UNIVERSITY OF SOUTHAMPTON PHYS6005W1

SEMESTER 1 EXAMINATION 2012/13

Cosmology

Duration: 120 MINS

*Answer **all** questions in **Section A**,*

*one **and only one** question in **Section B***

*and one **and only one** question in **Section C***

Each Section carries 1/3 of the total marks for the exam paper and you should aim to spend about 40 mins on each.

A Sheet of Physical Constants will be provided with this examination paper.

An outline marking scheme is shown in brackets to the right of each question.

Only university approved calculators may be used.

The default system of units is SI. You may give expressions in natural units, but you should state when you start to use them and note when you change unit systems. Throughout the paper the scale factor is normalized in such a way that at the present time $a_0 = 1$.

Section A

A1. Consider Friedmann cosmological models. Write down the equation relating the proper distance d_{pr} to the comoving distance $d_{(0)}$ of an astronomical object at rest in the comoving system. Using this equation, derive the relation between the proper velocity $v_{\text{pr}}(t)$, the Hubble parameter $H(t)$ and the proper distance. How is the Hubble parameter defined in terms of the scale factor $a(t)$? What is the name of this famous relation? [5]

A2. Consider an empty Universe within Friedmann cosmology. Is it an open, flat or closed Universe? Motivate your answer. Using the value of Hubble's constant $H_0 = 70\,\text{km}\,\text{s}^{-1}\,\text{Mpc}^{-1}$ and using $1\,\text{Mpc} = 3 \times 10^{19}\,\text{Km}$, calculate the age of the Universe, properly defined, in years (2 significant figures). [5]

A3. What is the definition of redshift of an astronomical source in terms of λ_{em} and λ_0, respectively the emitted and the observed wavelength associated to some characteristic spectral line? Consider Friedmann cosmological models. How can the cosmological redshift be calculated in terms of the scale factor $a(t)$? [3]

A4. Explain what is meant by Big Bang Nucleosynthesis (1 sentence). Consider T_{in}, the initial (and highest) temperature during the expansion of the Universe within the ΛCDM model. In order to reproduce correctly the measured primordial abundances, $k_B\,T_{\text{in}}$ has to be assumed to be much above a certain value. What is this value (1 significant figure) and what is its physical significance? [3]

A5. Consider an Inflationary stage described by a de Sitter expansion between an initial time $t_{\rm i}$ and a final time $t_{\rm f}$.

- Write down the expression $a(t)$ for the time evolution of scale factor between $t_{\rm i}$ and $t_{\rm f}$.

- How is the number of e-folds N defined?

- What is the value of the ratio between the final value, $H_{\rm f} \equiv H(t_{\rm f})$, and the initial value, $H_{\rm i} \equiv H(t_{\rm i})$, of the Hubble parameter?

[4]

Section B

B1. Consider one-fluid Friedmann cosmological models with an equation of state $p = w\,\varepsilon$ and $w = \mathrm{const} > -1$.

(a) Write down the cosmological fluid equation in any equivalent form you prefer. Show that the energy density ε scales with the scale factor as

$$\varepsilon = \frac{\varepsilon_0}{a^{3\,(1+w)}}\,.$$ [4]

(b) Show that the Friedmann equation for a flat Universe can be written, in terms of the Hubble constant, as

$$H^2 = H_0^2\,\frac{\varepsilon(t)}{\varepsilon_0}\,.$$ [4]

(c) In the case of a flat Universe, derive the expression for the evolution of the scale factor with time, $a(t)$, and for the age of the Universe, t_0, properly defined, in terms of the Hubble constant. [6]

(d) Show that, for a flat Universe, the energy density scales with time as

$$\varepsilon(t) = \varepsilon_0\,\left(\frac{t_0}{t}\right)^2\,,$$

independently of the value of w. [3]

(e) Consider now multi-fluid models with $\varepsilon = \sum_{i=1,N}\varepsilon_i$, $p = \sum_{i=1,N} p_i$ and $p_i = w_i\,\varepsilon_i$ with $w_i = \mathrm{const}$ and $w_1 < w_2 < \ldots < w_N$. Write down the expression for the dependence of the energy density with the scale factor $\varepsilon(a)$. What is the asymptotic limit for $a \to 0$? What is the asymptotic limit for $a \to \infty$? [3]

B2. Consider Friedmann cosmological models.

