SEMESTER 2 EXAMINATION 2014-2015

PARTICLE PHYSICS

Duration: 120 MINS (2 hours)

This paper contains 9 questions.

Answer **all** questions in **Section A** and **only two** questions in **Section B**.

Section A carries 1/3 of the total marks for the exam paper and you should aim to spend about 40 mins on it.

Section B carries 2/3 of the total marks for the exam paper and you should aim to spend about 80 mins on it.

An outline marking scheme is shown in brackets to the right of each question.

A Sheet of Physical Constants is provided with this examination paper.

Only university approved calculators may be used.

A foreign language word to word® translation dictionary (paper version) is permitted provided it contains no notes, additions or annotations.

Section A

A1. A free relativistic spin-zero particle can be described by the Klein-Gordon equation

$$(\Box + m^2)\,\phi = 0$$

where m is the particle's mass and ϕ is the plane wave representing the particle's state. We recall that the expression of the d'Alembertian operator is:

$$\Box = (\partial^0)^2 - \vec{\nabla}^2$$

where ∂^0 and $\vec{\nabla}$ are the partial derivatives with respect to time and space, respectively.

Discuss the two main problems that arise when the solution, ϕ, of the Klein-Gordon equation is interpreted as a single-particle wave function.

The Klein-Gordon equation admits the following candidate for the probability density:

$$\rho = i(\phi^* \partial^0 \phi - \phi \partial^0 \phi^*).$$

Show that ρ is not positive definite. [5]

A2. Draw the Feynman diagrams describing the following scattering process at lowest order:

$$e^+(s_1, p_1)e^-(s_2, p_2) \to \bar{\nu}_e(s_3, p_3)\nu_e(s_4, p_4)$$

where s_i and p_i (i=1,2,3,4) label the spin and the 4-momentum, respectively. What is the force responsible for this interaction? [4]

A3. Briefly explain the concept of renormalization. Sketch the one-loop diagrams in QED which need to be renormalized and describe the corresponding observables, if any, which they are related to. [5]

A4. What property of the strong interactions prevents quarks and gluons from existing as free particles? Explain the underlying concept via the behaviour of the interaction strength as a function of the energy of quarks and gluons. [3]

A5. Concisely describe the Higgs mechanism and its importance in particle physics. Sketch the function representing the Higgs potential and comment on its shape and its physical consequences. [3]

TURN OVER

Section B

B1. The Dirac equation describes relativistic massive particles with half-integer spin, that is fermions.

(a) The γ matrices are defined as:

$$\gamma^0 = \beta; \qquad \vec{\gamma} = \beta\vec{\alpha}$$

where $\vec{\alpha} = (\alpha_1, \alpha_2, \alpha_3)$ and β are the four-dimensional matrices appearing in the Dirac Hamiltonian

$$\hat{H}_D = \vec{\alpha} \cdot \vec{p} + \beta m$$

with $\vec{p}$ and m being the momentum and the mass of the particle.

By using four-vector notation to express the four-dimensional energy-momentum operator in time-position space:

$$\hat{p}^\mu = (\hat{H}_D, \hat{\vec{p}}) = i\partial^\mu = i(\partial^0, -\vec{\nabla})$$

write down the Dirac equation in time-position space in terms of the γ matrices. [5]

(b) The Dirac Hamiltonian, $\hat{\mathrm{H}}_D$, commutes with the total angular momentum, $\vec{J} = \vec{L} + \vec{\Sigma}/2$, where $\vec{\Sigma}/2$ represents the intrinsic angular momentum or spin operator $\hat{S}$. The expression of the spin operator $\hat{S}$ is:

$$\hat{S} = \frac{\vec{\Sigma}}{2} = \frac{1}{2}\begin{pmatrix} \vec{\sigma} & 0 \\ 0 & \vec{\sigma} \end{pmatrix}$$

where the three components of $\vec{\sigma}$ are the two-dimensional Pauli matrices, listed here below

$$\sigma_1 = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} \qquad \sigma_2 = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix} \qquad \sigma_3 = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}.$$

Write down the eigenvalue equation for the operator $\hat{S}^2$ acting on the state ψ and show that show that its eigenvalues are degenerate and equal to 3/4. [5]

(c) Using the above result, show that the solution of the Dirac equation, ψ, describes spin-1/2 particles. [5]

(d) The Dirac equation admits plane wave solutions of the following form:

$$\psi = \frac{N}{\sqrt{E+m}} u_r(p) e^{-i(Et - \vec{p}\cdot\vec{x})}$$

where $u_r(p)$ is called a spinor and the index $r = 1, 2$. In the above mentioned formula, N is a normalisation constant and E and $\vec{p}$ are the particle's energy and momentum, respectively. The explicit expression of the $u_r(p)$ spinor is:

$$u_r(p) = \sqrt{E+m} \begin{pmatrix} \chi_r \\ \frac{\vec{\sigma}\cdot\vec{p}}{E+m}\chi_r \end{pmatrix}$$

where the χ_r (r=1, 2) are the orthonormal two-component column vectors:

$$\chi_1 = \begin{pmatrix} 1 \\ 0 \end{pmatrix}; \quad \chi_2 = \begin{pmatrix} 0 \\ 1 \end{pmatrix}.$$

Taking the momentum $\vec{p}$ directed along the $\hat{z}$-axis, show that the spinor $u_2(p)$ describes spin-down fermions. [5]

TURN OVER

B2. Consider the Maxwell's equations:

$$\vec{\nabla} \cdot \vec{E} = \rho; \qquad \vec{\nabla} \cdot \vec{B} = 0; \qquad \vec{\nabla} \times \vec{E} = -\frac{\partial \vec{B}}{\partial t}; \qquad \vec{\nabla} \times \vec{B} = \vec{J} + \frac{\partial \vec{E}}{\partial t}$$

where $\vec{E}$ and $\vec{B}$ are the electric and magnetic fields, respectively, and ρ and $\vec{J}$ are the charge and the current density.

