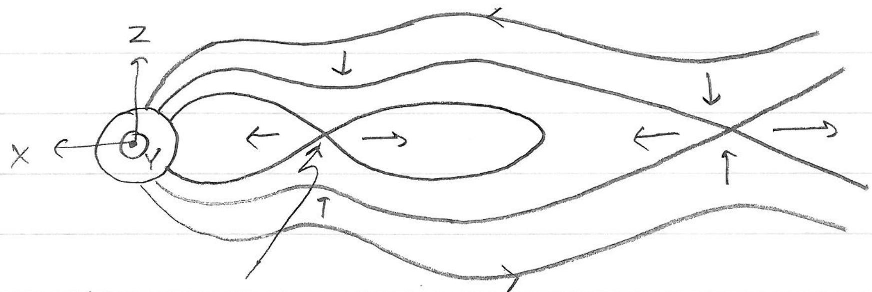
Reconnection efficiency $= \dfrac{62}{497} = 0.125 = 12.5\%$

[1]

B1 e)

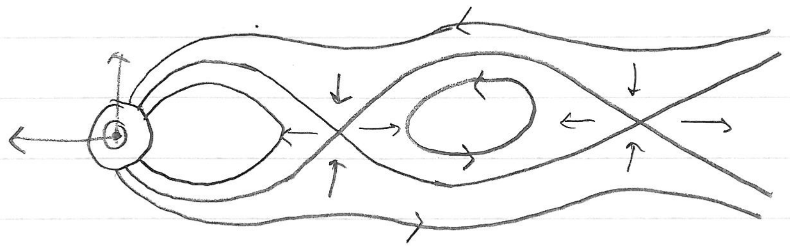


[1]



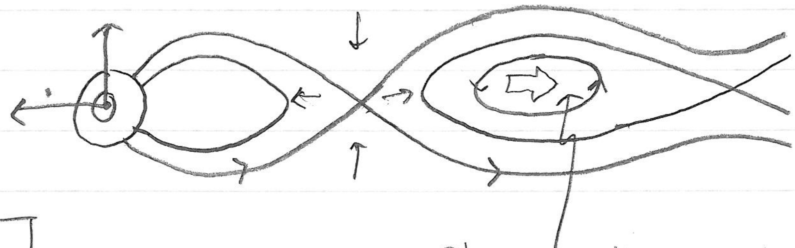
[1]

Near Earth Neutral Line
(NENL) forms, reconnecting closed field lines



[1]

NENL starts reconnecting open field lines



[1]

Bookwork

Plasmoid is "bubble" of plasma
disconnected from the Earth

After formation, the plasmoid is carried away
by the solar wind flow.

[1]

B2 a) Direction towards sun:

The stand-off distance of a magnetosphere towards the Sun is set by pressure balance between the dynamic pressure of the solar wind and the magnetic pressure inside the magnetosphere.

Dynamic pressure $= m_{sw} n_{sw} V_{sw}^2$ so depends on the mean solar wind ion mass, the solar wind number density and the solar wind velocity.

Magnetic pressure $P_m = \frac{B_m^2}{2\mu_0}$

so depends on the dipole field of the planet

[2]

Radius of cross-section:

The radius of cross-section is determined by pressure balance between the tail field and the static pressure of the solar wind, which is the sum of the thermal and magnetic pressures. The tail field depends on the total open flux.

Static pressure is $\frac{B_{sw}^2}{2\mu_0} + N_{sw} K T_{sw}$ and so depends on the solar wind number density and temperature and IMF strength.

Inside tail magnetic pressure $P_{TL} = \frac{B_{TL}^2}{2\mu_0}$

[2]

Bookwork

B2 b) The length of the connected tail in the anti-sunward direction depends on the solar wind speed and how long open field lines remain open. A key factor is how far down the tail the cross tail current sheet reconnection X line is.

