SECTION A

A1 Free solutions are of form $\phi = N e^{-i(Et - \vec{p}\cdot\vec{x})}$ [1]

Substituting into KG eq'n we recover

$$(E^2 - p^2 + m^2)\,\phi^2 = 0 \qquad [1]$$

$$\Rightarrow \quad E = \pm\sqrt{p^2 - m^2}$$

Negative E solutions are unphysical & a problem [1]

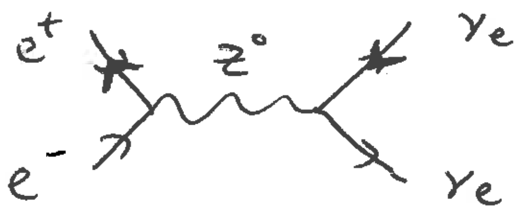
Also prob. density can be −ve [1]

$$\rho = i(\phi^* \partial^0 \phi - \phi\, \partial^0 \phi^*)$$

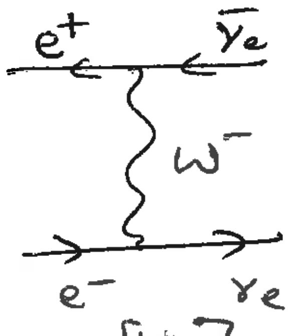
evaluate on solution $\rho = 2|N|^2 E$ [1]

can be −ve

A2



[1½]



[1½]

W & Z are mediators of the weak force [1]

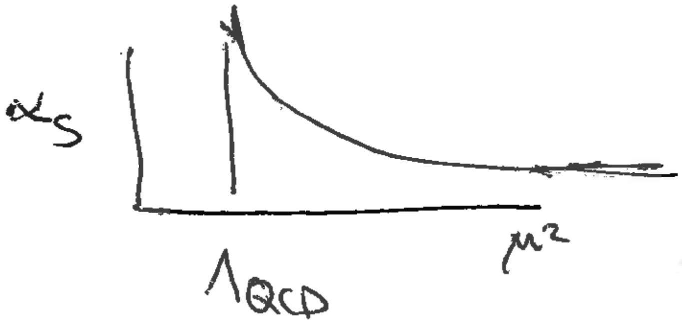
A3 In perturbation theory ∞ ~~diagrams~~ contributions to scattering processes ~~[illegible]~~ result. These ∞ can be removed by absorbing them into constants [2] ~~of theory~~

QED  [2]

In QED e, m_e, N_γ, N_e are renormalised [1]

charge e^- mass normalisations

A4] The strong interactions are asymptotically free [1]

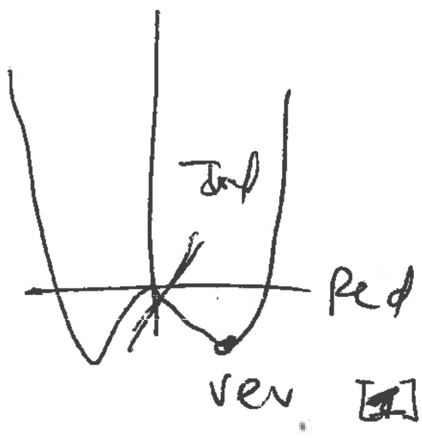


[1]

As quarks are separated the force grows until restoring force wins – quarks can not be freed [1]

A5] The higgs mechanism is a spontaneous symmetry breaking process [1]

$$V = -\mu^2|\phi|^2 + \lambda|\phi|^4$$



Particles can acquire mass, [2]
proportional to vev, without breaking [1]
gauge invariance.

B1

Dirac : $i\frac{\partial\psi}{\partial t} = \hat{H}_D\psi = (-i\vec{\alpha}.\vec{\nabla} + \beta m)\psi$ [1]

multiply both sides by γ^0:

$$i\gamma^0\partial^0\psi = (-i\vec{\alpha}\gamma^0\vec{\nabla} + \gamma^{0^2}m)\psi$$ [1]

$$i\gamma_0\partial^0\psi = (-i\gamma^i\nabla^i + m)\psi = (-i\gamma_i\partial^i + m)\psi$$ [2]

$$(i\gamma_\mu\partial^\mu - m)\psi = 0 = (i\not{\partial} - m)\psi$$ [1]

TOT = 5

b) $\hat{S}^2\psi = \lambda\psi = \left(\frac{\vec{\Sigma}}{2}\right)^2\psi$ [1]

$$\left(\frac{\vec{\Sigma}}{2}\right)^2\psi = \frac{1}{4}\begin{pmatrix}\vec{\sigma} & 0\\ 0 & \vec{\sigma}\end{pmatrix}\begin{pmatrix}\vec{\sigma} & 0\\ 0 & \vec{\sigma}\end{pmatrix}\psi = \frac{1}{4}\begin{pmatrix}\vec{\sigma}^2 & 0\\ 0 & \vec{\sigma}^2\end{pmatrix}\psi$$ [1]

$$\vec{\sigma}.\vec{\sigma} = \vec{\sigma}^2 = \sum_{i=1}^{3}\sigma_i^2 = 3I$$ [2]

$$\hat{S}^2\psi = \frac{3}{4}I\psi = \frac{3}{4}\psi$$ [1]