(a) Briefly state the physical laws encapsulated by each equation.

Defining the four-vector potential, $A^\mu = (\Phi, \vec{A})$, one can rewrite the $\vec{E}$ and $\vec{B}$ fields as:

$$\vec{E} = -\frac{\partial \vec{A}}{\partial t} - \vec{\nabla}\Phi; \qquad \vec{B} = \vec{\nabla} \times \vec{A}.$$

The second and third Maxwell's equations are automatically satisfied when expressed in terms of A^μ. The first and fourth equations are instead not trivial.

Write down the first and fourth Maxwell's equations in terms of Φ and $\vec{A}$.

(Note: $\vec{\nabla} \times (\vec{\nabla} \times \vec{A}) = -\nabla^2 \vec{A} + \vec{\nabla} \cdot (\vec{\nabla} \cdot A)$). [6]

(b) Performing the gauge transformation:

$$A^\mu \to A^{'\mu} = A^\mu - \partial^\mu \psi$$

where ψ is a scalar function and imposing the Lorentz condition

$$\partial_\mu A^\mu = 0$$

show that the Maxwell's equations simplify to

$$\Box A^\mu = J^\mu$$

with $J^\mu = (\rho, \vec{J})$ and the Box operator is given by $\Box = \partial_\mu \partial^\mu = (\partial^2/\partial t^2 - \nabla^2)$.

[5]

(c) A type of particle is described by certain solutions of the above equation in free space. What are these particles? What is their spin? [4]

(d) Show that the gauge invariance of the Maxwell's equations imposes the photon to be massless or in other terms that a massive photon would violate that symmetry. [5]

B3. On July 2012, the Higgs boson was discovered at the CERN Large Hadron Collider (LHC).

(a) What is the mass, spin and charge of the discovered Higgs particle? Sketch the three main processes in which the Higgs boson was produced at the LHC, by making use of the Feynman pictorial representation (i.e. Feynman diagrams). Name all the particles involved. [7]

(b) The Standard Model of particle physics is a gauge theory with symmetry group SU(3)×SU(2)×U(1). It has twelve associated gauge bosons: the photon, three weak bosons and eight gluons. The symmetry group SU(2)×U(1) is broken to the U(1) group of QED by the Higgs mechanism. During this process, called ElectroWeak Symmetry Breaking (EWSB), the weak gauge bosons acquire mass.

Consider the neutral gauge bosons, $W^{3\mu}$ and B^{μ}, of the SU(2) and U(1) group respectively. Their mass matrix can be written as:

$$(W^{3\mu} \;\; B^{\mu})\frac{v^2}{4}\begin{pmatrix} g_W^2 & g_W g_Y \\ g_W g_Y & g_Y^2 \end{pmatrix}\begin{pmatrix} W^{3\mu} \\ B^{\mu} \end{pmatrix}$$

where v is the vacuum expectation value of the Higgs and g_W and g_Y are the coupling constants associated to the $SU(2)$ and $U(1)$ gauge groups, respectively.

Compute the mass eigenvalues of the neutral ElectroWeak gauge bosons as a function of g_W, g_Y and v. [4]

(c) The expression of the unitary matrix, U, performing the change of basis from the states $(W^{3\mu} \;\; B^{\mu})$ to the physical states $(A^{\mu} \;\; Z^{\mu})$ where the mass matrix is diagonal is given by:

$$U = \begin{pmatrix} g_Y & -g_W \\ g_W & g_Y \end{pmatrix}\frac{1}{\sqrt{g_W^2 + g_Y^2}}$$

Find the expressions of the A^μ and Z^μ eigenstates which are associated to the eigenvalues found in (b). Express such eigenstates as a function of g_W, g_Y and the original states $W^{3\mu}$ and B^μ. Specify which physical bosons are described by these new states. [5]

(d) Represent pictorially the interaction between the Higgs and the fermions. Write down the coupling constant at the interaction vertex when the Higgs gets a vacuum expectation value, v, different from zero. Indicate explicitly the left-handed and/or right-handed nature of the involved fermions. [4]

TURN OVER

B4. At the CERN Large Hadron Collider (LHC) two protons collide head-on, each one having an energy equal to 6.5 TeV: E_p = 6.5 TeV.

(a) What are the elementary constituents of the proton? What are the charges that characterize them? Describe the meaning of Parton Distribution Function (PDF). In addition, by making use of the fraction of longitudinal momentum of the proton taken away by any of its constituents, give the expression of the center-of-mass energy for a generic hard scattering subprocess initiated by any two quarks emitted off the two protons at the LHC. [6]

(b) Consider the specific hard-scattering subprocess

$$u\bar{u} \to d\bar{d}$$

where u and d are the up and $down$-quark, respectively. Draw the four Feynman diagrams describing this process, indicating the forces involved and the force carriers which represent them. [4]

(c) Neglecting the weak force in the Feynman diagrams above, label the four external quarks by making use of the Feynman rules for the corresponding spinors:

$$u_r(p), \bar{u}_r(p), v_r(p), \bar{v}_r(p)$$

with r=1, 2, giving your reasons. Explain the meaning of these symbols. In addition, write down the Feynman rules for the propagators and vertices involved. [6]

(d) The down-quarks produced in the final state evolve when passing through the detector at the LHC. Give a definition of Parton Shower (PS) and sketch diagrammatically its meaning (consider only the strong force). What does one observe in the end? [4]

END OF PAPER