[2]

The length of the disconnected tail beyond the X line depends on the solar wind speed and the Alfvén speed at which the field line kink straightens.

[1]

Bookwork

c) In solar wind bulk flow of ions dominates so neglect magnetic field pressure and thermal pressure:

$$P_{sw} = N_{sw} m_{sw} V_{sw}^2$$

Assume dipole field for the Earth with

$$B_m = \frac{M_E}{r^3}\left(3\sin^2\lambda + 1\right)^{1/2}$$

where M_E is the magnetic moment of Earth

At nose of magnetosphere $\theta = 0$, so

$$B_m = \frac{M_E}{r^3}$$ [1]

In magnetosphere, magnetic field energy density or pressure dominates

$$P_m = \frac{B_m^2}{2\mu_0}$$

At equilibrium pressures balance at nose so

$$KP_{sw} = P_m$$

where the constant K accounts for the blunt nose factor (nose is not infinitely large planar obstacle so particles not perfectly reflected) and the deviation of the magnetic field from its dipolar value due to currents in the magnetosheath. Assume $K=2$. [1]

(other values ok)

B2 c) cont. $$kN_{sw} m_{sw} V_{sw}^2 = \frac{B_m^2}{2\mu_0} = \frac{M_E^2}{2r^6\mu_0}$$ [1]

For satellites in geostationary orbit to encounter magnetosheath, stand-off distance $r = 6.6 R_E$. [1]

Assume solar wind pressure due to protons only. [1]

$$V_{sw}^2 = \frac{M_E^2}{2kN_{sw} m_{sw} r^6 \mu_0}$$

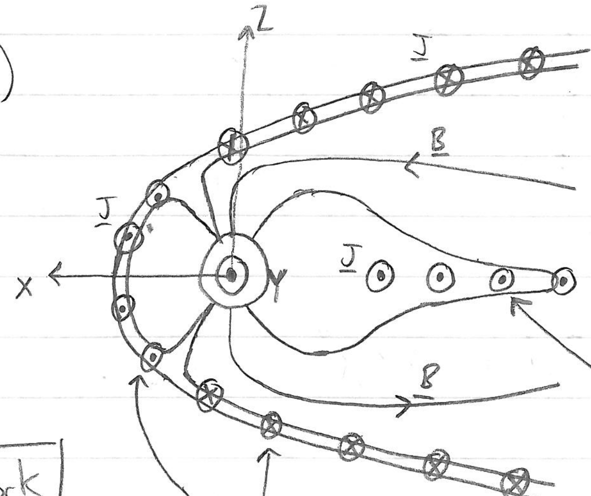
For stand off distance $= 6.6 R_E$, $$V_{sw} = \frac{7.8\times10^{15}}{(6.6\times 6370\times10^3)^3}\sqrt{\frac{1}{2\times2\times10^6\times1.67\times10^{-27}\times4\pi\times10^{-7}}}$$

$$= 1.15\times10^6 \text{ m/s}$$

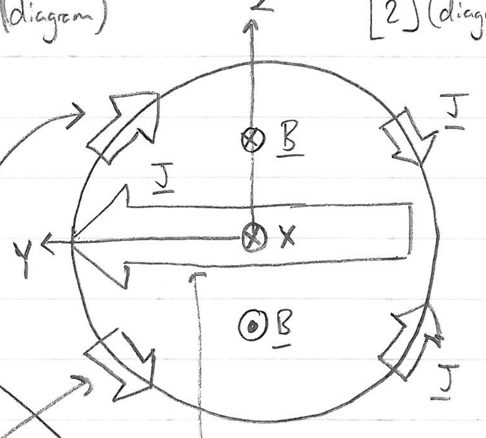
$$= \underline{1145 \text{ km/s}}$$ [2]

Similar problem done in lecture

d) [2] (diagram)



[2] (diagram)



Bookwork

Chapman-Ferraro (magnetopause) currents

Cross-tail current sheet

[2]