TOT = 5

c) From QM: $\hat{J}^2|J,m\rangle = J(J+1)|J,m\rangle$ [2]

$$\hat{S}^2\psi = \frac{3}{4}\psi \overset{[1]}{=} \frac{3}{4}|s,s_z\rangle \overset{[1]}{=} s(s+1)|s,s_z\rangle$$ [redacted]

$\frac{3}{4} = s(s+1)$ and $s = \frac{1}{2}$ solving it. [1]

TOT = 5

d) $\vec{p} = (0, 0, p_3)$ [1/2]

$$\frac{\Sigma_3}{2} u_2(p) \overset{[1]}{=} \hat{S}_3 u_2(p) \overset{[1]}{=} \frac{1}{2} \begin{pmatrix} \sigma_3 & 0 \\ 0 & \sigma_3 \end{pmatrix} \begin{pmatrix} \chi_2 \\ \frac{\sigma_3 p_3}{E+m} \chi_2 \end{pmatrix} \sqrt{E+m}$$

$$S_3 u_2(p) = \frac{1}{2} \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & -1 \end{pmatrix} \begin{pmatrix} 0 \\ \sqrt{E+m} \\ 0 \\ \frac{-p_3}{\sqrt{E+m}} \end{pmatrix}$$ [1]

$$S_3 u_2(p_2) \overset{[1]}{=} \frac{1}{2} \begin{pmatrix} 0 \\ -\sqrt{E+m} \\ 0 \\ \frac{p_3}{\sqrt{E+m}} \end{pmatrix} \overset{[1/2]}{=} -\frac{1}{2} u_2(p)$$

Tot = 5

B2

a) i $\vec{\nabla}\cdot\vec{E} = \rho$ is the Gauss's law. 1/2

ii $\vec{\nabla}\cdot\vec{B} = 0$ expresses absence of monopoles 1/2

iii $\vec{\nabla}\times\vec{E} = -\frac{\partial\vec{B}}{\partial t}$ is the Faraday's law 1/2

vi $\vec{\nabla}\times\vec{B} = \vec{J} + \frac{\partial\vec{E}}{\partial t}$ is the ~~[illegible]~~ Ampere law (modified) 1/2

~~[illegible]~~ =

1/2

~~[illegible]~~

1° Eq: $\vec{\nabla}\cdot\left(-\frac{\partial\vec{A}}{\partial t} - \vec{\nabla}\phi\right) = \rho = -\frac{\partial}{\partial t}(\vec{\nabla}\cdot\vec{A}) - \nabla^2\phi$ [1]

4° Eq: $\vec{\nabla}\times(\vec{\nabla}\times\vec{A}) = J + \frac{\partial}{\partial t}\left(-\frac{\partial\vec{A}}{\partial t} - \vec{\nabla}\phi\right) = -\nabla^2\vec{A} + \vec{\nabla}\cdot(\vec{\nabla}\cdot\vec{A})$ [2]

i.e. $\frac{\partial^2}{\partial t^2}\vec{A} - \nabla^2\vec{A} = J - \vec{\nabla}\cdot\left(\vec{\nabla}\cdot\vec{A} + \frac{\partial\phi}{\partial t}\right)$ [1]

TOT = 6

b) $\partial_\mu A^\mu = \partial_0 A^0 + \partial_i A^i = \partial^0\phi + \vec{\nabla}\cdot\vec{A} = 0$ [1]

So: $\vec{\nabla}\cdot\vec{E} = \rho$ ~~[illegible]~~ or $-\frac{\partial}{\partial t}(\vec{\nabla}\cdot\vec{A}) - \nabla^2\phi = \rho$

~~because~~ from point (a), becomes:

$$-\frac{\partial}{\partial t}\left(-\frac{\partial\phi}{\partial t}\right) - \nabla^2\phi = \left(\frac{\partial^2}{\partial t^2} - \nabla^2\right)\phi = \Box\phi = \rho$$ [2]

Eq. 4 is: $\frac{\partial^2\vec{A}}{\partial t^2} - \nabla^2\vec{A} = \vec{J} = \Box\vec{A}$ [1]

Globally: $\Box A^\mu = J^\mu = (\rho, \vec{J})$ [1]

TOT = 5

c) In free space $\Box A^\mu = 0$ i.e. the Klein-Gordon Eq which describes integer-spin (zero included) particles. These particles are photons. [2]

[1]

~~$A^\mu = N\epsilon_\lambda^\mu e^{-ip.x}$~~ i.e.

