SEMESTER 2 EXAMINATION 2013-2014

PARTICLE PHYSICS

Duration: 120 MINS (2 hours)

This paper contains 9 questions.

Answer **all** questions in **Section A** and **only two** questions in **Section B**.

Section A carries 1/3 of the total marks for the exam paper and you should aim to spend about 40 mins on it.

Section B carries 2/3 of the total marks for the exam paper and you should aim to spend about 80 mins on it.

An outline marking scheme is shown in brackets to the right of each question.

A Sheet of Physical Constants is provided with this examination paper.

Only university approved calculators may be used.

A foreign language translation dictionary (paper version) is permitted provided it contains no notes, additions or annotations.

Section A

A1. For a free relativistic particle, the energy is given by the equation:

$$E^2 = |\vec{p}|^2 + m^2$$

where $\vec{p}$ is the 3-momentum and m the mass of the particle. By using the correspondence between 4-momentum, $p^\mu = (E, \vec{p})$, and differential operator in position space:

$$p^\mu \to i\partial^\mu,$$

derive the Klein-Gordon equation for the free particle in position space. Write down its plane wave solutions. State the two main problems arising when one tries to interpret that elementary solution as a wave function. [5]

A2. What is the spin of a relativistic particle that is described by the Dirac equation? Give your reasons by comparing it to a particle described by the Klein-Gordon equation. [2]

A3. Draw the four Feynman diagrams describing the following scattering process at lowest order:

$$e^+(s_1, p_1)\ \ e^-(s_2, p_2) \to e^+(s_3, p_3)\ \ e^-(s_4, p_4)$$

where s_i and p_i ($i = 1, 2, 3, 4$) label the spin and the 4-momentum, respectively. Describe which interactions/forces produce this scattering. [4]

A4. Explain the concept of renormalization in Quantum Electrodynamics, and draw the main loop diagrams that visualize it. [5]

A5. What is asymptotic freedom in QCD? How is this property of strong interactions related to the fact that we do not observe free quarks and gluons? [4]

Section B

B1. The charged and neutral weak gauge bosons, $W^{\pm}$ and Z, acquire mass via the Higgs mechanism. This happens because the electroweak gauge symmetry is broken by the vacuum of the theory. To do this, an SU(2) doublet scalar field

$$\Phi = \begin{pmatrix} \phi_1 + i\phi_2 \\ \phi_3 + i\phi_4 \end{pmatrix}$$

is introduced with potential

$$V(\Phi) = -\frac{\mu^2}{2}|\Phi|^2 + \frac{\lambda}{4}|\Phi|^4.$$

Without loss of generality, we can look at the vacuum where $\Phi = \begin{pmatrix} v + i\eta \\ 0 \end{pmatrix}$, where v and η do not depend on the space-time coordinates.

(a) If μ^2 is negative, what does the potential look like? Sketch a diagram of V as a function of v and η. For what value of Φ is the potential V a minimum? [3]

(b) How does the situation change if μ^2 is positive? As before, sketch a diagram of V as a function of v and η. For what values of Φ is the potential V minimized? How many minima are there and what is the value of the potential at those points? Explain why the potential $V(\Phi)$ exhibits the electroweak gauge symmetry, while the chosen ground state does not. [7]

(c) Weak interactions are described by an SU(2) gauge theory. How many generators are there? Give a matrix representation for them. State Goldstone's theorem, and discuss its consequences for the weak interactions. List the gauge bosons associated with the generators of the SU(2) gauge group and describe how they get a mass via the Higgs mechanism. [6]

(d) Write down the three main processes used to discover the Higgs boson at the CERN Large Hadron Collider (LHC). Draw the Feynman diagrams corresponding to the Higgs signal. [4]

TURN OVER

B2. Consider the scattering process

$$e^-(s_1, p_1)\ \ e^+(s_2, p_2) \to \mu^-(s_3, p_3)\ \ \mu^+(s_4, p_4)$$

where s_i and p_i ($i = 1, 2, 3, 4$) label the spin and 4-momentum of the fermions involved. Suppose that the two initial particles collide head-on, each one having an energy equal to 5 GeV: $E(e^-) = E(e^+) = 5$ GeV. The scattering is mediated by two forces, one of which is electromagnetism.

(a) Draw the Feynman diagrams which contribute to this process, and list the particles which are the mediators of the interaction. As more than one force is involved, discuss their relative strength for the given process. [5]

(b) Taking into account only the electromagnetic interaction in the Feynman diagrams above, label all incoming and outgoing particles by making use of the Feynman rules for the corresponding spinors:

$$u_r(p), \bar{u}_r(p), v_r(p), \bar{v}_r(p)$$

with $r = 1, 2$, giving your reasons. Explain the meaning of the r index. In addition, write down the Feynman rules for each interaction vertex in the diagrams. Finally, give the expression of the propagators. [4]

(c) Give the definition of the Mandelstam variables s, t, u in terms of the corresponding 4-momenta and show that they obey the following rule:

$$s + t + u = 2m_e^2 + 2m_\mu^2$$

for the given process. Explain why, in general, cross sections are given in terms of these variables. In particular, explain the meaning of the s variable. [6]