(a) Write down the Friedmann equation relating the Hubble parameter to the energy density $\varepsilon(t)$ and to the curvature parameter k. How is the critical energy density $\varepsilon_c(t)$ defined? How is the energy density parameter $\Omega(t)$ defined? [4]

(b) Consider the acceleration equation

$$\frac{\ddot{a}}{a} = -\frac{4\pi G}{3c^2}(\varepsilon + 3p) .$$

Derive the fluid equation from the Friedmann equation and from the acceleration equation. [5]

(c) Show that for a multi-fluid model with $\varepsilon = \sum_i \varepsilon_i$ and $p = \sum_i p_i$, with $p_i = w_i \varepsilon_i$ and $w_i = \text{const}$, this can be recast, at the present time, as

$$\ddot{a}_0 = -\frac{1}{2} H_0^2 \sum_i \Omega_{i,0} (1 + 3w_i) ,$$

where $\Omega_{i,0} = \varepsilon_{i,0}/\varepsilon_{c,0}$ is the energy density parameter of the i^{th} fluid at the present time. [3]

(d) How is the deceleration parameter at the present time, q_0, defined? [2]

(e) Find an expression for the deceleration parameter in the ΛCDM model in terms of the energy density parameters $\Omega_{i,0} = \varepsilon_{i,0}/\varepsilon_{c,0}$. Estimate the value of q_0 in the ΛCDM model (1 significant figure) using $\Omega_{M,0} \simeq 0.265$. Consider now the deceleration parameter $q(t)$ at an arbitrary time t. Assuming that the ΛCDM model continues to hold in future indefinitely, what is the asymptotic value of $q(t)$ for $t \to \infty$? [6]

Section C

C1. (a) Using a value for the radiation energy density parameter at the present time $\Omega_{R,0} \simeq 0.75 \times 10^{-4}$, for the matter energy density parameter $\Omega_{M,0} \simeq 0.265$ and for the Boltzmann constant $k_B = 0.86 \times 10^{-4}\,\mathrm{eV\,K^{-1}}$, determine (1 significant digit) the value $k_B\,T_{\mathrm{eq}}^{RM}$, where T_{eq}^{RM} is the value of the temperature at the matter-radiation equality time. [3]

(b) List the three main observational features of the Cosmic Microwave Background Radiation (CMBR) spectrum. [5]

(c) Consider the angular power spectrum of the CMBR anisotropies defined as the set of the multipole moments C_l. How is the multipole number l related to a certain angular scale θ of the temperature fluctuations? What is the angular scale corresponding to the dipole? Assuming a flat Universe, estimate the value of l_1, the multipole number corresponding to the position of the so called 'first peak' in the angular power spectrum.

Note: Use $z_{\mathrm{dec}} = 1100$ as a value for the redshift at the time of photon decoupling and $c_s = c/\sqrt{3}$ for the speed of sound. [5]

(d) What is the main particle reaction coupling photons to matter during the early Universe prior the period of recombination? [2]

(e) What is meant by 'recombination' and what is the most relevant particle process? How is the recombination temperature $k_B\,T_{\mathrm{rec}} \simeq 0.32\,\mathrm{eV}$ defined? Using $t_0 = 13.7\,\mathrm{Gyr}$ for the current age of the Universe and $k_B\,T_0 = 2.35 \times 10^{-4}\,\mathrm{eV}$ for the CMBR temperature, estimate the age of the Universe at the recombination time neglecting the cosmological constant. [5]

C2. (a) Explain why in our Galaxy, at distances R from the Galactic centre much bigger than $\sim 4\,\mathrm{Kpc}$, the observation that $v(R) = \mathrm{const}$ is evidence for the presence of Dark Matter. [5]

(b) State the virial theorem and discuss how it can be used to estimate the total mass of clusters of galaxies in terms of their mean squared velocity and size. [6]

(c) Discuss how, by combining

- the evidence of a flat Universe from the observation of Cosmic Microwave Background (CMB) anisotropies,
- the results on the cosmological redshifts from the Supernovae type Ia data, which found $\Omega_{\Lambda,0} \simeq 1.6\,\Omega_{M,0} + 0.3$,
- the measurement of the baryonic contribution to the energy density in terms of the parameter $\Omega_{B,0}\,h^2$, both from CMB anisotropies and from Big Bang Nucleosynthesis,
- the measurement of the Hubble constant from the Hubble Space Telescope,

it has been possible to infer indirectly the existence of a dominant non-baryonic component of Dark Matter. (You may assume that the radiation component gives a negligible contribution to the total energy density of the Universe at the present time and that the acceleration of the Universe can be described by a cosmological constant-like fluid.) [6]

(d) Suppose that the Dark Matter is made of a new elementary particle of mass $M_{DM} = 1\,\mathrm{GeV}$. Calculate (order-of-magnitude) how many DM particles per cubic meter would be necessary to explain the measurement $\Omega_{DM,0}\, h^2 \sim 0.1$ using $\varepsilon_{\rm c,0}\, h^{-2} \sim 10\,\mathrm{GeV\,m^{-3}}$, where $\varepsilon_{\rm c,0}$ is the critical energy density at the present time. [3]

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