~~photons with the polarization vector ϵ_λ^μ~~

with $S = 1$ [1]

TOT = 4

d) If we had $(\Box + m_\gamma^2)A^\mu = 0$ and performed a gauge transformation (remaining in Lorentz gauge): we would get: [1]

$$(\Box + m_\gamma^2)(A^\mu - \partial^\mu \psi) = \Box A^\mu - \partial^\mu \Box\psi + m_\gamma^2 A^\mu - m_\gamma^2 \partial^\mu \psi$$

[2]

In Lorentz gauge $\partial_\mu A'^\mu = 0 = \partial_\mu(A^\mu - \partial^\mu \psi) = -\Box\psi = 0$ [1]

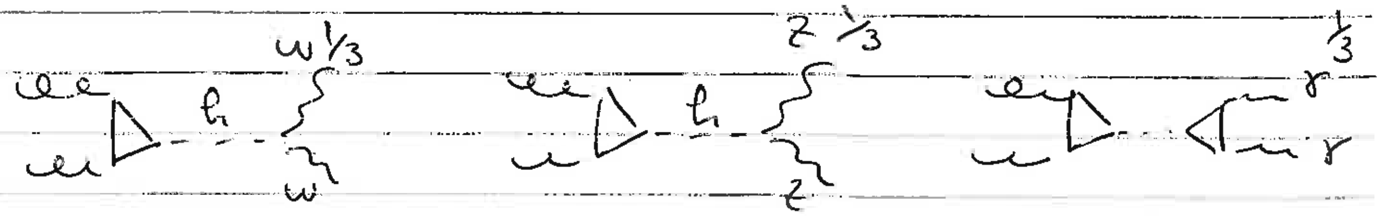
So: $(\Box + m_\gamma^2)A^\mu - m_\gamma^2 \partial^\mu \psi = (\Box + m_\gamma^2)A'^\mu$

and the term $-m_\gamma^2 \partial^\mu \psi$ violate gauge invariance [1]

TOT = 5

B3

a) $m_h \approx 125$ GeV [1], $S = 0$ [1], $Q = 0$ [1]



[1] [1] [1]

TOT = 7

b) Mass eigenvalues:

$$\det\begin{pmatrix} \frac{v^2 g_W^2}{4} - \lambda & \frac{v^2}{4} g_W g_Y \\ g_W g_Y \frac{v^2}{4} & \frac{v^2}{4} g_Y^2 - \lambda \end{pmatrix} = 0 \quad [1]$$

$$\left(\frac{g_W^2 v^2}{4} - \lambda\right)\left(\frac{v^2 g_Y^2}{4} - \lambda\right) - \frac{v^4 g_W^2 g_Y^2}{16} = 0 = \lambda^2 - \lambda\left(g_W^2 + g_Y^2\right)\frac{v^2}{4} \quad [1]$$

with solutions $\lambda = 0$ and $\lambda = \left(g_W^2 + g_Y^2\right)\frac{v^2}{4}$ [1]

the mass eigenvalues are $m_1 = 0$, $m_2 = \frac{v}{2}\sqrt{g_W^2 + g_Y^2}$ [1]

TOT = 4

c) $$U\begin{pmatrix} W_3^\mu \\ B^\mu \end{pmatrix} = \begin{pmatrix} A^\mu \\ Z^\mu \end{pmatrix} = \begin{pmatrix} g_Y & -g_W \\ g_W & g_Y \end{pmatrix}\frac{1}{\sqrt{g_Y^2 + g_W^2}}\begin{pmatrix} W_3^\mu \\ B^\mu \end{pmatrix} \quad [2]$$

$$\frac{g_Y}{\sqrt{g_Y^2 + g_W^2}} W_3^\mu - \frac{g_W}{\sqrt{g_Y^2 + g_W^2}} B^\mu = A^\mu \quad [1] \quad ; \quad \frac{g_W}{\sqrt{g_Y^2 + g_W^2}} W_3^\mu + \frac{g_Y}{\sqrt{g_Y^2 + g_W^2}} B^\mu = Z^\mu \quad [1]$$

A^μ and Z^μ are the photon and neutral Z^0 [1].