B3 a) Boltzmann distribution

$$n_e = n_0 e^{-q\phi/kT}$$ [1]

Perturbation caused by introducing q_t means electrons move (assume ions don't, $n_i = n_0$) [1]

Polarisation electric field is established, which is given by Gauss's law [1]

b) $$\underline{\nabla} . \underline{E} = \frac{\rho}{\varepsilon_0} = \frac{q}{\varepsilon_0}(n_i - n_e)$$

or Poisson's equation

$$\underline{E} = -\underline{\nabla}\phi$$

$$\therefore \nabla^2\phi = \frac{-q}{\varepsilon_0}(n_i - n_e)$$ [2]

Since ions don't have time to move, $n_i = n_0$

In plasma can assume that $kT \gg q\phi$ [1]
(high temperature)

$$n_e = n_0\left(1 + \frac{q\phi}{kT} + \ldots\right)$$

$$\therefore \nabla^2\phi = \frac{-q}{\varepsilon_0}\left(n_0 - n_0 - n_0\frac{q\phi}{kT}\right)$$

$$\therefore \nabla^2\phi = \frac{\phi n_0 q^2}{\varepsilon_0 kT}$$ Q.E.D. [2]

Problem done in lectures

B3 c) Given solutions in spherical coords

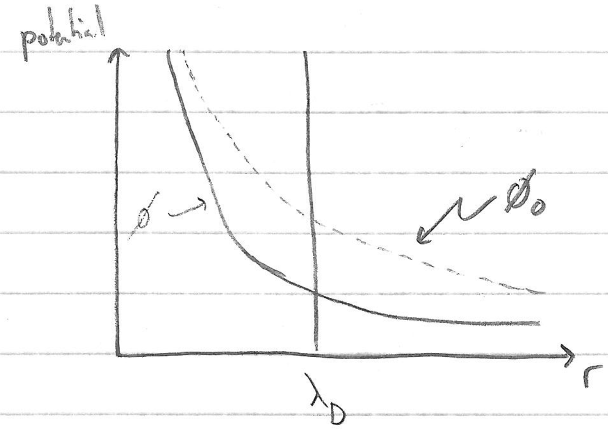
$$\phi = \phi_0 e^{\pm r/\lambda_D}$$

At $r = \infty$, $\phi \to 0$ and no applied field to disturb equilibrium balance between ions and electrons, so choose dying exponential e^{-r/λ_D}

[2]

At $r \sim 0$ no shielding so should have Coulomb potential of point charge

$$\therefore \phi_0 = \frac{q}{4\pi\varepsilon_0 r}$$



[2]

λ_D is shielding distance over which the Coulomb potential is killed off by the polarisation of the plasma.

[2]

$\boxed{\text{Done in lectures}}$

d) $$\lambda_D = \sqrt{\frac{\varepsilon_0 kT}{n_0 e^2}} = \sqrt{\frac{8.85\times10^{-12} \times 1.38\times10^{-23} \times 5\times10^6}{10^4 \times 1.6\times10^{-19} \times 1.6\times10^{-19}}}$$

$= 1544\ m$ [1]

Much smaller than length scale of system so first plasma criterion satisfied [1]

$\boxed{\text{Problem sheet}}$

B3 e) $n_0 \lambda_D^3 \gg 1$

$$n_0 \left(\frac{\varepsilon_0 kT}{n_0 q^2} \right)^{3/2} \gg 1$$

$$\frac{\varepsilon_0 kT}{n_0^{1/3} q^2} \gg 1 \qquad [1]$$

Mean potential energy of particle due to nearest neighbour is inversely proportional to mean inter-particle distance, thus $\propto n_0^{1/3}$ [1]

Kinetic energy is proportional to kT [1]

$\therefore$ 2nd plasma criterion is equivalent to

mean kinetic energy $\gg$ mean potential energy

So particles need enough kinetic energy to be free. [1]

Done in lectures