SEMESTER 2 EXAMINATION 2012/13

PARTICLE PHYSICS

Duration: 120 MINS

This paper contains 9 questions.

Answer **all** questions in **Section A** and **only two** questions in **Section B**.

Section A carries 1/3 of the total marks for the exam paper and you should aim to spend about 40 mins on it.

Section B carries 2/3 of the total marks for the exam paper and you should aim to spend about 80 mins on it.

An outline marking scheme is shown in brackets to the right of each question.

A Sheet of Physical Constants is provided with this examination paper.

Only university approved calculators may be used.

A foreign language translation dictionary (paper version) is permitted provided it contains no notes, additions or annotations.

Section A

A1. The Dirac equation can be written as

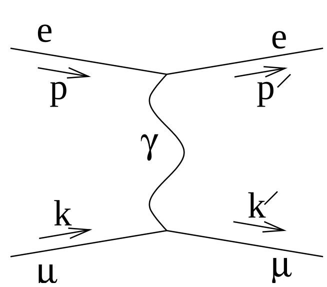
$$(i\gamma^{\mu}\partial_{\mu} - m)\,\psi = 0.$$

Show, by acting on the Dirac Equation with $\gamma^{\nu}\partial_{\nu}$, that for ψ to also satisfy the Klein Gordon equation the γ^{μ}'s must obey the relation

$$\{\gamma^{\mu}, \gamma^{\nu}\} = 2g^{\mu\nu}.$$

[4]

A2. What are the Feynman rules that should be associated with the following Feynman diagram for $e^{-}\ \mu^{-}$ scattering? The arrows show the flow of four-momentum.



[4]

A3. By ensuring that the probability that a particle described by a wave function ψ in space is invariant, show that one can make transformations

$$\psi \to \psi' = U\psi$$

only if U is a unitary operator. [4]

A4. List and describe the four components of a typical collider detector. [4]

A5. State the Goldstone theorem. [4]

TURN OVER

Section B

B1. The Dirac equation may be written as

$$(i\gamma_\mu \partial^\mu - m)\,\psi = 0$$

where ($i = 1, 2, 3$)

$$\gamma_0 = \beta, \qquad \gamma_i = \beta\alpha_i$$

with, in the Dirac representation,

$$\alpha_i = \begin{pmatrix} 0 & \sigma_i \\ \sigma_i & 0 \end{pmatrix}, \qquad \beta = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$$

and

$$\sigma_1 = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}, \qquad \sigma_2 = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix}, \qquad \sigma_3 = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}.$$

(a) Show that $\psi^\dagger\psi$ satisfies an appropriate continuity equation and hence may be interpreted as the probability density. [7]

(b) Show explicitly that

$$(\sigma.\mathbf{p})^2 = |\mathbf{p}|^2.$$

[4]

(c) Hence show that there are plane wave solutions of the form

$$\psi = \begin{pmatrix} \chi(\mathbf{p}) \\ \phi(\mathbf{p}) \end{pmatrix} e^{-i(Et-\mathbf{p}.\mathbf{x})}$$

with both positive and negative energies. [6]

(d) How does the Feynman Stueckelburg interpretation account for the negative energy solutions in nature? [3]

TURN OVER

B2. (a) A major piece of experimental evidence for there being three quark colours is the measurement of the R ratio

$$R = \frac{\sigma(e^+e^- \to \text{hadrons})}{\sigma(e^+e^- \to \mu^+\mu^-)}.$$

Explain why this ratio of cross-sections allows us to measure the number of colours. [6]

(b) Calculate the R ratio above the charm quark mass threshold. [2]

(c) Explain why QCD has an invariance under the action of unitary 3x3 matrices. [4]

(d) How many degrees of freedom do the 3x3 unitary matrices have? [2]

(e) Explain what the algebra

$$3 \otimes \bar{3} = 8 \oplus 1$$

means when related to a bound state of a quark and an anti-quark. [6]

B3. Consider a gauge theory with two $U(1)$ gauge symmetries and a single scalar field ϕ. The scalar field couples to the two gauge bosons with equal but opposite charges. The equations of motion for the two gauge fields are therefore

$$\partial^\nu \partial_\nu A^\mu_{(1)} = iq\phi^* D^\mu \phi - iq(D^\mu \phi)^* \phi$$
$$\partial^\nu \partial_\nu A^\mu_{(2)} = -iq\phi^* D^\mu \phi + iq(D^\mu \phi)^* \phi$$

where the covariant derivative is $D^\mu = \partial^\mu + iqA^\mu_{(1)} - iqA^\mu_{(2)}$. The scalar field in addition has a potential

$$V = -\frac{1}{2}\mu^2|\phi|^2 + \frac{1}{4}\lambda|\phi|^4$$

where μ and λ are positive couplings.

(a) Show that the potential is gauge invariant. [2]

(b) Sketch the scalar potential and find the value v that ϕ takes at the potential minimum. [4]

(c) Show that the gauge bosons acquire mass and that their mass squared matrix may be written as

$$(A^\mu_{(1)}, A^\mu_{(2)}) 2v^2 \begin{pmatrix} q^2 & -q^2 \\ -q^2 & q^2 \end{pmatrix} \begin{pmatrix} A^\mu_{(1)} \\ A^\mu_{(2)} \end{pmatrix}.$$

[6]

(d) Hence show that there is a massless physical gauge field that is an equal superposition of $A^\mu_{(1)}$ and $A^\mu_{(2)}$. [8]

TURN OVER

B4. (a) Where was the Higgs boson discovered? What was the initial state of the particle collider concerned? [2]

(b) By using a Feynman diagram, sketch the production mechanism used to discover such a particle. Identify all particles produced at each stage of the process and assign the correct particle/antiparticle labels. [4]

(c) Given that the Higgs boson was found to have a mass of 125 GeV, list the possible decay channels for such a state and discuss the pros and cons of each of these in terms of their discovery potential. [7]

(d) The Higgs boson data initially displayed anomalies with respect to the Standard Model predictions. In which production and/or decay channels were such anomalies more prominent? If these deviations from the Standard Model are to be attributed to new physics effects, how would the latter enter the physics processes involved? [3]

(e) How was the Higgs boson mass measured from the data when the particle was discovered? what are the best observables to use to extract information about its quantum numbers, i.e., spin, charge and parity? [4]

END OF PAPER