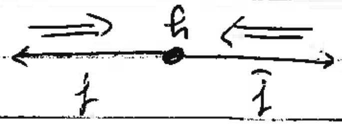
TOT = 5

d)

[1]

$g \propto \frac{m_f}{v}$ [1]

$f = f_L$ and $\bar{f} = \bar{f}_R$



[2]

TOT = 4

B4

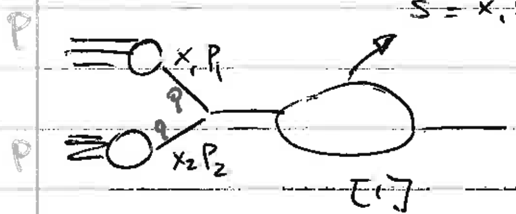
a) quarks and gluons. Quarks are divided into valence and sea quarks. [2]

Their charges are $Q_u = +\frac{2}{3}$, $Q_d = -\frac{1}{3}$, $Q_{\bar{u}} = -\frac{2}{3}$, $Q_{\bar{d}} = \frac{1}{3}$, $Q_g = 0$ [1]

PDF: probability that quarks and gluons are emitted off protons [1]

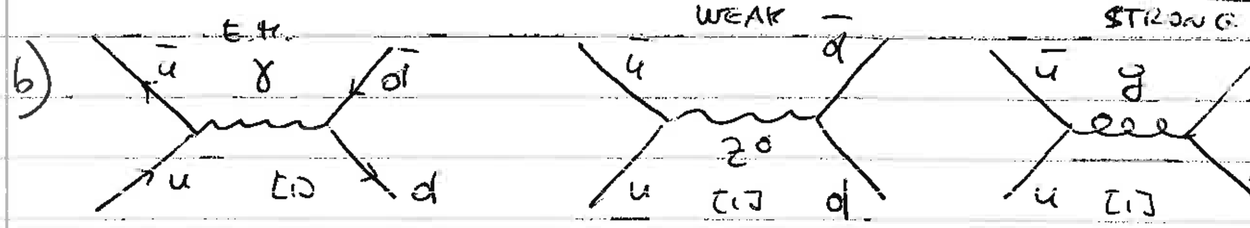
$\hat{s} = x_1 x_2 S$ or $E_{CM} = \sqrt{x_1 x_2}\, E_{collider}$

[1]

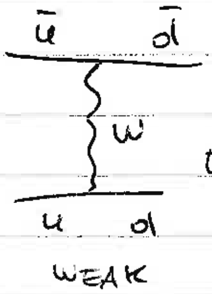


[1]

TOT = [6]



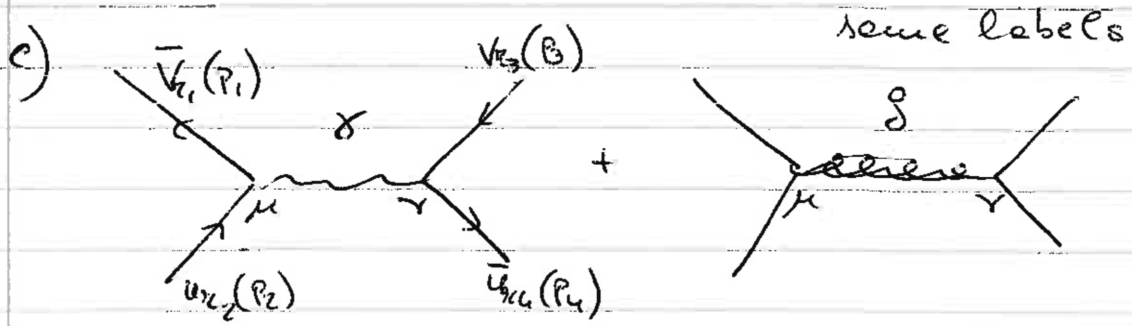
b)



[1]

TOT = [4]

c)



[1]

$u_r(p)$ = spinor for incoming particle ½

$\bar{u}_r(p)$ = " for outgoing " ½

$\bar{v}_r(p)$ = spinor for incoming antiparticle ½

$v_r(p)$ = spinor for outgoing antiparticle ½

Propagators: $\frac{-ig^{\mu\nu}}{q^2}$ [1]

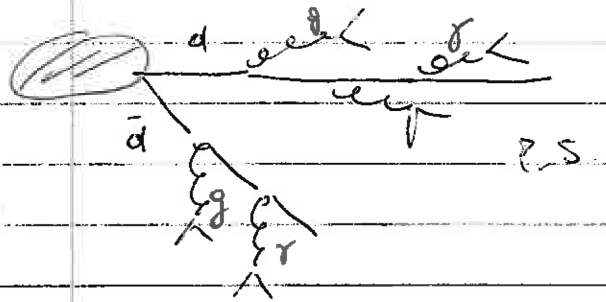
quark charge

Vertices: $-i(Q_q e)\gamma_\mu \Big|_{QED}$ and $-iT^a\gamma_\mu \Big|_{QCD}$ [2]

TOT= 6

d) Down quarks are produced asymptotically free at high energy. [1]

They will radiate via P.S. loosing energy [1]



[1]

One observes hadrons (colorless) [1]

TOT= 4