UNIVERSITY OF SOUTHAMPTON PHYS6012W1

SEMESTER 1 EXAMINATION 2013/14

Coherent Light, Coherent Matter

Duration: 120 MINS

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*Answer **all** questions in **Section A** and **only two** questions in **Section B**.*

Section A carries 1/3 of the total marks for the exam paper and you should aim to spend about 40 mins on it. Section B carries 2/3 of the total marks for the exam paper and you should aim to spend about 80 mins on it.

A Sheet of Physical Constants will be provided with this examination paper.

An outline marking scheme is shown in brackets to the right of each question.

Only university approved calculators may be used.

Number of Pages 6

Section A

A1. Calculate the lowest temperature that can be achieved by the Doppler cooling method using the D_2 line of Sodium (m=23 amu) at 589 nm, which has a radiative lifetime of 16 ns. Calculate also the average velocity and de Broglie wavelength of the atoms at this temperature. [5]

A2. Write down the relationship between the standard deviation Δn of the photon number distribution and the average photon number $\bar{n}$ for light sources described by sub-Poissonian, Poissonian and super-Poissonian statistics. Draw a diagram comparing the three different distributions for a given mean photon number of n = 100. [5]

A3. A quantum dot emitting at 930 nm is placed at the centre of a resonant micropillar cavity containing material of refractive index 3.5 (GaAs). The modal volume is 1.8×10^{-18} m^3 and the spectral width $\Delta\lambda$ of the resonant mode is 0.18 nm. Calculate the Purcell factor. What is the physical significance of this result? [6]

A4. An attenuated light beam from an Ar laser operating at 514 nm (2.41 eV) with a power of 0.1 pW is detected by a photo-counting system of quantum efficiency 10$\%$, with the time interval set at 0.5 s. Calculate the average number of photons counted. [4]

Section B

B1. This question is about the second order correlation function used to quantify the temporal coherence of classical light. Consider a light source with constant average intensity such that $\langle I(t)\rangle = \langle I(t+\tau)\rangle$ and for which the time-scale of the intensity fluctuations is determined by the coherence time τ_c.

(a) Write down the definition of the second order correlation function $g^{(2)}(\tau)$. How is it different from the first order correlation function $g^{(1)}(\tau)$? [2]

(b) Show that in general $g^{(2)}(\tau) = 1$, for $\tau \gg \tau_c$. [Hint: Start with the general expression for the intensity: $I(t) = \langle I\rangle + \Delta I(t)$ and substitute into $g^{(2)}(\tau)$.] [5]

(c) Discuss $g^{(2)}(\tau)$ for the case $\tau = 0$. Discuss perfectly coherent light and chaotic light in this case. [6]

(d) Draw a diagram of the dependence of $g^{(2)}$ on τ for chaotic and for perfectly coherent light. [2]

(e) Evaluate $g^{(2)}(0)$ for a monochromatic light wave with a sinusoidal intensity modulation such that $I(t) = I_0(1 + A\sin(\omega t))$, with $|A| \leq 1$. [5]

B2. This question is about squeezed states of light and classical waves at a 50:50 beam splitter.

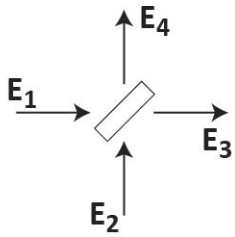

Figure 1: Input and output field on a 50:50 beam splitter.

(a) Explain what amplitude squeezing is and how it is represented by a phasor. Why will light with very strong quadrature squeezing not exhibit amplitude squeezing, no matter how the axes of the uncertainty ellipse are chosen? Use quadrature uncertainty diagrams for a coherent state and strongly quadrature-squeezed light as part of your argument. [6]

(b) Draw and explain why strongly amplitude-squeezed light have an uncertainty area shaped like a banana on a quadrature uncertainty diagram. [2]

(c) Squeezed light can be analysed by homodyne detection using a beam splitter. Consider now classical waves and a 50:50 beam splitter as shown in Fig.1. Let the phase shifts on the transmission and reflection be written ϕ_i^t and ϕ_i^r, respectively, where $i = 1, 2$. Assume that E_1 and E_2 are real. Draw diagrams for the possible phase shifts on transmission and reflection for both input fields and verify that the total output fields must be in the form:

$$E_3 = \frac{1}{\sqrt{2}}[E_1 \exp(i\phi_1^t) + E_2 \exp(i\phi_2^r)],$$
$$E_4 = \frac{1}{\sqrt{2}}[E_1 \exp(i\phi_1^r) + E_2 \exp(i\phi_2^t)].$$

[8]

(d) Consider the same situation as in (c) and, by consideration of conservation of energy, show that:

$$\cos(\phi_2^r - \phi_1^t) + \cos(\phi_1^r - \phi_2^t) = 0.$$

[Hints: The input fields are real. Start by considering the quantity $(E_3 E_3^* + E_4 E_4^*)$. Then use the fact that energy conservation means that the total input power must be equal to the total output power.] [4]

B3. This question is about non-classical light: photon number states and coherent states.

(a) The field quadratures in the photon number presentation can be represented using the photon creation $\hat{a}^\dagger$ and annihilation $\hat{a}$ operators as:

$$\hat{X}_1 = \frac{1}{2}(\hat{a}^\dagger + \hat{a}),$$
$$\hat{X}_2 = \frac{i}{2}(\hat{a}^\dagger - \hat{a}).$$

Evaluate the commutator $[\hat{X}_1, \hat{X}_2]$, and hence find the uncertainty product $(\Delta\hat{X}_1)^2(\Delta\hat{X}_2)^2$, given that $[\hat{a}, \hat{a}^\dagger] = 1$. [6]

(b) Show that the coherent state:

$$|\alpha\rangle = \exp(-|\alpha|^2/2)\sum_{n=0}^{\infty}\frac{\alpha^n}{(n!)^{1/2}}|n\rangle,$$

is correctly normalized, where $|n\rangle$ is the photon number state. Start with evaluating the Dirac bracket $\langle\alpha|\alpha\rangle$ and make use of the orthonormality of number states $\langle n|n'\rangle = \delta_{nn'}$. [Hint: You may need to recall, the Taylor expansion of $e^{+|\alpha|^2} = \sum_{n=0}^{\infty}\frac{(\alpha^*\alpha)^n}{(n!)}$]. [5]

(c) For the coherent state $|\alpha\rangle$ with $\alpha = |\alpha|e^{i\phi}$, show that $\langle\alpha|\hat{X}_1|\alpha\rangle = |\alpha|\cos\phi$, and $\langle\alpha|\hat{X}_2|\alpha\rangle = |\alpha|\sin\phi$. Then show that uncertainty $\Delta X_1 = 1/2$, and draw a phasor diagram to illustrate this result. [Hint: Make use of $\hat{a}|\alpha\rangle = \alpha|\alpha\rangle$ and $\langle\alpha|\hat{a}^\dagger = \langle\alpha|\alpha^*$ and $(\Delta O)^2 = \langle\hat{O}^2\rangle - \langle\hat{O}\rangle^2$.] [6]

(d) Show that a photon number state $|n\rangle$ is an eigenstate of the operator $(\hat{X}_1^2 + \hat{X}_2^2)$ with eigenvalue $(n + 1/2)$, by using equations: $\hat{a}^\dagger|n\rangle = (n+1)^{1/2}|n+1\rangle$ and $\hat{a}|n\rangle = n^{1/2}|n-1\rangle$. [3]

END OF PAPER