(d) Neglecting the mass of the fermions and considering only the electromagnetic interaction, the differential cross section in the solid angle for the above process has the following expression:

$$\frac{d\sigma}{d\Omega} = \frac{1}{64\pi^2 s} 2e^4 \left[\frac{1}{2}(1 + \cos^2\theta)\right]$$

where e is the electron charge, s is the Mandelstam variable, and θ is the scattering angle of the outgoing muon, μ^-, relative to the incoming electron. Compute the total cross section, and give its expression in terms of the electromagnetic alpha: α_{em}. [5]

TURN OVER

B3. Quarks are described by three component colour wave functions, which are solutions of the Dirac equation:

$$(i\gamma^{\mu}\partial_{\mu} - m)\psi = 0$$

where m is the mass of the quark, γ_{μ} are the four γ-matrices and $\partial_{\mu} = (\partial_0, \vec{\nabla})$. We require that the Dirac equation is invariant under the generalized gauge transformation:

$$\psi \to \psi' = e^{ig_s\theta^a(x)T^a}\psi$$

where g_s is the strong force's coupling, $\theta_a(x)$ are parameters which depend on the 4-vector $x_{\mu} = (t, \vec{x})$, and T^a are the Gell-Mann matrices.

(a) Assuming that the $\theta^a(x)$ parameters are infinitesimal and Taylor expanding the above transformation up to $O(g_s)$, show that the Dirac equation is not invariant under such a transformation. We now introduce in the Dirac equation the following covariant derivative:

$$\partial_{\mu} \to D_{\mu} = \partial_{\mu} + ig_sG^a_{\mu}T^a$$

where G^a_{μ} are the gluon wave functions. Show how the gluon wave function should simultaneously transform in order to leave the modified Dirac equation invariant under the generalized gauge transformations. [6]

(b) Draw the Feynman diagrams for the process

$$u\bar{u} \to c\bar{c}$$

where u and c are the *up* and *charm*-quark, respectively, neglecting electromagnetic and weak forces. Specify the Feynman rules involved for external particles, propagators and vertices. [6]

(c) Suppose that the exchanged gluon, mediator of the strong force, is

$$G^a_{\mu} = G^5_{\mu}$$

which is associated with the Gell-Mann matrix

$$T^5 = \frac{1}{2}\begin{pmatrix} 0 & 0 & 1 \\ 0 & 0 & 0 \\ 1 & 0 & 0 \end{pmatrix}.$$

How does G^5_μ act on the quark wave function ψ expressed as a 3-component vector in colour space? Label the spinors in the Feynman diagram accordingly, specifying the colour index. Is the gluon coloured? [5]

(d) If gluons are coloured, could they interact directly with each other? Discuss the difference in this respect between gluons and photons. [3]

TURN OVER

B4. The Dirac equation describes relativistic fermions.

(a) Starting from the Dirac Hamiltonian

$$H_D = \vec{\alpha} \cdot \vec{p} + \beta m$$

where $\vec{\alpha} = (\alpha_1, \alpha_2, \alpha_3)$ and β are matrices, $\vec{p}$ is the 3-momentum, and m the mass of the particle, show that the Dirac equation has the following expression in position space:

$$i\frac{\partial\psi}{\partial t} = (-i\vec{\alpha} \cdot \vec{\nabla} + \beta m)\psi.$$

The free particle described by the wave function ψ must satisfy the correct relativistic energy-momentum relation. Show that this requirement imposes the following constraints on the $\vec{\alpha}$ and β matrices:

$$\alpha_i\alpha_j + \alpha_j\alpha_i = 2\delta_{ij}$$

$$\beta\alpha_i + \alpha_i\beta = 0$$

$$\beta^2 = 1$$

for $i, j = 1, 2, 3$. By making use of the γ-matrices:

$$\gamma^0 = \beta; \qquad \vec{\gamma} = \beta\vec{\alpha}$$

show that the Dirac equation becomes:

$$(i\gamma^\mu\partial_\mu - m)\psi = 0.$$ [7]

(b) We define the 4-vector current $J_\mu = (\rho, \vec{J})$ to be:

$$\rho = \psi^\dagger\psi; \qquad \vec{J} = \psi^\dagger\vec{\alpha}\psi.$$

Show that it satisfies the continuity equation

$$\partial_\mu J^\mu = 0.$$ [3]

(c) The Dirac equation admits plane wave solutions of the form:

$$\psi = \begin{pmatrix} \chi(\vec{p}) \\ \phi(\vec{p}) \end{pmatrix} e^{-i(Et-\vec{p}\cdot\vec{x})}$$

where $\chi(\vec{p})$ and $\phi(\vec{p})$ are two-component spinors. Using the Dirac representation of the matrices:

$$\vec{\alpha} = \begin{pmatrix} 0 & \vec{\sigma} \\ \vec{\sigma} & 0 \end{pmatrix}, \qquad \beta = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$$

where each entry denotes a two-by-two block and 1 denotes the two-by-two identity matrix, and assuming that the relativistic particle is at rest, show that both positive and negative energy solutions exist. [6]

(d) What was the Dirac interpretation of the negative energy states? Compare the Dirac interpretation with the Feynman-Stueckelberg picture, which was adopted from then on. Sketch a diagram to illustrate a physical process where the Feynman-Stueckelberg picture can be visualized. [4]

END OF PAPER